.MATH 1300 A MIDTERM # 1 Fall-2016.

Professors: Termeh Kousha

Last Name: _____ **First Name:** _____

ID# Marking Scheme and Solutions

Instructions: This midterm exam consists of 4 multiple choice questions and 2 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 50 points.
Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. You may use the backs of pages.

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For long answer questions, YOU MUST SHOW YOUR WORK.

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Multiple Choice Answers:

#1	#2	#3	#4
B	D	E	E

Multiple Choice Questions (1-4)

Question 1 *Solve the following logarithmic equation.*

$$\log_3(x+2) - \log_3(x) = 2$$

A) $\frac{e+1}{e}$ B) $\frac{1}{4}$ C) $\frac{4}{9}$ D) $\frac{1}{9}$ E) $\frac{e}{e+9}$ F) 4

$\log_3 \frac{x+2}{x} = 2$ $\quad \frac{x+2}{x} = 9$ $\quad x+2 = 9x$

$8x = 2$ $\quad x = \frac{1}{4}$

Question 2 *Find a and b so that $f(x)$ is continuous everywhere.*

$$f(x) = \begin{cases} -bx + a & x < -1 \\ 2x + 6 & x = -1 \\ ax + 2 & x > -1 \end{cases}$$

A) $a = 1$ and $b = 4$

B) $a = 1$ and $b = -1$

C) $a = b = 4$

D) $a = -2$ and $b = 6$

E) $a = -2$ and $b = 2$

F) $a = 4$ and $b = 2$

$\lim_{x \to -1^-} f(x) = f(-1) = \lim_{x \to -1^+} f(x)$

@ $x = -1$ $\quad -b(-1) + a = 2(-1) + 6 = a(-1) + 2$

$b + a = 4 = -a + 2$

$a = -2$

$b = 6$

Question 3 *Find the slope of the tangent line to the graph* $f(x) = (4-2x)\ln(2x^2+x-1)$ *when* $x = 1$.

A) $5 + \ln(2)$

B) $\frac{5}{3} + \ln(4)$

C) $2 + \ln(2)$

D) $\frac{2}{3} - 2\ln(2)$

E) $5 - \ln(4)$

F) $\frac{5}{2} - \ln(4)$

$$f'(x) = -2\ln(2x^2+x-1) + (4-2x)\frac{4x+1}{2x^2+x-1}$$

$$f'(1) = -2\ln(2) + 2\frac{5}{2}$$

$$= 5 - \ln 4$$

Question 4 *Find the following limit.*

$$\lim_{x\to 3^-} \frac{x(x-3)}{|x-3|}.$$

(Note: This is a one-sided limit)

A) 1 **B)** -1 **C)** 0 **D)** 3 **E)** -3 **F)** The limit does NOT exist.

$$\lim_{x\to 3^-} \frac{x(x-3)}{|x-3|} = \lim_{x\to 3^-} \frac{x(x-3)}{-(x-3)} = \lim_{x\to 3^-} -x = -3$$

Long Answer Questions (5-6)

Question 5 (20 points)

zero point if they don't use the limit (for direct answer). -2 points for forgetting $\lim_{h\to 0}$

A) (7 points) *Using* **only** *the definition of derivative as a limit, calculate* $f'(x)$ *where*

$$f(x) = \sqrt{x-3}.$$

2 points $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h-3}-\sqrt{x-3}}{h}$

2 points $= \lim_{h\to 0} \frac{\sqrt{x+h-3}-\sqrt{x-3}}{h} \cdot \frac{\sqrt{x+h-3}+\sqrt{x-3}}{\sqrt{x+h-3}+\sqrt{x-3}} = \lim_{h\to 0} \frac{(x+h-3)-(x-3)}{h(\sqrt{x+h-3}+\sqrt{x-3})}$ $\quad -x+3$

2 points $= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h-3}+\sqrt{x-3})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h-3}+\sqrt{x-3}} = \underbrace{\frac{1}{2\sqrt{x-3}}}$

