RYERSON UNIVERSITY

DEPARTMENT OF MATHEMATICS

Term Test - MTH 207 - Fall 2016

Last Name (Print):_____ First Name:_____. Student Number: _____

Signature:_____. Section Number:_____.

Date: Nov 4th, 2016 10:45 - 12:45

Instructions:

1. Enter your section number NOW.
2. Have your student card available on your desk.
3. calculators and other aids are not permitted. Verify that your test booklet has 8 pages.
4. DO NOT ASK FOR ANY INTERPRETATION OF ANY QUESTIONS ON THIS TEST.
5. This test consists of 2 parts.
   (a) **Part A** has 20 multiple-choice questions. There are no part marks in the multiple-choice section. Each correct response receives 1.5 points. An incorrect response or no response receives 0 point. Write down your responses for each of these questions on page 5. Only the responses entered there will be graded.
   (b) **Part B** has 3 full-solution questions.
      i. Unless otherwise instructed, make sure you include all significant steps in your solutions to questions in Part B, presented in the correct order. Unjustified answers will be given little or no credit. Cross out or erase all rough work not relevant to your solution.
      ii. Present your solutions neatly and legibly in the space provided. If you need more space, use the back of the page. Indicate this fact on the original page, making sure that your solution cannot be confused with any rough work which may be there.

For Instructor's use only.

Question(s)	Score
1-20	
21	
22	
23	
Total	

Part A – Multiple Choice Questions.

There is only one correct answer for each question. **Write down your answers on Page 5.**

Question 1 Let $f(x) = \ln(|5x - 1| - 4)$. Then the domain of f is

(a) $(1, \infty)$ (b) $(-\infty, -3/5)$ (c) $(-3/5, 1)$ (d) $(-\infty, 1)$ (e) none of these

Question 2

$$\log_{\sqrt{\pi}} 125 + \log_{\sqrt{\pi}} \frac{1}{25} + \log_{\sqrt{\pi}} \frac{\pi}{5} =$$

(a) 0 (b) 1 (c) 2 (d) 4 (e) none of these

Question 3 Let $f(x) = \frac{x}{x^2+1}$. Then the range of f is

(a)$\mathbb{R}$ (b) $(-\infty, 1/2)$ (c) $(-\infty, -1/2) \cup (1/2, \infty)$ (d) $(-1/2, 1/2)$ (e) none of these

Question 4 If

$$\log_4(x + 14) - \log_4 x = 1/2,$$

then x equals

(a) 14 (b) 14/15 (c) $-14/3$ (d) $\sqrt{14}$ (e) none of these

Question 5 Let $f(x) = \ln(2e^x + 3)$. Then $f^{-1}(x)$ equals

(a) $2/\ln(e^x - 3)$ (b) $\ln(e^x - 3)$ (c) $\ln\left(\frac{1}{2}(e^x - 3)\right)$ (d) $\frac{1}{2}\ln(e^x - 3)$ (e) none of these

Question 6 Let $f(x) = 2^x + 3^x + 5^x - 10$. Then $f^{-1}(0)$ equals

(a) $1/f(0)$ (b) -1 (c) 0 (d) $-1/f(0)$ (e) none of these

Question 7 If $\sin x = 1/3$, then $\tan^2 x$ equals

(a) 1/2 (b) 1/8 (c) 8 (d) 3/8 (e) none of these

Question 8 Suppose that $\lim_{x\to 1} f(x) = 3$ and $\lim_{x\to 1} g(x) = -4$. If $a \in \mathbb{R}$ and

$$\lim_{x\to 1}(af(x) + 2ag(x)) = \lim_{x\to 1}(f(x) - 3ag(x)),$$

then a equals

(a) 3/23 (b) $-3/17$ (c) 3/7 (d) $-3/19$ (e) none of these

Question 9 Suppose that f is continuous on $\mathbb{R}$ and $f(-2) = 7, f(5) = -2, f(1) = 4$ and $f(0) = -2016$. Then the number of distinct x such that $f(x) = 0$ is

