Carleton University Department of Civil and Environmental Engineering

ECOR 1101: MECHANICS I

Midterm Examination, May 20th, 2016

Name(s): SOLUTION.   Student #: JOSH-

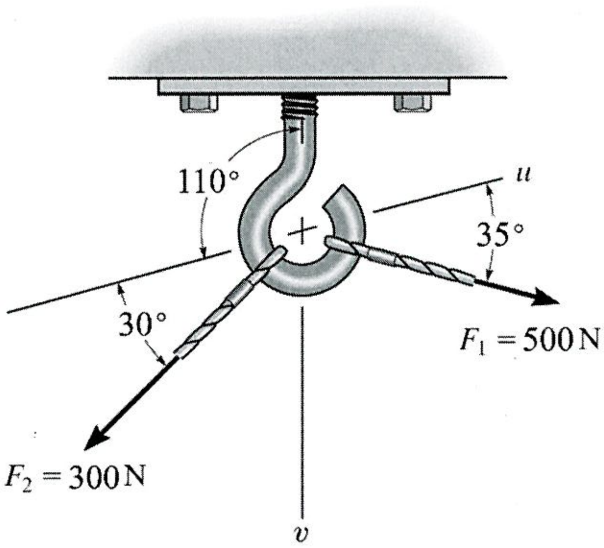
Notes:

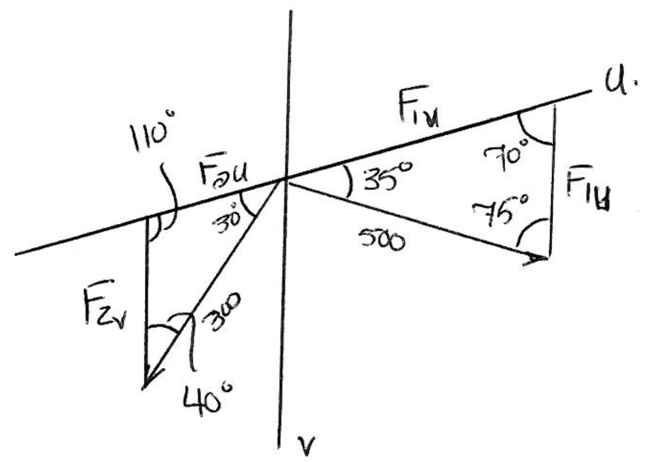
1) Time limit: **120 minutes** (6 questions, 20 minutes for each question)
2) This examination paper has 7 pages, including this title page.
3) Answer all questions. There is no choice.
4) Answer all questions in the space provided **on this examination paper.** You may use the reverse side of these pages for your rough work if necessary.
5) Write your name and student number on the top of **each page.** We will not discuss grading later if that has not been done.
6) Do not separate the pages.
7) Show **all** necessary steps to support your answers.
8) If you feel that any information is incorrect or incomplete, make a reasonable assumption, state it clearly, and proceed.
9) List the three principles of engineering below number 10) for 3 bonus marks.
10) **Authorized memoranda:** non-programmable calculator.

Problem	**Mark**
Question 1 [*20 marks*]	
Question 2 [*20 marks*]	
Question 3 [*20 marks*]	
Question 4 [*20 marks*]	
Question 5 [*20 marks*]	
Question 6 [*20 marks*]	
Total [*120 marks*]	

Question 1 [*20 marks*]

a) Resolve the forces F_1 and F_2 into components along the u and v axes. [*10 marks*]
b) Find the resultant force and its direction measured counter-clockwise from the positive u-axis. [*10 marks*]





(A) Components along u & v: (10)

