Question 1

a)

$x_1 =$ Number of Desks
$x_2 =$ Number of Chairs

Maximize $z = 400x_1 + 100x_2$

Subject to

$$8x_1 + 10x_2 \leq 80$$
$$2x_1 + 6x_2 \leq 36$$
$$x_1, x_2 \geq 0$$

b) The optimal solution is $x_1 = 10$ and $x_2 = 0$ (i.e. 10 desks and no chairs).

The optimal value of the objective function is $z = 4000$

c) Slack

$$8x_1 + 10x_2 + s_1 \leq 80$$
$$2x_1 + 6x_2 + s_2 \leq 36$$

Labour

$$8(10) + 10(0) - 80 + s_1 = 0$$
$$80 - 80 + s_1 = 0$$
$$s_1 = 0$$

Wood

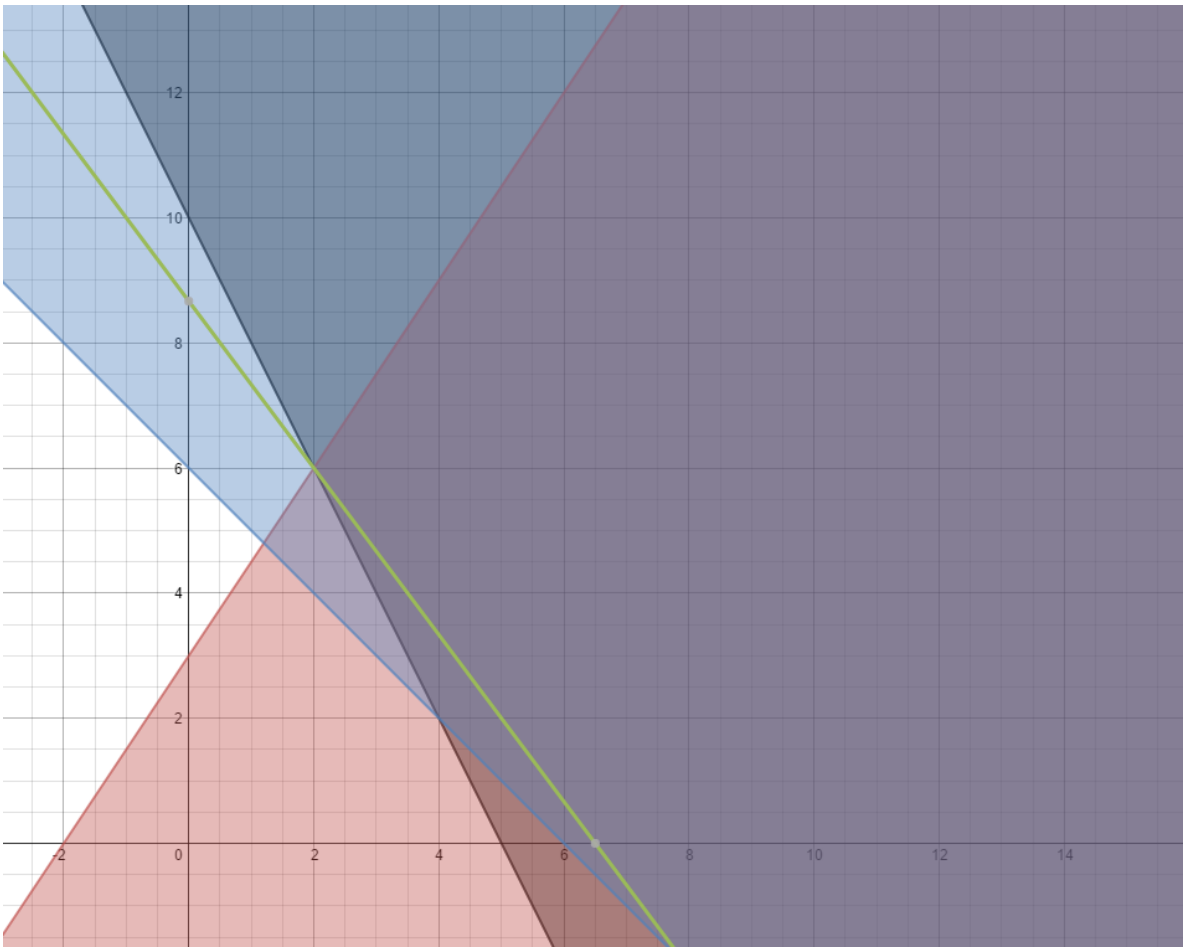
$$2(10) + 6(0) - 36 + s_2 = 0$$
$$20 - 36 + s_2 = 0$$
$$s_2 = 16$$

d) There is **no labour** left over but there is **16 pounds of wood** left.

e) The constraint on labour is the only one that is binding because the optimal solution reaches the maximum amount of labour available but not the max amount of wood. If there was more labour there would be a more optimal solution.

f) You would probably have some chairs being made if their profit was $500 in order to use up all the resources available.

