Assignment #1
Linear Programming (LP)
Formulation, Graphical Method and Excel Solver
Solutions and Marking Scheme

ADM2302 students are reminded that submitted assignments must be neat, readable, and well-organized. Assignment marks will be adjusted for sloppiness, poor grammar and spelling, as well as for technical errors. While working together is encouraged, plagiarism on assignments will not be accepted. The assignment is to be submitted electronically as a **single PDF file** via blackboard learn by Sunday October 9th prior to 23:59. Front page of the PDF document has to include title of the assignment, course code and section, student name and student number. Second page is *the individual statement of integrity that must be signed.*

Note: *Each student must provide an individual original submission of completed Assignment #1*. Please also note: Assignment #1 copies that are submitted jointly (i.e., by more than one author) will not be graded.

E-mail questions related to the assignment should be sent to the Teaching Assistant or posted on the Blackboard Learn course website "Discussion Area" (viewed by all).

TOTAL: 70 points

General marking rules

- After inputting the final grade on the 1st page, if the statement of integrity is not available or not signed by a given student then deduct 7 points. On the front page, show the "original grade" – 7 = "new grade".

- Don't penalize twice for an error that occurs at the start and it does affect the results that follows.

- Provide a brief explanation that would allow the students to understand were the error was committed and know what the right solution is.

- **Please provide me with the most common mistakes, so I can provide feedback to the students and go over the concepts in class.**

Problem 1 (20 points)

Personal Mini Warehouses is planning to open a new branch in Manitoba. In doing so, the company must determine how many storage rooms of each size to build. The company builds large and small rental spaces. After careful analysis of the expected monthly earnings of both types of rental space, as well as constraints (budget, area required for each size space, and projected rental limitations), Personal has formulated the following Linear Programming (LP) model:

x_1 = number of large spaces to develop
x_2 = number of small spaces to develop

Maximize monthly earnings $Z = 50x_1 + 20x_2$ (profit)
subject to
$2x_1 + 4x_2 \leq 400$ (advertising budget in $)
$100x_1 + 50x_2 \leq 8000$ (space area in square metres)
$x_1 \leq 60$ (rental limit expected)
$x_1, x_2 \geq 0$ (non negativity)

Briefly explain or define each of these parts of the model:

a. The 50 in the objective function.
b. The product of the 20 and x_2 in the objective function (e.g. $20x_2$)
c. The 8000 square meters in the space area constraint.
d. The 100 in the space area constraint.
e. The product of 2 and x_1 in the advertising budget constraint (e.g. $2x_1$)

Solve this LP problem using the graphical method:

f. Graph the constraints and identify the feasible region.
g. Determine the optimal solution and the maximum profit (show your calculations). **Include "managerial statements" that communicate the results of the analyses. (i.e. describe verbally)**
h. Determine the amount of slack for each of the constraint.
i. There is a single optimal solution to this problem. However, if the objective function had been parallel to one of the constraints, there would have been two equally optimal solutions. If the profit of x_1 remains at $50, what profit of x_2 would cause the objective function to be parallel to the space area constraint? Explain how you determined this.

Solution:

Defining parts of a linear programming model **Part a. to e.:**

1 point each for a total of 5 points

a. The $50 refers to the monthly earnings per unit of large space storage room developed

b. This is the total amount of profit/monthly earnings realized from small spaces storage room developed.