(a) exactly 1 (b) exactly 2 (c) exactly 3 (d) 4 or more (e) none of these

Question 10 If $g(x) = \frac{2}{x+3}$, then

$$\lim_{x\to -1} \frac{g(x) - 1}{x + 1} =$$

(a) $-1/2$ (b) -1 (c) 1 (d) ∞ (e) none of these

Question 11

$$\lim_{x\to 2^-}\left(\frac{1}{x-2}-\frac{1}{x^2-2x}\right)=$$

(a) $-\infty$ (b) ∞ (c) 0 (d) -1 (e) none of these

Question 12 For any real number r, it can be shown that

$$\lim_{x\to\infty}\frac{x^r}{e^x}=0.$$

Hence,

$$\lim_{x\to\infty}\frac{e^x+x^{100}}{e^{2x}+x^2}=$$

(a) 0 (b) ∞ (c) 1/2 (d) 98 (e) none of these

Question 13 If $\sin^4 x-\cos^4 x=1/2$, then $\sin^2 x$ equals

(a) $\sqrt{3}/2$ (b) $\sqrt{3}/4$ (c) 3/2 (d) 3/4 (e) none of these

Question 14

$$\lim_{t\to 0}\frac{\tan 5t}{\sin(3t)}=$$

(a) 1 (b) 0 (c) 3/5 (d) 1/15 (e) none of these

Question 15 Suppose that f and g are functions with the same domain and the range of f is $(1,\infty)$. If

$$g(x)=-2f(x)+3,$$

then the range of g is

(a) $(1,\infty)$ (b) $(-\infty,-1)$ (c) $(3,-1)$ (d) $(-\infty,1)$ (e) none of these

Question 16

$$\lim_{x\to\infty}x\left(\sqrt{1+\frac{1}{x}}-1\right)=$$

(a) 0 (b) 1 (c) 1/2 (d) ∞ (e) none of these

Question 17 Let

$$f(x)=\left(\frac{1}{2}\right)^{x^2-1}.$$

Then the range of f is

(a) $[1/2,\infty)$ (b) $(0,\infty)$ (c) $[2,\infty)$ (d) $(0,2]$ (e) none of these

Question 18

$$\lim_{t\to 16} \frac{\sqrt[4]{t}-2}{t-16} =$$

(a) 1 (b) ∞ (c) 1/32 (d) 0 (e) none of these

Question 19 Suppose that $\lim_{x\to 0} \frac{f(x)}{x} = -3/4$. Then

$$\lim_{x\to 1} \frac{f(x^2+2x-3)}{x^2+x-2} =$$

(a) -1 (b) 1 (c) $-9/8$ (d) $-3/4$ (e) none of these

Question 20 If b, c are constants, then it can be shown that

$$\lim_{x\to\infty} \left(\sqrt{x^2+bx+c}-x\right) = \frac{b}{2}.$$

Using this fact,

$$\lim_{x\to\infty} \left(\sqrt{x^2+4x+1}-\sqrt{x^2+18x-5}+\sqrt{x^2-8x-17}-\sqrt{x^2+22x+17}\right) =$$

(a) 17 (b) 0 (c) -22 (d) 4 (e) none of these

Enter your answers (A, B, C, D, or E) for questions 1-20 below. Please use non-erasable INK only.

1.

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Part B - Full solution questions

Question 21 (8 Marks) Let f be defined as follows:

$$f(x) = \begin{cases} x + 2 & \text{if } x < 2 \\ ax^2 - bx + 1 & \text{if } 2 \leq x < 3 \\ 3x - a + b & \text{if } x \geq 3 \end{cases}$$

Determine the values of the constants a and b that make f continuous at each point in $\mathbb{R}$.

Question 22 (6 marks) Suppose that f is a function such that

$$\lim_{x\to\infty}[f(x)+2x^2]=1.$$

Determine, if possible,

$$\lim_{x\to\infty}\frac{f(x)}{x^2+1}.$$

Justify your answer

Question 23 (6 marks) Let f be continuous on $[0,1]$ and suppose that $0 \leq f(x) \leq 2$ for all $x \in [0,1]$. Show that there is some $t \in [0,1]$ such that $f(t) = 3t$.