Question 2



The optimal solution is $x_1 = 2$ and $x_2 = 6$

The optimal value is $z = 52$

Question 3

a)

Decision Variables**:**

x_1 = Number of Medical Beds
x_2 = Number of Surgical Beds

Objective function:

Revenue = Revenue per medical bed (i.e. the number of patients one bed will have in a year times the revenue per patient) + Revenue per surgical bed.

Maximize $z = (\frac{365}{9})2375x_1 + (\frac{365}{4})1600x_2$

Constraints:

Tests per year	$133.8\dot{3}x_1 + 237.25x_2 \leq 13500$
X-Rays per year	$40.\dot{5}x_1 + 273.75x_2 \leq 7200$
Operations per year	$91.25x_2 \leq 2750$
Number of Beds	$x_1 + x_2 \leq 85$

Subject to

$$133.8\dot{3}x_1 + 237.25x_2 \leq 13500$$
$$40.\dot{5}x_1 + 273.75x_2 \leq 7200$$
$$91.25x_2 \leq 2750$$
$$x_1 + x_2 \leq 85$$
$$x_1, x_2 \geq 0$$

b)

	A	B	C	D	E	F
1						
2		Surgery Centre				
3						
4						
5		x1	x2			
6		Medical Patient Beds	Surgery Patient Bed			
7	Number of Beds	68.90708755	16.09291245			
8	Revenue per patient	$2,375	$1,600	$8,986,658		
9	Number of days stayed per patient	9	4			
10	Number of patients per year	40.55555556	91.25			
11	**Constraints:**					
12	Tests per year	133.8333333	237.25	13040.1087	<=	13500
13	X-Rays per year	40.55555556	273.75	7200	<=	7200
14	Operations per year	0	91.25	1468.478261	<=	2750
15	Beds	1	1	85	<=	85
16				LHS	SIGN	RHS
17						

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$D$8	Revenue per patient	$0	$8,986,658

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$7	Number of Beds Medical Patient Beds	0	68.90708755	Contin
$C$7	Number of Beds Surgery Patient Bed	0	16.09291245	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$12	Tests per year	13040.1087	$D$12<=$F$12	Not Binding	459.8913043
$D$13	X-Rays per year	7200	$D$13<=$F$13	Binding	0
$D$14	Operations per year	1468.478261	$D$14<=$F$14	Not Binding	1281.521739
$D$15	Beds	85	$D$15<=$F$15	Binding	0

Here's the excel sheet for the rounded values of x1 and x2 since you can't have a fraction of a bed.

	A	B	C	D	E	F	G
1							
2			Surgery Centre				
3							
4							
5		x1	x2				
6		Medical Patient B	Surgery Patient Bed				
7	Number of	69	16				
8	Revenue per patient	$2,375	$1,600				
9	# Days stayed per patient	9	4				
10	# of patients per year	40	91	$8,884,600			
11	**Constraints:**						
12	Tests per bed	132	236.6	12893.6	<=	13500	
13	X-Rays per bed	40	273	7128	<=	7200	
14	Operations per bed	0	91	1456	<=	2750	
15	Beds	1	1	85	<=	85	

c) After solving a linear programming model based on this hospital's needs, the results are showing that the optimal amount of medical beds and surgical beds in order to maximize our revenue is, 69 medical beds and 16 surgical beds to bring in a optimized revenue of $8,884,600

Question 4

a)

Decision Variables:

x_1 = Number of nurses starting at 12 a.m. (leaving at 8 a.m.)
x_2 = Number of nurses starting at 2 a.m. (leaving at 10 a.m.)
x_3 = Number of nurses starting at 4 a.m. (leaving at 12 p.m.)
x_4 = Number of nurses starting at 6 a.m. (leaving at 2 p.m.)
x_5 = Number of nurses starting at 8 a.m. (leaving at 4 p.m.)
x_6 = Number of nurses starting at 10 a.m. (leaving at 6 p.m.)
x_7 = Number of nurses starting at 12 a.m. (leaving at 8 p.m.)
x_8 = Number of nurses starting at 2 p.m. (leaving at 10 p.m.)
x_9 = Number of nurses starting at 4 p.m. (leaving at 12 a.m.)

x_{10} = Number of nurses starting at 6 p.m. (leaving at 2 a.m.)
x_{11} = Number of nurses starting at 8 p.m. (leaving at 4 a.m.)
x_{12} = Number of nurses starting at 10 p.m. (leaving at 6 a.m.)