$F_{2u} = \dfrac{300}{\sin 110°} \sin 40° = -205$ N

$F_{2v} = \dfrac{300}{\sin 110°} \sin 30° = 160$ N

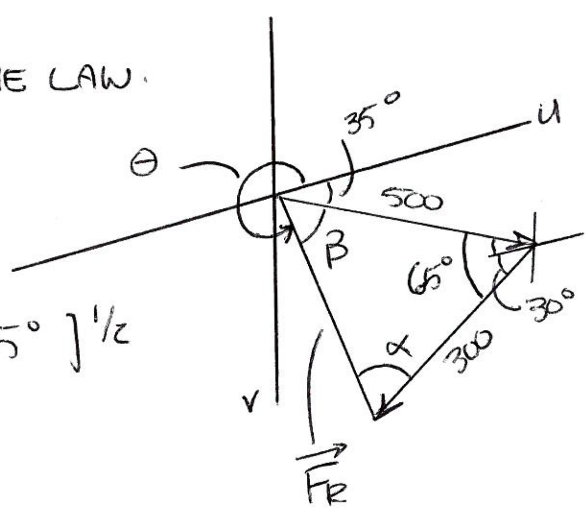
$F_{1u} = \dfrac{500}{\sin 70} \sin 75° = 514$ N

$F_{1v} = \dfrac{500}{\sin 70} \sin 35° = 305$ N

(5 marks for each)

(B) Resultant force.

2 sides & angle in between: cosine law.



$F_R = \left[500^2 + 300^2 - 2(500)(300)\cos 65°\right]^{1/2}$

$= \underline{462\text{ N}}$ ← (5)

$\dfrac{500}{\sin \alpha} = \dfrac{462}{\sin 65}$ → $\alpha = 78.8°$

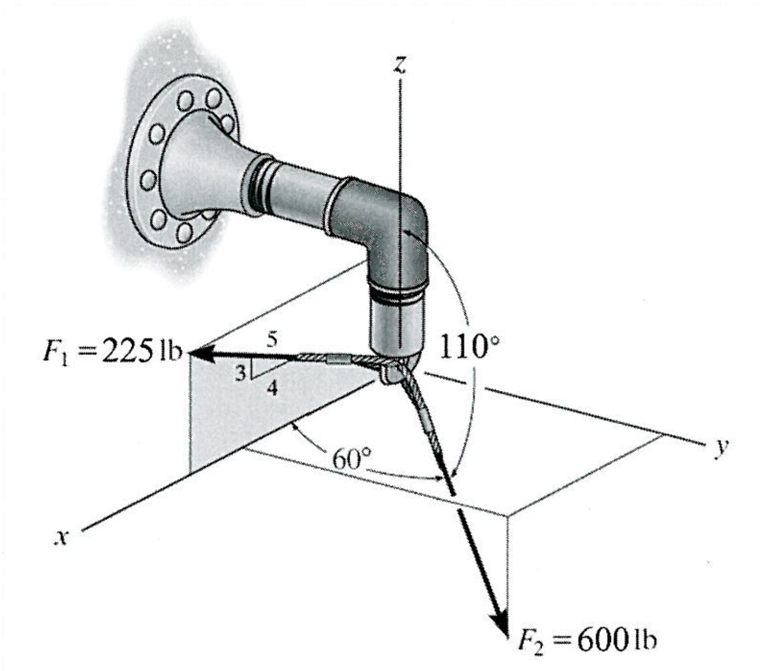
$\beta = 180° - 78.8° - 65°$
$= 36.2°$

$\theta = 360° - 36.2° - 35° = \underline{289°}$ ← (5)

This angle is measured counter-clockwise from the positive u-axis to F_R.

Question 2 [*20 marks*]

a) What is the angle $\vec{F}_2$ makes with the y-axis. [*2 marks*]
b) Express each force acting on the pipe assembly in Cartesian vector form. [*6 marks*]
c) Determine the magnitude and direction cosines of the resultant force of the two forces acting on the pipe assembly. [*12 marks*]



(A) DIRECTION COSINE:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

FOR $\alpha = 60°$
$\gamma = 110°$

$\underline{\beta = 37.3°}$ ← (2)

(B) $\vec{F_1} = 225(4/5)\hat{i} + 225(3/5)\hat{k} = [180\hat{i} + 135\hat{k}]$ lb.