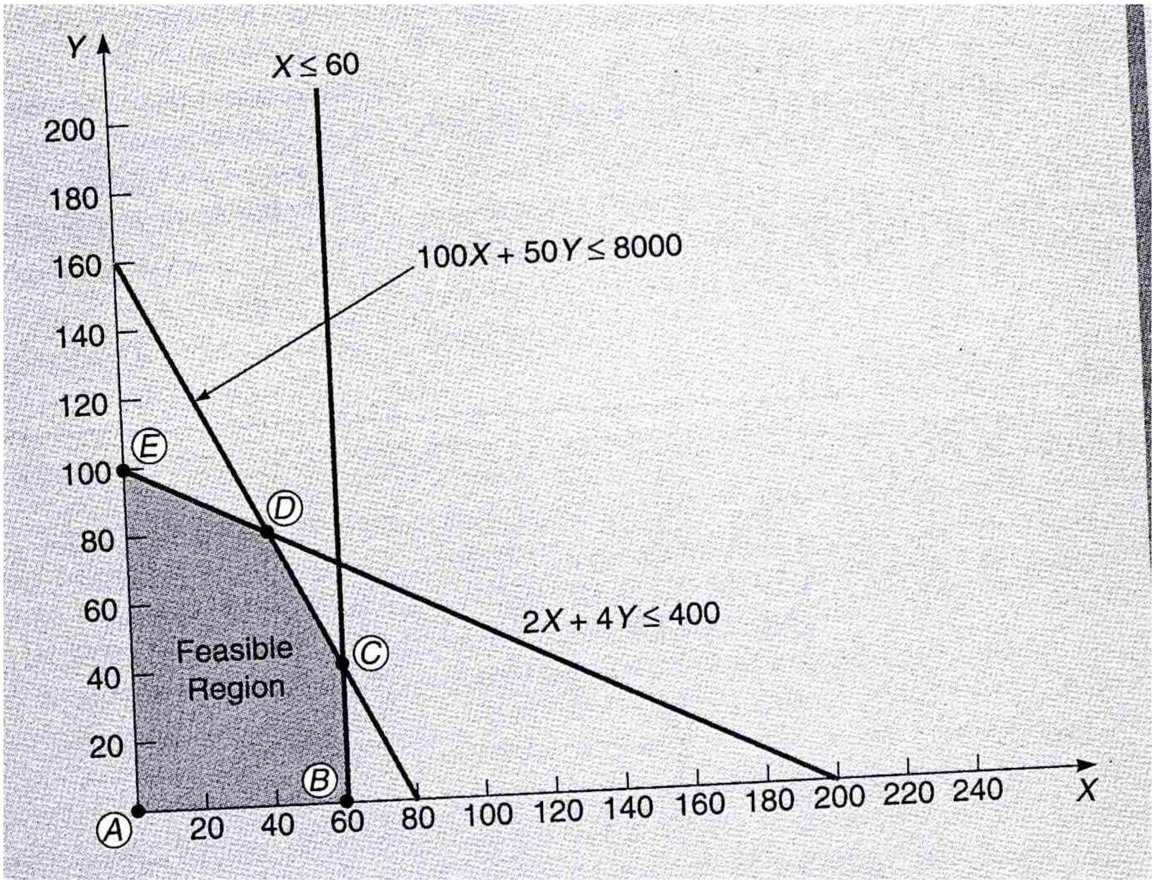
c. "8000" square meters is the total number space area available for both large and small spaces storage rooms to develop.

d. 100 square meters is the required area per unit of large space storage room to develop.

e. The total amount in $ devoted to advertise for the large space storage room.

f. ***(5 points)***

3 points: treat the constraint correctly **Note that x_2 is equivalent to Y in the graph.**
2 points find the feasible region



g. (4 points)

1 point for "managerial statement", e.g., *the optimal solution (as given by graph) is: (1) to develop 60 large spaces and 40 small spaces storage rooms; (2) for a total monthly earnings of $3,800.*

3 points: solve correctly using the algebraic method (two equations with two unknown) and not by eye balling