1 point final Answer

$\lim =$

(-1) (-1)

B) (7 points) *For which values* x*, the graph of the following function has horizontal tangent lines? (i.e.* $f'(x) = 0$*)*

$$f(x) = (3x-2)^2 e^{2x-4}$$

3 points $f'(x) = 2(3x-2)(3)e^{2x-4} + (3x-2)^2 e^{2x-4} \cdot 2$

2 points $= (3x-2)2e^{2x-4}\big(3 + (3x-2)\big) = 2(3x-2)e^{2x-4}(3x+1)$

$e^{2x-4} \neq 0$ $\qquad = 0$

2 points. $\begin{cases} 3x-2=0 & 3x=2 & x=2/3 \\ 3x+1=0 & 3x=-1 & x=-1/3 \end{cases}$

C) (6 points) *Suppose* $4,000$ *dollars is invested at a rate of* 6 *percent. Find the time needed for an initial deposit to double?*

$P_0 = 4000$

$r = 6\%$

2 points for the formula $\quad P(t) = P_0 e^{rt}$

$2P_0 = P_0 e^{0.06t}$

$2 = e^{0.06t}$

2 points for taking the ln $\quad \ln 2 = 0.06t$

$t = \underbrace{\frac{\ln 2}{0.06}}$ or $t = \frac{\ln 2 \cdot 100}{6}$

2 points.

Question 6 (10 points) *Suppose that x and y are related by the equation*

$$y^3 + (x-1)y = 28 - e^{x-1}.$$

6 points A) *Use implicit differentiation to find* $\frac{dy}{dx}$.

$$3y^2 y' + y + (x-1)y' = -e^{x-1}$$

← 4 points for correct derivative

$$y'(3y^2 + (x-1)) = -e^{x-1} - y$$

$$y' = \frac{-e^{x-1} - y}{3y^2 + (x-1)}$$

2 points for isolating y

4 points B) *Find the equation of tangent line at the point* $(1,3)$.

$$y'\Big|_{(1,3)} = \frac{-e^{1-1} - 3}{3(9) + 1 - 1} = \frac{-1-3}{27} = \frac{-4}{27}$$

(2 points for slope)

If the answer in part (a) is wrong, but based on that part (b) is correct, They will get full points for part (b).

2 points for the equation

$$y = mx + b$$
$$3 = -\frac{4}{27}(1) + b$$
$$b = \frac{81}{27} + \frac{4}{27} = \frac{85}{27}$$

$$y = -\frac{4}{27}x + \frac{85}{27}$$

Space for additional work

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Multiple Choice Answers:

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D	D	E	E

Multiple Choice Questions (1-4)

Question 1 *Solve the following logarithmic equation.*

$$\log_4(x+1) - \log_4(x) = 2$$

A) $\frac{e+1}{e}$ B) $\frac{2}{3}$ C) $\frac{15}{16}$ (D) $\frac{1}{15}$ E) $\frac{e}{e+16}$ F) $\frac{16}{5}$

$\log_4 \frac{x+1}{x} = 2$ $\quad \frac{x+1}{x} = 16$ $\quad x+1 = 16x$

$15x = 1$

$x = \frac{1}{15}$

Question 2 *Find a and b so that $f(x)$ is continuous everywhere.*

$$f(x) = \begin{cases} ax+2 & x < -1 \\ 5+x & x = -1 \\ -bx+a & x > -1 \end{cases}$$

A) $a = 1$ and $b = 4$

B) $a = 1$ and $b = -1$

C) $a = b = 4$

(D) $a = -2$ and $b = 6$

E) $a = -2$ and $b = 2$

F) $a = 4$ and $b = 2$

$\lim_{x \to -1^-} f = \lim_{x \to -1^+} f = f(-1)$

$a(-1)+2 = -b(-1)+a = 5-1$

$-a+2 = b+a = 4$

$a = -2 \quad b = 6$

Question 3 *Find the slope of the tangent line to the graph* $f(x) = (-2x+4)\ln(x^2+2x-1)$ *when* $x = 1$.

