

Midterm Test, MATH 265 Lec. B, Winter 2017

All problems have equal value. Each is worth 6 marks. Show all your steps. Only approved calculators are allowed.

Problem 1: Use double integral in polar coordinates to find the area of the region bounded by the circles $x^2 + y^2 = 16$ and $x^2 + (y - 2)^2 = 4$ and the line $y=0$ in the positive quadrant.

Problem 2: Sketch the region of integration D in the positive quadrant bounded by the curves: $x = y^4$, $x = 2 - y^4$, and evaluate the integral:

$$\iint_D \sqrt{y - \frac{1}{5} y^5} dA .$$

Problem 3: Find the mass of the solid obtained from the unit ball centered at the origin by removing the cone $z = -\sqrt{x^2 + y^2}$, if the density at a point is equal to the distance from the origin.

Problem 4: Find the volume of the solid bounded by the cylinder $x^2 + y^2 + 8x = 0$, by the paraboloid $z = x^2 + y^2$ and the plane $z = 0$.

Problem 5: Evaluate the integral

$$\iint_D (x + y) \sin(x^2 - y^2) \cos(x^2 - y^2) dA ,$$

where D is bounded by the curves:

$$x + y = 0, \quad x + y = \pi, \quad x - y = 0, \quad x - y = 2.$$

Hint: if you eventually obtain a difficult iterated integral try to change the order of integration.

GOOD LUCK !!!