MATH 1102. Test 1.

Instruction: Explain your solutions in detail.
Each problem 1-4 is worth 10 points. The test will be marked out of 40.
This question sheet is *two-sided*.

1. Find the reduced row-echelon form of the following matrix and write down the general parametric solution for corresponding the system of linear equations:

$$\left(\begin{array}{ccccc|c} 0 & 1 & 2 & 1 & 2 & 1 \\ 1 & 3 & 3 & 3 & 3 & 3 \\ 2 & 5 & 4 & 4 & 5 & 5 \end{array}\right).$$

2. (a) Write down the table of inverses in $\mathbb{Z}_{11}$ (no need to show your work or justify your results here).

(b) Solve the following system of linear equations over the field $\mathbb{Z}_{11}$ (you are not allowed to use fractions in your answer):

$$\begin{cases} 3x_1 + 2x_2 + 3x_3 = 2 \\ 7x_1 + 5x_2 + 7x_3 = 3 \\ 4x_1 + 3x_2 + 9x_3 = 4 \end{cases}$$

3. (i) Use Euclidean algorithm to find GCD(89,32).

(ii) Find a pair of integers u, v such that

$$89u + 32v = 1.$$

(iii) Find 32^{-1} in $\mathbb{Z}_{89}$.

4. Let F be a field. Fix an element h of this field with the property that $h \neq g^2$ for all $g \in F$. Let T be a variable, and consider the set S of expressions

$$S = \{a + bT\}, \qquad \text{where} \;\; a, b \in F.$$

Define addition in S by

$$(a + bT) + (c + dT) = (a + c) + (b + d)T, \quad a, b, c, d \in F.$$

Define multiplication in S by imposing a relation $T^2 = h$:

$$(a + bT)(c + dT) = (ac + bdh) + (ad + bc)T.$$

Show that the distributive law $x(y + z) = xy + xz$ holds in S.

Bonus problem. (5 points) In the setting of Problem 4, show that every non-zero element of S has a multiplicative inverse in S.