$\vec{F_2} = 600\cos 60°\hat{i} + 600\cos 37.3\hat{j} + 600\cos 110°\hat{k}$

$= [300\hat{i} + 477\hat{j} - 205\hat{k}]$ lb. (3)

(C) $\vec{F_R} = \Sigma\vec{F}$

$= \vec{F_1} + \vec{F_2}$

$= [180 + 300]\hat{i} + [0 + 477]\hat{j} + [135 - 205]\hat{k}$

$= [480\hat{i} + 477\hat{j} - 70\hat{k}]$ lb. (3)

$|\vec{F_R}| = [480^2 + 477^2 + 70^2]^{1/2} = 680.5$ lb (3)

$\alpha = \cos^{-1}\left(\frac{480}{680.5}\right) = 45.1°$ (2)

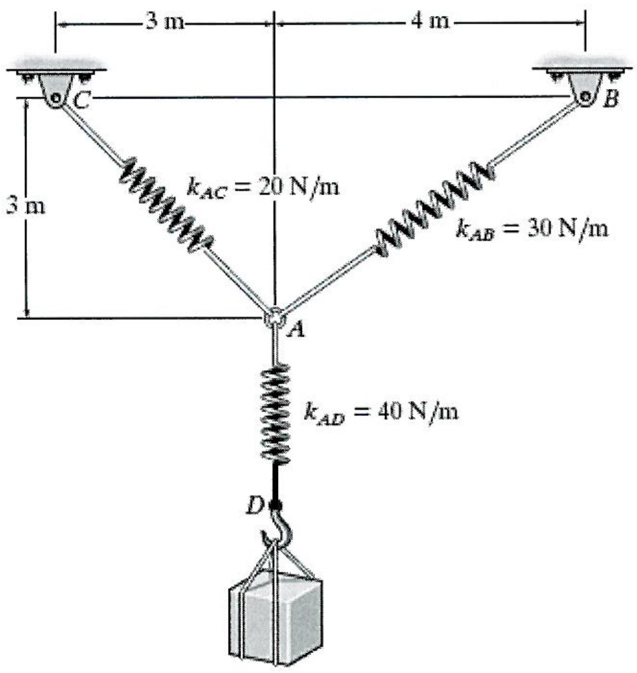
$\beta = \cos^{-1}\left(\frac{477}{680.5}\right) = 45.5°$ (2)

$\gamma = \cos^{-1}\left(\frac{-70}{680.5}\right) = 95.9°$ (2)

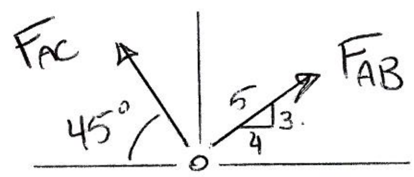
Question 3 [*20 marks*]

a) If the deformation of each spring is not allowed to exceed 200mm, determine the maximum mass of the box that can be safely supported. Include a complete FBD in your solution. [*16 marks*]

b) Using the mass from part a), how much does each spring deform in the equilibrium position? [*4 marks*]



FBD AT A (3)



$F_{AD} = W = Mg$

(A) EQUATIONS OF EQUILIBRIUM:

$\xrightarrow{+} \sum F_x = 0 \qquad -F_{AC}\cos 45° + F_{AB}(4/5) = 0 \qquad (1) \quad (2)$

$+\uparrow \sum F_y = 0 \qquad F_{AC}\sin 45° + F_{AB}(3/5) = gM. \qquad (2) \quad (2)$

ASSUME SPRING AC REACHES 200mm FIRST.

$F_{AC} = k_{AC}\Delta s = 20(0.2) = 4N \qquad \underline{F_{AC} = 4.00N} \leftarrow (2)$

FROM (1) $\qquad F_{AB}(4/5) = F_{AC}\cos 45°$

$F_{AB} = 4\cos 45°(5/4) = \underline{3.54N} \leftarrow (2)$

FROM (2) $\qquad Mg = F_{AC}\sin 45° + F_{AB}(3/5)$

$= 4\sin 45° + 3.54(3/5)$

$= \underline{4.95N} \leftarrow (2)$

$M = \underline{0.505 \text{ kg}} \leftarrow (3)$

(B) DEFORMATIONS: $\qquad \Delta_{AC} = 2.00mm.$

$\Delta_{AB} = \frac{3.54}{30} = \underline{0.118m} < 0.2m$ ok! (2)

$\Delta_{AD} = \frac{4.95}{40} = \underline{0.124m.} < 0.2m$ ok! (2)

