1) Evaluate the limit if it exists. **[2 marks]**

$a)\ \lim_{x\to\infty} \frac{-4x^3 - 3x + 1}{2x^3 - 3x^2} = \lim_{x\to\infty} \frac{-\frac{4x^3}{x^3} - \frac{3x}{x^3} + \frac{1}{x^3}}{2\frac{x^3}{x^3} - \frac{3x^2}{x^3}} = -\frac{4}{2} = -2$

b) $a)\ \lim_{x\to 5} \frac{x^2-25}{x-5} = \lim_{x\to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}} = 10$

2) Find the derivatives of the given functions.

a. $y = (3x^6 - 3x)^5(4x^5 - 9)^4$ **[4 marks]**

$y' = 5(3x^6-3x)^4(18x^5-3)(4x^5-9)^4 +$
$4(3x^6-3x)^5(4x^5-9)^3(20x^4)$

b. $y = 6\sqrt[3]{x^9 - 5x^4} = 6(x^9-5x^4)^{\frac{1}{3}}$ **[4 marks]**

$y' = 2(x^9-5x^4)^{-2/3}(9x^8-20x^3)$

$= \frac{2(9x^8-20x^3)}{\sqrt[3]{(x^9-5x^4)^2}}$

c. Find $\frac{dy}{dx}$ if $y = \cos(5x + 2)$ **[4 marks]**

$$\frac{dy}{dx} = -\sin(5x+2)(5)$$
$$= -5\sin(5x+2)$$

d. $y = \sqrt{\tan(6x^3)}$ $= \tan(6x^3)^{\frac{1}{2}}$ **[4 marks]**

$$y' = \frac{1}{2}\tan(6x^3)^{-\frac{1}{2}} \cdot \sec^2(6x^3)(18x^2)$$
$$= \frac{9x^2\sec^2(6x^3)}{\sqrt{\tan(6x^3)}}$$

3) Find the equation of the tangent to the given curve, if x = 2. **[3 marks]**

$y = 3x^2 + 4x - 11$

$y' = 6x + 4\ ;\ m_t = 6(2) + 4 = 16$

$y(2) = 12 + 8 - 11 = 9$

$9 = 16(2) + b;\ \ 9 = 32 + b$

$b = -23$

$\therefore\ y = 16x - 23$

4) A company projects that its total savings S (in dollars) by converting to a solar-heating system with a solar-collector area A (in m^2) will be $S = 20A - 5A^3$. Find the area that should give the maximum savings. **[3 marks]**

$\frac{dS}{dA} = 20 - 15A^2$

$0 = 20 - 15A^2$

$A = \sqrt{\frac{4}{3}}$

$S'' = -30A = -30\left(\sqrt{\frac{4}{3}}\right) < 0$

$\Downarrow$

$\therefore A = \sqrt{\frac{4}{3}}$ is Max

5) If a derivative of a function is: $\frac{dy}{dx} = 4x + 2$, determine the equation of the function if it passes through the point (3, 4)

$y = \int(4x+2)dx = 2x^2 + 2x + C$

$4 = 2(3)^2 + 2(3) + C$

$4 = 18 + 6 + C$

$4 = 24 + C$

$C = -20$

$\therefore y = 2x^2 + 2x - 20$

6) Given the function $y = x^3 + 3x^2$, find

a. The domain, and range of the function y **[2 marks]**

$D: -\infty < x < \infty$

$R: -\infty < x < \infty$

b. The x-intercepts and y-intercepts **[1 mark]**

$y_{int} = 0^3 + 3(0)^2 = \emptyset$

$x_{int}: \quad 0 = x^3 + 3x^2$

$= x^2(x+3) \Rightarrow x_{int} = 0, -3$

c. The critical points **[1 mark]**

$y = x^3 + 3x^2$

$y' = 3x^2 + 6x$

$3x^2 + 6x = 0$

$3x(x+2) = 0$

$x = 0, x = -2$

$y'' = 6x + 6$

$y''(0) = 6 > 0 \Rightarrow x = 0 - \underline{min}$

$y''(-2) = -12 + 6 = -6 < 0 \Rightarrow -2 \underline{- Max}$

d. Draw the table of increase and decrease of y and find the maximum and minimum points. **[1 mark]**

x	$x < -2$	-2	$-2 < x < 0$	0	$x > 0$
y'	+	0	—	0	+
y	↗ incr		decr ↘		↗ incr
	$x = -3$	Max	$x = -1$	min	$x = 1$

$y' =$

$y(-2) = -8 + 12 = 4$

$y(0) = \emptyset$

Max = (−2,

e. Find the points of inflection. **[1 mark]**

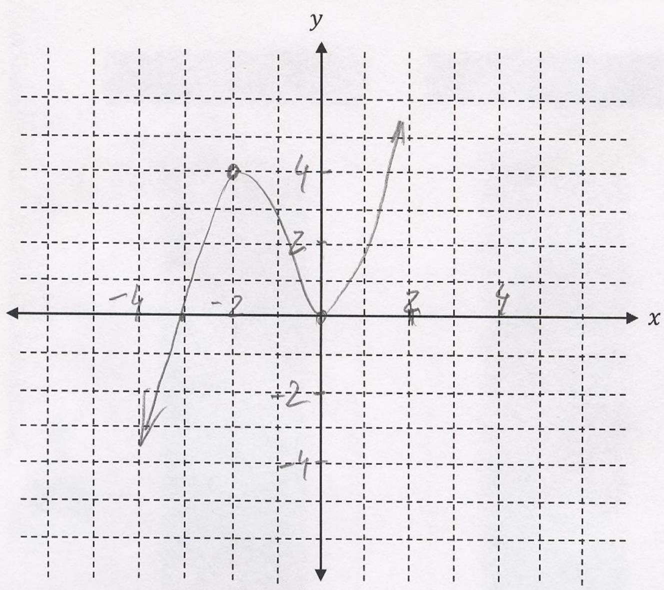
$y'' = 6x + 6 = 0$

$6x = -6$

$x = -1 \quad y(-1) = -1 + 3 = 2$

$PI = (-1, 2)$

f. Sketch the graph of y. Clearly indicate on the graph the x-intercepts, the y-intercepts, the maximum, the minimum and the inflection points. **[1 marks]**