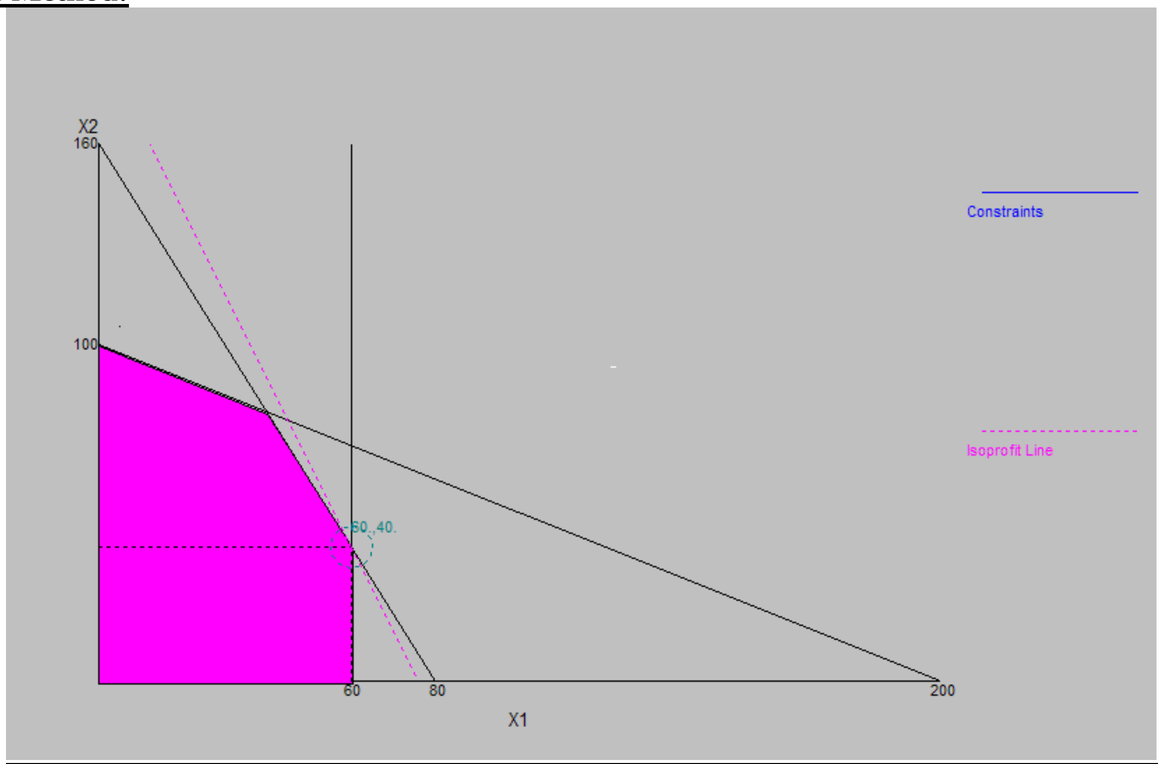
Please note that the students have to demonstrate that they used either the isoprofit method (drawing the objective function line) or the corner point method (by checking all the five corner point and picking the one that leads to the highest profit). If not then deduct 2 points.

- If the student recommend a solution that is infeasible then **deduct 3 points**

<u>Corner Point Method:</u> Note that x_2 is equivalent to Y in the table below.

Corner Point	Values of X, Y	Objective Function Value ($)
Ⓐ	(0, 0)	0
Ⓑ	(60, 0)	3000
Ⓒ	(60, 40)	3800
Ⓓ	(40, 80)	3600
Ⓔ	(0, 100)	2000

<u>Isoprofit Line Method:</u>



h. 3 points (Deduct 1 point per mistake)

The amount of slack for each constraint is:

Advertising budget slack = 400 – 280 = $120
Space area slack = 8000 – 8000 = 0 square meters
Rental limit slack = 60 – 60 = 0

i. 3 points

The coefficient would have to be at 25. (1 point)

Deduct 2 points if no explanation is provided.

Method 1:

The coefficient would have to be at 25. This is because the ratio of the coefficients must be the same. For space area constraint, the ratio is 0.5 (slope =- 50/100=-1/2), so for the objective function to be the same, the coefficient for X2 would be 25 (slope of the objective function has to be equal also to -25/50=-1/2 or -0.5).