Objective Function:

Minimize

$$z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

Subject to

$$x_1 + x_{10} + x_{11} + x_{12} \geq 30$$
$$x_1 + x_2 + x_{11} + x_{12} \geq 20$$
$$x_1 + x_2 + x_3 + x_{12} \geq 40$$
$$x_1 + x_2 + x_3 + x_4 \geq 50$$
$$x_2 + x_3 + x_4 + x_5 \geq 60$$
$$x_3 + x_4 + x_5 + x_6 \geq 80$$
$$x_4 + x_5 + x_6 + x_7 \geq 80$$
$$x_5 + x_6 + x_7 + x_8 \geq 70$$
$$x_6 + x_7 + x_8 + x_9 \geq 70$$
$$x_7 + x_8 + x_9 + x_{10} \geq 60$$
$$x_8 + x_9 + x_{10} + x_{11} \geq 50$$
$$x_9 + x_{10} + x_{11} + x_{12} \geq 50$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12} \geq 0$$

b)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Hospital															
2																
3																
4		x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12			
5	Starting Time	12am	2am	4am	6am	8am	10am	12pm	2pm	4pm	6pm	8pm	10pm			
6	Number of nurses	20	0	20	10	30	20	20	0	30	20	0	0	170		
7	**Constraints:**															
8	12am-2am needs	1									1	1	1	40	>=	30
9	2am-4am needs	1	1									1	1	20	>=	20
10	4am-6am needs	1	1	1									1	40	>=	40
11	6am-8am needs	1	1	1	1									50	>=	50
12	8am-10am needs		1	1	1	1								60	>=	60
13	10am-12pm needs			1	1	1	1							80	>=	80
14	12pm-2pm needs				1	1	1	1						80	>=	80
15	2pm-4pm needs					1	1	1	1					70	>=	70
16	4pm-6pm needs						1	1	1	1				70	>=	70
17	6pm-8pm needs							1	1	1	1			70	>=	60
18	8pm-10pm needs								1	1	1	1		50	>=	50
19	10pm-12am needs									1	1	1	1	50	>=	50
20														LHS	SIGN	RHS
21																

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$N$6	Number of nurses	0	170

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Number of nurses 12am	0	20	Contin
$C$6	Number of nurses 2am	0	0	Contin
$D$6	Number of nurses 4am	0	20	Contin
$E$6	Number of nurses 6am	0	10	Contin
$F$6	Number of nurses 8am	0	30	Contin
$G$6	Number of nurses 10am	0	20	Contin
$H$6	Number of nurses 12pm	0	20	Contin
$I$6	Number of nurses 2pm	0	0	Contin
$J$6	Number of nurses 4pm	0	30	Contin
$K$6	Number of nurses 6pm	0	20	Contin
$L$6	Number of nurses 8pm	0	0	Contin
$M$6	Number of nurses 10pm	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$N$8	12am-2am	40	$N$8>=$P$8	Not Binding	10
$N$9	2am-4am	20	$N$9>=$P$9	Binding	0
$N$10	4am-6am	40	$N$10>=$P$10	Binding	0
$N$11	6am-8am	50	$N$11>=$P$11	Binding	0
$N$12	8am-10am	60	$N$12>=$P$12	Binding	0
$N$13	10am-12pm	80	$N$13>=$P$13	Binding	0
$N$14	12pm-2pm	80	$N$14>=$P$14	Binding	0
$N$15	2pm-4pm	70	$N$15>=$P$15	Binding	0
$N$16	4pm-6pm	70	$N$16>=$P$16	Binding	0
$N$17	6pm-8pm	70	$N$17>=$P$17	Not Binding	10
$N$18	8pm-10pm	50	$N$18>=$P$18	Binding	0
$N$19	10pm-12am	50	$N$19>=$P$19	Binding	0