MATH1104H Solutions Tutorial 1

a) $\begin{bmatrix} 1 & 3 & -1 & 1 & 1 \\ 1 & 4 & 2 & -2 & 3 \\ 2 & 7 & 1 & -1 & 6 \end{bmatrix} \xrightarrow[R_3=R_3-2R_1]{R_2=R_2-R_1} \begin{bmatrix} 1 & 3 & -1 & 1 & 1 \\ 0 & 1 & 3 & -3 & 2 \\ 0 & 1 & 3 & -3 & 4 \end{bmatrix}$

$\xrightarrow{R_3=R_3-R_2} \begin{bmatrix} 1 & 3 & -1 & 1 & 1 \\ 0 & 1 & 3 & -3 & 2 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$ so no solution.

b) $\begin{bmatrix} 0 & 4 & 7 & 10 & 6 \\ -3 & -7 & -17 & 5 & -11 \\ 1 & 2 & 5 & -3 & 3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 5 & -3 & 3 \\ -3 & -7 & -17 & 5 & -11 \\ 0 & 4 & 7 & 10 & 6 \end{bmatrix}$

$\xrightarrow{R_2=R_2+3R_1} \begin{bmatrix} 1 & 2 & 5 & -3 & 3 \\ 0 & -1 & -2 & -4 & -2 \\ 0 & 4 & 7 & 10 & 6 \end{bmatrix} \xrightarrow{R_3=R_3+4R_2} \begin{bmatrix} 1 & 2 & 5 & -3 & 3 \\ 0 & -1 & -2 & -4 & -2 \\ 0 & 0 & -1 & -6 & -2 \end{bmatrix}$

$$\begin{aligned} x_1 + 2x_2 + 5x_3 - 3x_4 &= 3 \\ -x_2 - 2x_3 - 4x_4 &= -2 \\ -x_3 - 6x_4 &= -2 \end{aligned}$$

Using back-substitution we obtain $x_3 = -6x_4 + 2$, $x_2 = 8x_4 - 2$ and $x_1 = 17x_4 - 3$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_4 \begin{bmatrix} 17 \\ 8 \\ -6 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ -2 \\ 2 \\ 0 \end{bmatrix}$$

c) $\begin{bmatrix} 3 & -3 & 4 & 6 \\ 2 & -1 & 2 & -1 \\ 5 & 2 & -3 & 0 \end{bmatrix} \xrightarrow{R_1=\frac{1}{3}R_1} \begin{bmatrix} 1 & -1 & 4/3 & 2 \\ 2 & -1 & 2 & -1 \\ 5 & 2 & -3 & 0 \end{bmatrix}$

$\xrightarrow[R_3=R_3-5R_1]{R_2=R_2-2R_1} \begin{bmatrix} 1 & -1 & 4/3 & 2 \\ 0 & 1 & -2/3 & -5 \\ 0 & 7 & -29/3 & -10 \end{bmatrix} \xrightarrow{R_3=R_3-7R_2} \begin{bmatrix} 1 & -1 & 4/3 & 2 \\ 0 & 1 & -2/3 & -5 \\ 0 & 0 & -5 & 25 \end{bmatrix}$

$$\begin{aligned} x_1 - x_2 + \tfrac{4}{3}x_3 &= 2 \\ x_2 - \tfrac{2}{3}x_3 &= -5 \\ -5x_3 &= 25 \end{aligned}$$

Using back-substitution we obtain $x_3 = -5$, $x_2 = -\frac{25}{3}$ and $x_1 = \frac{1}{3}$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ -\frac{25}{3} \\ -5 \end{bmatrix}$$

2. $\begin{bmatrix} 0 & 1 & -2 & 1 & 1 \\ 2 & -1 & 0 & -1 & 0 \\ 4 & 1 & -6 & 1 & 3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 & 1 \\ 4 & 1 & -6 & 1 & 3 \end{bmatrix}$

$\xrightarrow{R_3 = R_3 - 2R_1} \begin{bmatrix} 2 & -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 3 & -6 & 3 & 3 \end{bmatrix} \xrightarrow{R_3 = R_3 - 3R_2} \begin{bmatrix} 2 & -1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\xrightarrow{R_1 = R_1 + R_2} \begin{bmatrix} 2 & 0 & -2 & 0 & 1 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 = \frac{1}{2} R_1} \begin{bmatrix} 1 & 0 & -1 & 0 & 1/2 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

This gives $\begin{aligned} x_1 - x_3 &= \tfrac{1}{2} \\ x_2 - 2x_3 + x_4 &= 1 \end{aligned}$ which we turn into $\begin{aligned} x_1 &= x_3 + \tfrac{1}{2} \\ x_2 &= 2x_3 - x_4 + 1 \end{aligned}$

So we obtain the following formula that gives all solutions

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

3. $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 2 & k & k \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_2} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & k-2 & k-4 \end{bmatrix}$

For the system to be inconsistant, we need the last row to be 0 0 0 a, where $a \neq 0$.

This means that we need $k - 2 = 0$, that is $k = 2$.