Method 2:

Multiple optimal solutions mean that the corner point would lead to the same profit if plugged into the objective function line (e.g .C (60, 40) and D (40, 80) should lead to the same profit).

Let s be the unknown cost for x2 then

$50(60) + s(40) = 50(40) + s(80)$
$3000 – 2000 = 40 s$
$1000 = 40s \Rightarrow s = 1000/40 = 25$

Problem 2 (14 points)

Solve the following problem given these constraints and objective function:

Minimize $Z = 3x_1 + 6x_2$

subject to

$$3x_1 + 2x_2 \leq 18$$
$$x_1 + x_2 \geq 5$$
$$x_1 \leq 4$$
$$x_2 \leq 7$$
$$x_2/x_1 \leq 7/8$$
$$x_1, x_2 \geq 0$$

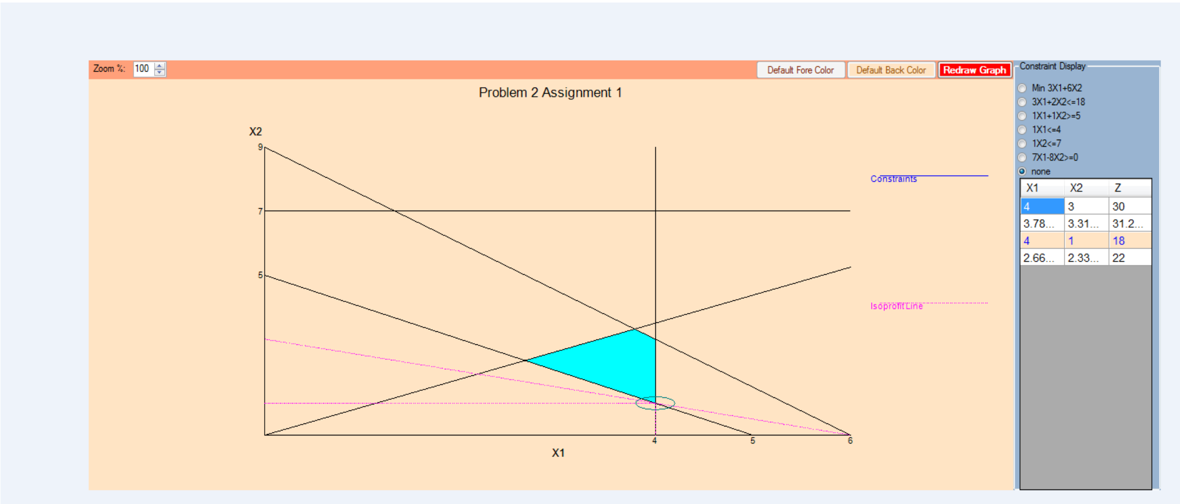
a. Graph the constraints and identify the feasible region. (8 points)
b. Determine the optimal solution and the minimum value of the objective function. (Show your calculation). (4 points)
c. What would be the effect on the solution if the constraint $x_2 \leq 7$ is changed to $x_2 \geq 7$? (2 points)

Solution:

a. (7 points)

6 points: treat the constraint correctly: 2 points for drawing $x_2/x_1 \leq 7/8$ ($7x_1 - 8x_2 \geq 0$) correctly.
1 point for the other constraint

2 points: find the feasible region



b. (4 points)

The optimal solution for the MINIMIZATION (as given by graph above) is:
(1) x1=4 and x2 = 1; (2) Z = 18 ***(1 point)***

3 points: solve correctly using the algebraic method (two equations with two unknown) and not by eye balling

- Please note that the students have to demonstrate that they used either the isocost method (drawing the objective function line) or the corner point method (by checking all the three corner point and picking the one that leads to the smallest value for Z)…if this is not the case then **deduct 2 points**
- If the student recommend a solution that is infeasible then **deduct 3 points**

c. (2 points): the problem become infeasible.