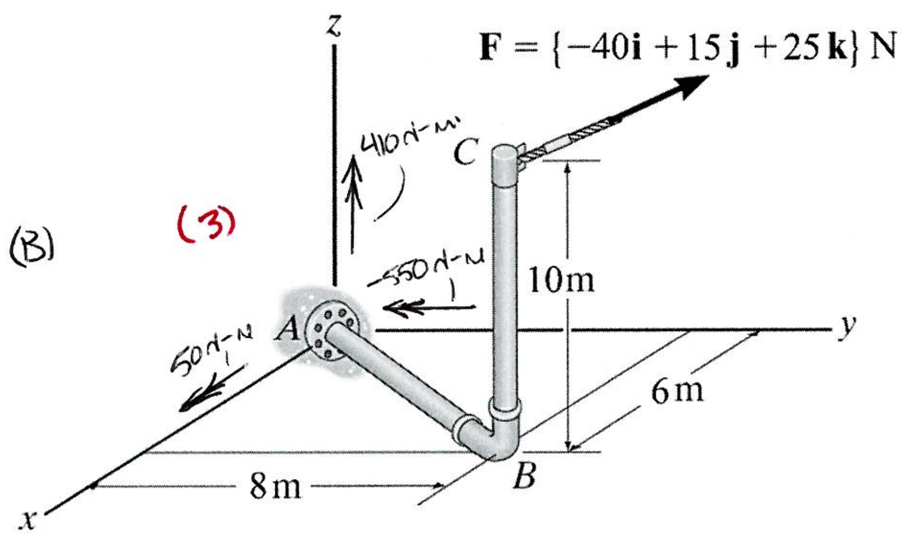
THEREFORE OUR ASSUMPTION WAS OK!

Question 4 [*20 marks*]

a) Determine the magnitude of the moments of the force $\vec{F}$ about the *x*, *y*, and *z*-axes using the Cartesian vector approach (moment about A). [*10 marks*]

b) Clearly illustrate the direction of the moments on the Figure shown below. [*3 marks*]

c) Find the moment of the force $\vec{F}$ about an axis extending from A to B, ie. AB axis. Express the result as a Cartesian vector. [*7 marks*]



(Draw the direction of the moment about each axis in this Figure)

(A)
$$\vec{r}_{AC} = [6\hat{i} + 8\hat{j} + 10\hat{k}]\,\text{M.} \quad (2)$$
$$\vec{F} = [-40\hat{i} + 15\hat{j} + 25\hat{k}]\,\text{N}$$

$$\vec{M}_A = \vec{r}_{AC} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 8 & 10 \\ -40 & 15 & 25 \end{vmatrix}$$

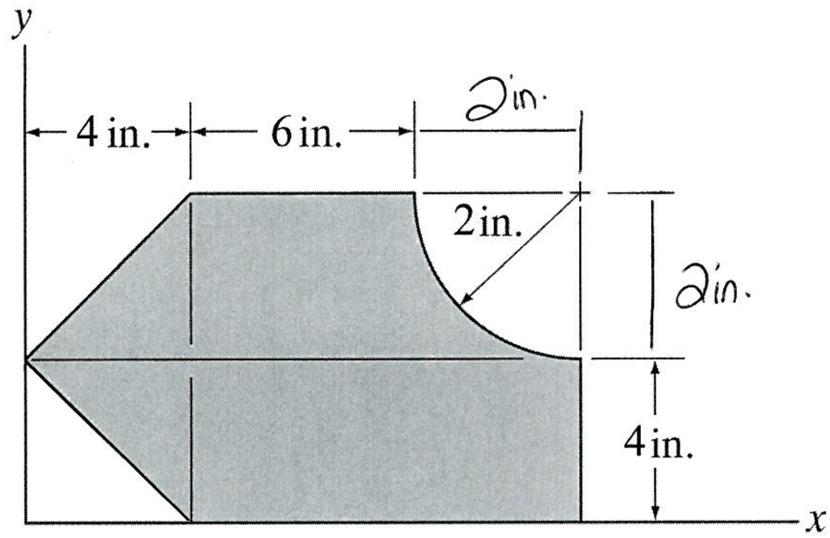
$$= [50\hat{i} - 550\hat{j} + 410\hat{k}]\,\text{N-M.} \quad (6)$$

