CARLETON UNIVERSITY

FINAL EXAMINATION
April 2014

DURATION: 3 HOURS **SCANTRON FORMS REQUIRED**

Department Name and Course Number: School of Mathematics and Statistics, MATH 1005 A, B, C, D, E, F
Course Instructor(s): Dr. S. Melkonian (Sec. A), Dr. B. Fodden (Sec. B and E), Dr. G. Li (Sec. C), Mr. M. Blenkinsop (Sect. D), Dr. M. Rafsanjani-Sadeghi (Sec. F).

AUTHORIZED MEMORANDA
NON-PROGRAMMABLE, NON-GRAPHIC CALCULATORS

This examination consists of 20 multiple-choice questions, worth 5 marks each.

1. If y is the solution of the initial-value problem $\dfrac{dy}{dx} = -3x^2y^2$, $y(0) = 1$, then $y(1) =$

   (a) 2 (b) 0 (c) $\dfrac{1}{2}$ (d) Undefined (e) None of these

2. If y is the solution of the initial-value problem $xy' + 2y = 3x$, $y(2) = 3$, then $y(1) =$

   (a) 0 (b) 1 (c) 10 (d) 5 (e) None of these

3. The general solution of the differential equation $x^2\dfrac{dy}{dx} = y^2 + xy$ is $y =$

   (a) $\dfrac{-1}{\ln|x| + C}$ (b) $\dfrac{1}{\ln|x| + C}$ (c) $\dfrac{x}{\ln|x| + C}$ (d) $\dfrac{-x}{\ln|x| + C}$ (e) None of these

4. The general solution of the exact equation $e^y + \cos(x) + (xe^y + 2y)\dfrac{dy}{dx} = 0$ is

   (a) $xe^y - \sin(x) = C$ (b) $xe^y - \sin(x) + y^2 = C$ (c) $xe^y + \sin(x) = C$
   (d) $xe^y + \sin(x) + y^2 = C$ (e) None of these

5. The integrating factor which makes the equation $1 + x\cos(y) + \sin(y)\dfrac{dy}{dx} = 0$ exact is

   (a) $I(x) = -x$ (b) $I(x) = e^{-\frac{x^2}{2}}$ (c) $I(x) = e^{\frac{x^2}{2}}$ (d) $I(y) = e^{\frac{y^2}{2}}$ (e) None of these

6. The solution of the initial-value problem $y'' + y' - 6y = 0$, $y(0) = 0$, $y'(0) = 10$, is

   (a) $2(e^{2x} - e^{-3x})$ (b) $-10(e^{2x} - e^{-3x})$ (c) $-2(e^{-2x} - e^{3x})$ (d) $-10(e^{-2x} - e^{3x})$
   (e) None of these

7. The general solution of the differential equation $x^2y'' - 3xy' + 4y = 0$ is

   (a) $e^{\frac{3}{2}x}\left[c_1\cos\left(\frac{\sqrt{7}}{2}x\right) + c_2\sin\left(\frac{\sqrt{7}}{2}x\right)\right]$ (b) $|x|^{\frac{3}{2}}\left[c_1\cos\left(\frac{\sqrt{7}}{2}\ln|x|\right) + c_2\sin\left(\frac{\sqrt{7}}{2}\ln|x|\right)\right]$
   (c) $c_1x^2 + c_2x^2$ (d) $x^2(c_1 + c_2\ln|x|)$ (e) None of these

8. A particular solution y_p of the equation $y'' + y = \cos(x)$ has the form

   (a) $A\cos(x)$ (b) $A\cos(x) + B\sin(x)$ (c) $Ax\cos(x)$ (d) $x[A\cos(x) + B\sin(x)]$
   (e) None of these

9. The general solution of the differential equation $y'' - 3y' - 4y = 6e^{2x}$ is $y =$

   (a) $c_1e^{-x} + c_2e^{4x} + e^{2x}$ (b) $c_1e^{-x} + c_2e^{4x} - e^{2x}$ (c) $c_1e^{-x} + c_2e^{4x} - xe^{2x}$
   (d) $c_1e^{-x} + c_2e^{4x} - x^2e^{2x}$ (e) None of these

10. Two linearly independent solutions of the system $\mathbf{x}' = A\mathbf{x}$, $A = \begin{pmatrix} 1 & 2 \\ -3 & -4 \end{pmatrix}$, are

    (a) $e^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $e^{-2t}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ (b) $e^{-2t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $e^{-t}\begin{pmatrix} -2 \\ 3 \end{pmatrix}$
    (c) $e^{t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $e^{2t}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ (d) $e^{2t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $e^{t}\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ (e) None of these

11. The sum of the series $\sum_{n=1}^{\infty} 2^{n+1}3^{2-n}$ is

    (a) $\frac{4}{3}$ (b) 2 (c) 54 (d) 36 (e) None of these

12. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)}$

    (a) Converges absolutely (b) Converges conditionally (c) Diverges
    (d) Diverges absolutely (e) None of these

13. Which of the following three series converge(s)?

    $$\text{(i)} \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}} \qquad \text{(ii)} \sum_{n=2}^{\infty} \frac{1}{n^2-1} \qquad \text{(iii)} \sum_{n=1}^{\infty} \frac{1}{n+1}$$

    (a) (i) and (iii) (b) (ii) and (iii) (c) (i) only (d) (ii) only (e) (iii) only

14. Which of the following three series converge(s) absolutely?

    $$\text{(i)} \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1} \qquad \text{(ii)} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \qquad \text{(iii)} \sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$$

(a) (i) and (ii) (b) (i) and (iii) (c) (ii) and (iii) (d) (i) only (e) (ii) only

15. The radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{3^n (x+1)^n}{n+1}$ is $R =$

(a) 3 (b) 1 (c) $\frac{1}{3}$ (d) ∞ (e) None of these

16. The interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n+1}}$ is $I =$

(a) $[1,3)$ (b) $(1,3]$ (c) $(1,3)$ (d) $[1,3]$ (e) None of these

17. The coefficient of $(x-2)^3$ in the Taylor series of $f(x) = \ln(x)$ about (centred at) 2 is

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{24}$ (d) $\frac{1}{12}$ (e) None of these

18. The coefficient of x^3 in the Maclaurin series of $f(x) = (1+x)^{-3}$ is

(a) -10 (b) 10 (c) -20 (d) 20 (e) None of these

19. Let $f(x) = x + 1$ for $0 \le x \le 1$. The Fourier sine series of f on $[0,1]$ is $\displaystyle\sum_{n=1}^{\infty} b_n \sin(n\pi x)$, where $b_n =$

(a) $\frac{2}{n\pi}\Big[1 - 2(-1)^n\Big]$ (b) $\frac{1}{n\pi}\Big[1 - 2(-1)^n\Big]$ (c) $\frac{-2(-1)^n}{n\pi}$ (d) $\frac{(-1)^{n-1}}{n\pi}$
(e) None of these

20. Let $f(x) = \left\{ \begin{array}{ll} x, & 0 \le x < 1 \\ 3, & 1 \le x \le 2 \end{array} \right\}$ for $0 \le x \le 2$. At $x = 7$, the Fourier sine series of f on $[0,2]$ converges to

(a) 2 (b) -2 (c) 1 (d) 3 (e) None of these