Problem 3 (9 points)

Solve the following linear programming problem graphically:

Maximize $Z = 110x_1 + 75x_2$
subject to

$$2x_1 + x_2 \geq 40$$

$-6x_1 + 8x_2 \leq 120$
$70x_1 + 105x_2 \geq 2{,}100$
$x_1, x_2 \geq 0$

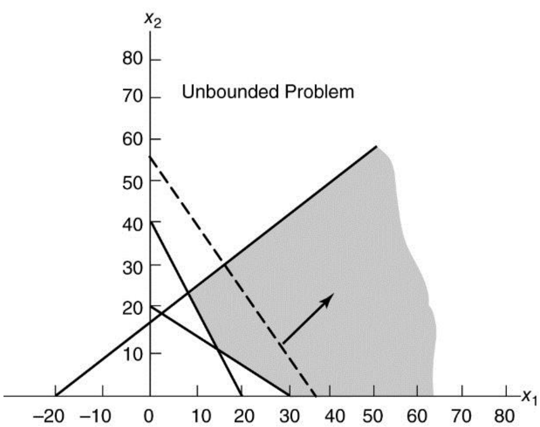
a. Graph the constraints and identify the feasible region. (6 points)
b. Explain the solution result. (Show your work). (3 points)

Solution:

a. (6 points)

4 points: treat the constraint correctly: 2 points for drawing $-6x_1 + 8x_2 = 120$ correctly (make sure it is the correct slope, i.e. they have to connect two points that on the equation line.
1 point for the other constraint

2 points: find the feasible region



b. (3 points): unbounded solution.
Student have to plot the isoprofit line and show that maximization is in the direction of the open area and thus unbounded solution. If they don't show that then deduct 2 points.

Problem 4 (27 points)

Universal Claims Processors processes insurance claims for large national insurance companies. Most claim processing is done by a large pool of computer operators, some of whom are permanent and some temporary. A permanent operator can process 16 claims per day, whereas a temporary operator can process 12 claims per day, and on average the company processes at least 450 claims each day. The company has 40 computer workstations. A permeant operator will generate about 0.5 claims with errors each day, whereas a temporary operator averages about 1.4 defective claims per day. The company wants to limit claims with errors to 25 per day. A permanent operator is paid $64 per day and a temporary operator is paid $42 per day. The company wants to determine the number of permanent and temporary operators to hire in order to minimize costs.

a. Formulate algebraically a linear programming model for the above problem. Define the decision variables, objective function, and constraints. (6 points)
b. Draw the feasible region for the linear programming model. (5 points)

c. Find the optimal solution(s) and optimal value of the objective function for the linear programing model. Justify why the solution is optimal. Describe also verbally how many permanent and temporary operators to hire (i.e. include a managerial statement). (4 points).
d. Explain the effect on the optimal solution and optimal value of the objective function of changing the daily pay for a permanent operators from \$64 to \$54. (2 points)
e. Explain the effect on the optimal solution and optimal value of the objective function of changing the daily pay for a temporary operators from \$42 to \$36. (2 points)

f. Suppose that Universal Claims Processors decided not to try to limit the number of defective claims each day. What would be the effect on the optimal solution? Formulate a linear programming model on a spreadsheet to find out. SOLVE using Excel solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). Describe verbally how many permanent and temporary operators to hire. (8 points)

Solution:

a.

Definition of Decision variables (1 point)

x_1: Number of permanent operators to hire

x_2: Number of temporary operators to hire

Minimize $Z = 64x_1 + 42x_2$ (labor cost, \$) (1 point)

subject to

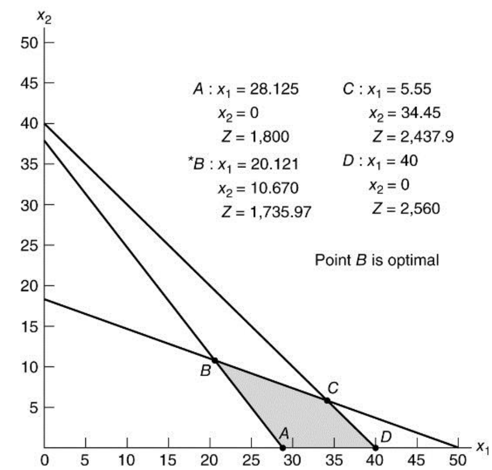
$16x_1 + 12x_2 \geq 450$ (claims processed per day) (1 point)