(C)
$$\vec{M}_{AB} = (\vec{u}_{AB} \cdot \vec{M}_A)\vec{u}_{AB}$$
$$= \left(\frac{6}{10}\hat{i} + \frac{8}{10}\hat{j}\right) \cdot \left(50\hat{i} - 550\hat{j} + 410\hat{k}\right)\vec{u}_{AB} \quad (2)$$
$$= -410\,\vec{u}_{AB} \quad (2)$$
$$= [-246\hat{i} - 328\hat{j}]\,\text{N-M.} \quad (2)$$

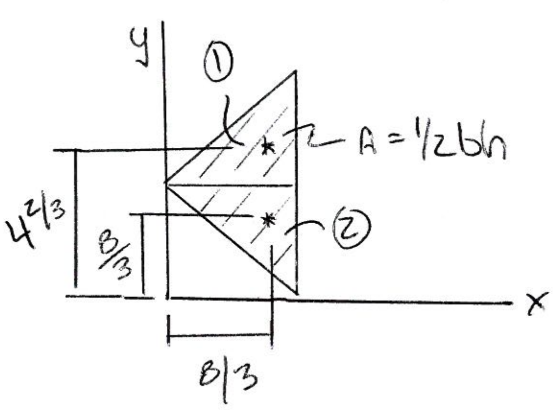
NOTE: $$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{6\hat{i} + 8\hat{j}}{[6^2 + 8^2]^{1/2}} = 6\hat{i} + 8\hat{j} \quad (1)$$

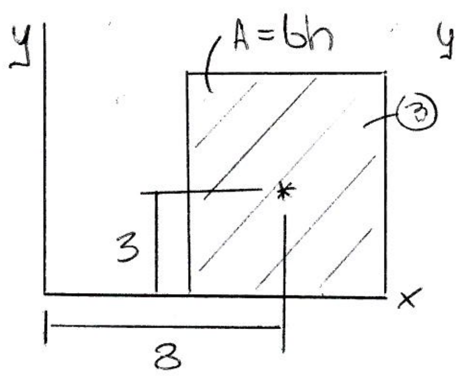
Question 5 [*20 marks*]

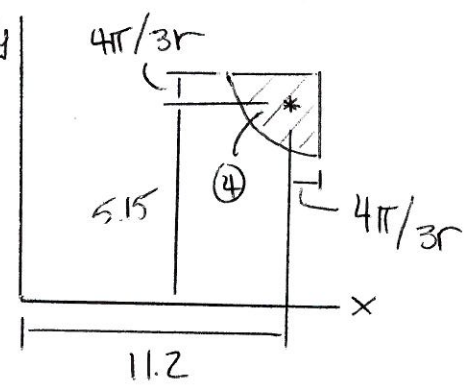
Locate the centroid $(\bar{x}, \bar{y})$ of the composite area shown in the Figure below.



Segment	A (in^2)	$\tilde{x}$ (in)	$\tilde{y}$ (in)	$A\tilde{x}$ (in^3)	$A\tilde{y}$ (in^3)	
1	8	8/3	8/3	64/3	64/3	(4)
2	4	8/3	$4\frac{2}{3}$	32/3	56/3	(4)
3	48	8	3	384	144	(4)
4	$-\pi$	11.2	5.15	$-56/5\pi$	$-103/20\pi$	(4)
	56.9 in^2			380.8 in^3	167.8 in^3	