g. Find the vertical asymptotes of $y = \frac{1}{x^2+2x-3} = \frac{1}{(x+3)(x-1)}$ $\quad x^2+2x-3 =$

S P F
2 -3 (3)(-1)

VA: $x=1,\ x=-3$

7) Find the given integrals.

a. $\int \sin(4x)dx = \frac{1}{4}\int 4\sin(4x)dx = \frac{1}{4}\int \sin u\,du$ **[3 marks]**

$u = 4x$

$du = 4dx$

$= -\frac{1}{4}\cos(4x) + C$

b. Find antiderivative of $(x) = x^3 + 4x^2 - 7x + 2$

$\int (x^3+4x^2-7x+2)dx = \frac{x^4}{4} + \frac{4x^3}{3} - \frac{7x^2}{2} + 2x + C$

b. $\int_1^3 (-6x+1)dx$ $= \left[-3x^2+x\right]_1^3 = -27+3-$ **[4 marks]**

$(-3+1) = -24+2 = -22$

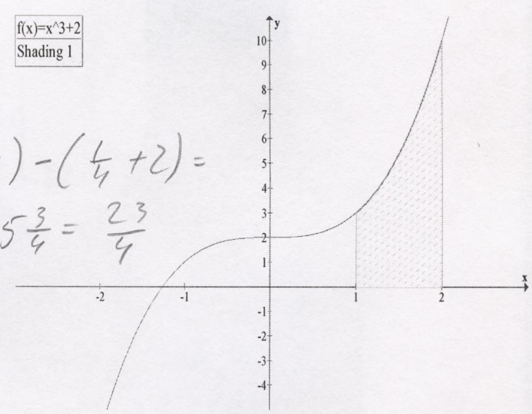
8) Find the exact area under the graph of $f(x) = x^3 + 2$ between $x = 1$ and $x = 2$.

[4 marks]

$A = \int_1^2 (x^3+2)dx =$

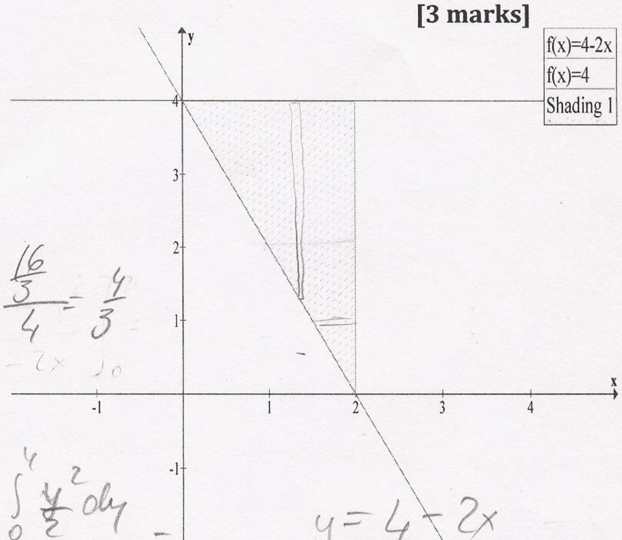
$= \left[\frac{x^4}{4} + 2x\right]_1^2 = (4+4) - (\frac{1}{4}+2) =$

$= 6 - \frac{1}{4} = 5\frac{3}{4} = \frac{23}{4}$



9) A thin plate is contained within the region bounded by $y = 4 - 2x$, $x = 2$, and $y = 4$.

a. Find the coordinates of the centroid. **[3 marks]**



$$\bar{x} = \frac{\int_0^2 x[4-(4-2x)]dx}{\int_0^2 [4-(4-2x)]dx} = \frac{\int_0^2 2x^2 dx}{\int_0^2 2x\, dx} = \frac{\frac{2}{3}x^3}{x^2}\Big|_0^2 = \frac{\frac{16}{3}}{4} = \frac{4}{3}$$

$$\bar{y} = \frac{\int_0^4 y\left(2 - 2 + \frac{y}{2}\right)dy}{\int_0^4 \left(\frac{y}{2}\right)dy} = \frac{\int_0^4 \frac{y^2}{2}dy}{\frac{y^2}{4}\Big]_0^4} =$$

$$= \frac{\frac{y^3}{6}\Big]_0^4}{4} = \frac{\frac{64}{6}}{4} = \frac{32}{3}\cdot\frac{1}{4} = \frac{8}{3}$$

$y = 4 - 2x$

$y - 4 = -2x$

$x = \frac{4-y}{2} = 2 - \frac{y}{2}$

b. Find the moment of inertia about the y-axis. **[2 marks]**

$$I_y = k\int_0^2 x^2(4 - 4 + 2x)dx = 2k\int_0^2 x^3 dx = 2k\frac{x^4}{4}\Big|_0^2 = \frac{k}{2}16 = 8k$$