2nd Midterm Review Sheet, Dr. E. Fink

Tests for convergence:

(a) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$,

(b) $\sum_{n=2}^{\infty} \frac{1}{n(ln(n))^2}$,

(c) $\sum_{k=1}^{\infty} \frac{1}{k^2+k^3}$,

(d) $\sum_{i=1}^{\infty} \frac{i}{\sqrt{i^5+1}}$,

(e) $\sum_{s=1}^{\infty} \frac{s^2-5s}{s^3+s+1}$,

(f) $\sum_{i=1}^{\infty} \frac{ln(i)}{i}$,

(g) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n!}$,

(h) $\sum_{n=1}^{\infty} (-1)^{n+3} \frac{n^2}{n^3+4}$,

(i) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+3}$,

(j) $\sum_{n=1}^{\infty} \frac{n!}{100^n}$

(k) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$

(l) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+4}$

Do the following series **converge absolutely**?

(a) $\sum_{n=1}^{\infty} (-1)^{n+3} \frac{n^2}{n^3+4}$

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^3+4}$

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{\cos^2 n}{n^{7/8}}$

(d) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+2}$

(e) $\sum_{n=0}^{\infty} (-1)^{n-1} \frac{\sqrt{n+2}}{\sqrt{n^2+7}}$

Find radius and Interval of Convergence:

1. $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$
2. $\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 3^n}$
3. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2+1}$
4. $\sum_{n=1}^{\infty} \frac{n \cdot (x+1)^n}{4^n}$

Taylor and MacLaurin series

Give the MacLaurin series and their radii of convergence for the following functions. Note that often it is easier to quote the popular series or substitute into one that you know. You might also use the method of integrating or deriving a series that you already know as shown in class.

1. $\frac{1}{1-x}$
2. e^x
3. $\sin x$
4. $\cos x$
5. $\arctan x$
   Recall that $\int \frac{dx}{1+x^2} = \arctan(x) + C$.
6. $\ln(1+x)$
7. $(1+x)^k$
8. $\cos(3x^2)$
9. $\ln(x^3)$
10. $e^{4/(x+3)}$
11. e^{-2x^2}
12. $x \cdot \cos(x)$
13. $\cos(x^2)$
14. $x \cdot \cos(\frac{1}{2}x^2)$

Compute the Taylor Series of the following functions at the given point $x = a$:

1. e^x at $x = 1$
2. $\ln(x + 1)$ at $x = 3$
3. $\cos(x)$ at $x = \pi/2$
4. $f(x) = x^6 + 7x^4 + 3x^2 + 1$ at $x = 1$.

Do you remember these facts?

1. What is the series $\sum_{n=1}^{\infty} \frac{1}{n}$ called? Does it converge?
2. What is the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ called? Does it converge?
3. What is the series $\sum_{n=0}^{\infty} q^n$ called? Does it converge? If so, what is its sum?
4. What is absolute convergence? Give an example of a series that converges but does not converge absolutely.
5. If a series converges absolutely, does it converge?
6. If a series converges, does it converge absolutely?
7. Give the formula for the Taylor Series of a function $f(x)$ centered at $x = a$.
8. Give the formula for the MacLaurin Series of a function $f(x)$.

Good luck with studying!