A) $1 + \ln(2)$

B) $\frac{8}{3} + \ln(4)$

C) $2 + \ln(2)$

D) $\frac{2}{3} - 2\ln(2)$

E) $4 - \ln(4)$

F) $\frac{5}{3} + \ln(4)$

$$f'(x) = -2\ln(x^2+2x-1) + (-2x+4)\frac{2x+2}{x^2+2x-1}$$

$$f'(1) = -2\ln(2) + 2\frac{4}{2}$$

$$= 4 - \ln 4$$

Question 4 *Find the following limit.*

$$\lim_{x\to 4^-} \frac{x(x-4)}{|x-4|}.$$

(Note: This is a one-sided limit)

A) 1 **B)** -1 **C)** 0 **D)** 4 **E)** -4 **F)** The limit does NOT exist.

$$\lim_{x\to 4^-} \frac{x(x-4)}{|x-4|} = \lim_{x\to 4^-} \frac{x(x-4)}{-(x-4)} = \lim_{x\to 4^-} -x = -4$$

Long Answer Questions (5-6)

Question 5 (20 points)

A) (7 points) *Using* **only** *the definition of derivative as a limit, calculate* $f'(x)$ *where*

$$f(x) = \sqrt{x-2}.$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h-2}-\sqrt{x-2}}{h} = \lim_{h\to 0} \frac{\sqrt{x+h-2}-\sqrt{x-2}}{h} \cdot \frac{\sqrt{x+h-2}+\sqrt{x-2}}{\sqrt{x+h-2}+\sqrt{x-2}}$$

$$= \lim_{h\to 0} \frac{(x+h-2)-(x-2)}{h(\sqrt{x+h-2}+\sqrt{x-2})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h-2}+\sqrt{x-2})} = \frac{1}{2\sqrt{x-2}}.$$

B) (7 points) *For which values* x*, the graph of the following function has horizontal tangent lines? (i.e.* $f'(x) = 0$*)*

$$f(x) = (-3x+1)^2 e^{2x+3}$$

$$f'(x) = 2(-3x+1)(-3)e^{2x+3} + (-3x+1)^2 2e^{2x+3}$$

$$= 2e^{2x+3}(-3x+1)\underbrace{(-3+(-3x+1))}_{-3-3x+1} = 0$$

$-3x+1=0$, $3x=1$, $x=1/3$ $\qquad$ $-2-3x=0$, $3x=-2$, $x=-2/3$

C) (6 points) *Suppose* $5,000$ *dollars is invested at a rate of* 4 *percent. Find the time needed for an initial deposit to double?*

$P_0 = 5000$

$r = 0.04$

$2P_0 = P_0 e^{rt}$

$2 = e^{0.04t}$

$\ln 2 = 0.04t$

$t = \frac{\ln 2}{0.04}$ or $t = 25\ln 2$

Question 6 (10 points) *Suppose that x and y are related by the equation*

$$e^{x-1} + (x-1)y = 28 - y^3.$$

A) *Use implicit differentiation to find* $\frac{dy}{dx}$.

$$e^{x-1} + y + (x-1)y' = -3y^2y'$$

$$(x-1)y' + 3y^2y' = -e^{x-1} - y$$

$$y'(3y^2 + x - 1) = e^{-x-1} - y$$

$$y' = \frac{-e^{x-1} - y}{3y^2 + x - 1}$$

B) *Find the equation of tangent line at the point* $(1, 3)$.

$$y'\Big|_{(1,3)} = \frac{-e^{1-1} - 3}{3(9) + 1 - 1} = \frac{-4}{27} =$$

$$y = mx + b$$

$$3 = \frac{-4}{27}x + b \qquad b = +\frac{4}{27} + \frac{81}{27} = \frac{85}{27}$$

$$y = \frac{-4}{27}x + \frac{85}{27}$$

Space for additional work