$A_1 = 1/2\,bh = 1/2(4)(4) = 8\,in^2$ $\quad$ $A_2 = 1/2(4)(2) = 4\,in^2$

$A_3 = bh = 6(8) = 48\,in^2$ $\quad$ $A_4 = -1/4\,\pi r^2 = -\pi$

FIND CENTROID

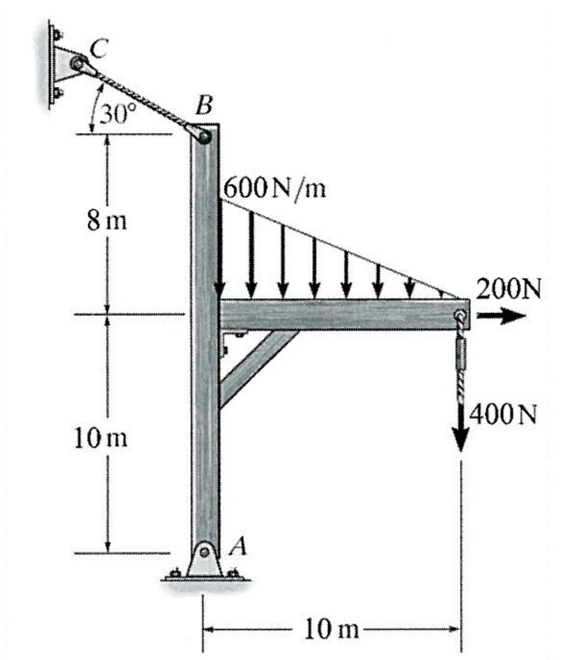
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = 6.69\,in \leftarrow (2) \qquad \bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = 2.95\,in \leftarrow (2)$$

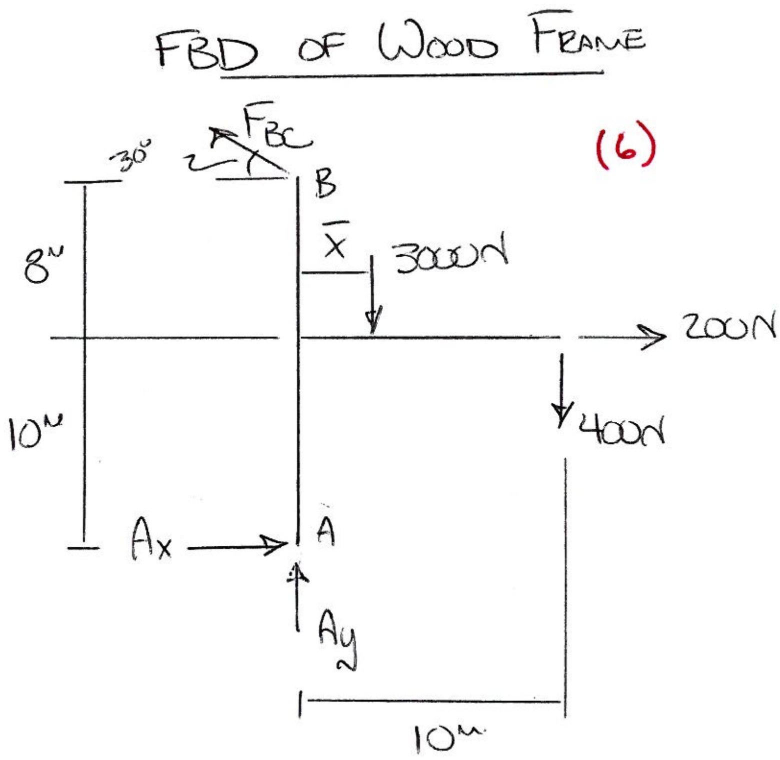
Question 6 [*20 marks*]

a) Replace the triangular distributed load by a single resultant force and indicate its position along the horizontal member. You may neglect the thickness of the members. [*2 marks*]

b) What is the tension developed in cable *BC* used to support the wood frame? Include a complete FBD of the wood frame in your solution. [*6 marks*]

c) Determine the horizontal and vertical components of reaction at the pin *A*. [*12 marks*]





(A) $F_R = \frac{1}{2}bh$

$= \frac{1}{2}(10)(600)$

$= 3000\,N = 3.00\,kN$ (1)

$\bar{x} = \frac{1}{3}(10) = 3.33\,m.$ (1)

(B) $\circlearrowleft + \sum M_A = 0$

$-400(10) - 200(10) - 3000(3.33) + F_{BC}\cos 30°(18) = 0$

$F_{BC} = \underline{1.03\,kN}$ ← (4)

(C) $+\uparrow \sum F_y = 0$ $\quad -400 - 3000 + F_{BC}\sin 30° + A_y = 0$

$A_y = \underline{2.89\,kN}$ ← (4)

$\overset{+}{\rightarrow} \sum F_x = 0$ $\quad -F_{BC}\cos 30° + 200 + A_x = 0$

$A_x = \underline{0.688\,kN}$ ← (4)