$x_1 + x_2 \leq 40$ (Number of workstations) (1 point)

$0.5x_1 + 1.4x_2 \leq 25$ (defective claims per day) (1 point)

$x_1, x_2 \geq 0$ (1 point)

b. and c.



Marking Scheme for part (b)

3 points: treat the constraint correctly

2 points find the feasible region

Marking Scheme for part (c)

1 point for "managerial statement", e.g., *the optimal solution (as given by graph) is: (1) to hire 20.12 permanent operators and 10.67 temporary operators for, (2) for a total cost of $1,735.97*

Note that an integer solution is required but for now the students do NOT have to provide such a solution if they do then it is ok as long as they provide a feasible integer solution.

3 points: solve correctly using the algebraic method (two equations with two unknown) and not by eye balling
Please note that the students have to demonstrate that they used either the isoCost method (drawing the objective function line) or the corner point method (by checking all the four corner point and picking the one that leads to the highest profit). If not then deduct 2 points.

- If the student recommend a solution that is infeasible then **deduct 3 points**

d. Changing the pay for a full-time claims solution to point *A* in the graphical solution where x_1 = 28.125 and x_2 = 0, i.e., there will be no part-time (temporary) operators. (2 points)

e. Changing the pay for a part-time operator from $42 to $36 has no effect on the number of full-time (permanent) and part-time (temporary) operators hired **(1 point),** although the total cost will be reduced to $1,671.95. **(1 point)**

f. Eliminating the constraint for defective claims would result in a new solution, x_1 = 0 and x_2 = 37.5, where only part-time operators would be hired. Refer to the Excel Spreadsheet formulation and answer report below.

(4points) for the correct formulation in Excel
(2 points) for the answer report and correct answer. It is ok to have the answer report in French language.
2 points for "managerial statement", e.g., *the optimal solution (as given by Excel Solver output) is: (1) Not to hire permanent operators and just hire 37.5 temporary operators for, (2) for a total cost of $1,575*
Note that an integer solution is required but for now the students do NOT have to provide such a solution if they do then it is ok as long as they provide a feasible integer solution.

	Permanent Emloyee (x1)	Temporary Employee (x2)			
Solution	0	37.5			
			Total		
Cost	$ 64.00	$ 42.00	$1,575.00		
			LHS		RHS
Claims processed per day	16	12	450	>=	450
Number of workstations	1	1	37.5	<=	40

Microsoft Excel 15.0 Answer Report
Worksheet: [Assignment 1 Problem 4 partf.xlsx]Problem 4 part f
Report Created: 05/10/2016 11:41:59 AM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0 Seconds.
Iterations: 2 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$D$4	Cost Total	$ -	$ 1,575.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$2	Solution Permanent Emloyee (x1)	0	0	Contin
$C$2	Solution Temporary Employee (x2)	0	37.5	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$7	Claims processed per day LHS	450	$D$7>=$F$7	Binding	0
$D$8	Number of workstations LHS	37.5	$D$8<=$F$8	Not Binding	2.5

Problem 1: B. Render, R.M. Stair, Jr., N. Balakrishnan, and B. Smith 2010. *Managerial Decision modeling with spreadsheets*, 2nd Canadian edition. Pearson Education Canada, Toronto, Ontario.

Problem 2, Problem 3 and problem 4: Taylor III, Bernard W. 2016. *Introduction to Management Science*, 12th edition. Pearson Education Inc.