ADM 2302X Mid-Term — Solution to Exam — Summer 2010

Question#1: Graphical Solution (12 Marks)

Consider the following linear programming problem:

Max $Z = 3X_1 + 4X_2$

Subject to

(1) $X_1 + 2X_2 <= 13$

(2) $-2X_1 + X_2 <= 4$

(3) $2X_1 - 3X_2 <= 12$

$X_1, X_2 >= 0$

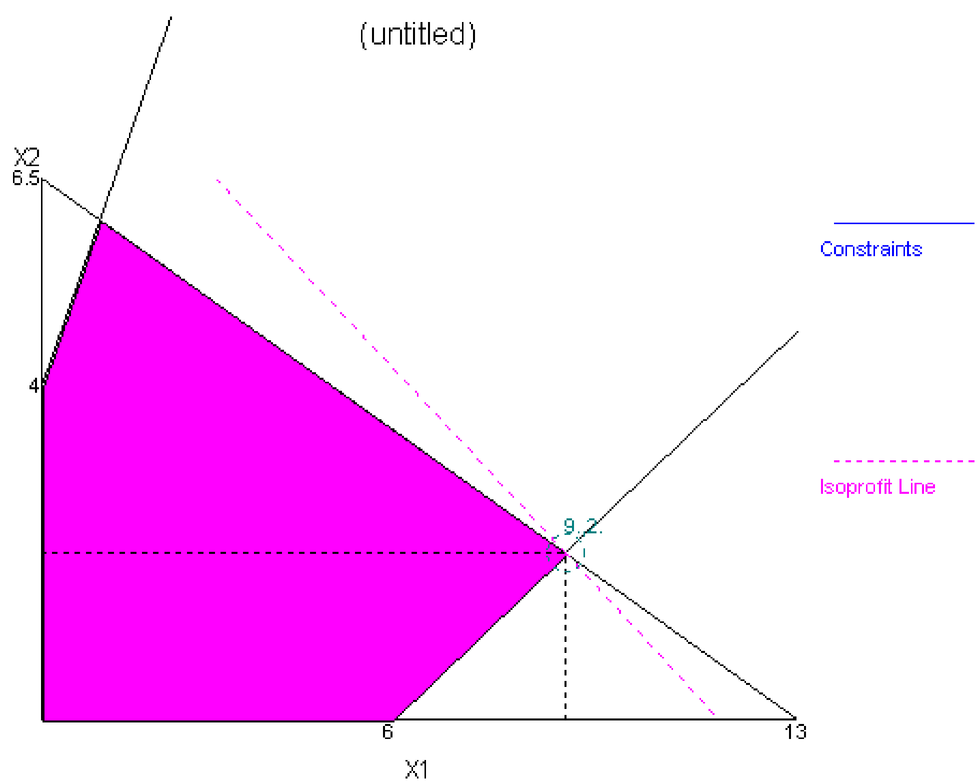
a. Graph the constraint lines and mark them clearly with the numbers (1), (2) and (3) to indicate which line corresponds to which constraint. Darken the Feasible Solution Region (FSR). Make sure you use a straight edge ruler and HB pencil. **(4 Marks)**

b. State the optimal solution by filling in the blanks below. Do not use values obtained from the graph **but get precise values by solving for the co-ordinates. Show all your work for at least one set of co-ordinates.** **(4 Marks)**

Optimal value of X_1 or X1* = ___**9**_______.

Optimal value of X_2 X2* = _____**2**______.

Optimal value of objective function Z or Z* = ____**35**________.



Corner Points or Vertices and the Evaluation of 'Z' value.

X_1	X_2	Vertex	$Z = 3X_1 + 4X_2$	
0	0	O	0	
6	0	A	18	
9	2	B	35	$Z_{max} = 35$
1	6	C	27	
0	4	D	16	

c. If the coefficient of X2 in the objective function was ‘a’, what would be the value of ‘a’ so that the iso-profit line would be parallel to constraint (1) and there would be multiple optimal solutions. What would be the new value of the objective function? **(4 Marks)**

3 **$X_1 + a\, X_2$ = (say Z = 24) or**

a **$X_2 = 24 - 3\, X_1$**

$X_2 = -\frac{3}{a} x_1 + \frac{24}{a}$ **{ Compare it with: y = m x + b}**

The slope of the ‘Z’ or “iso-profit” line is: $m_Z = -\frac{3}{a}$

The slope of Constraint#1 is: $m_1 = -\frac{1}{2}$

Thus, **$m_Z = m_1$ ==> $-\frac{3}{a} = -\frac{1}{2}$ or a = 6**

$Z = 3\, X_1 + 6\, X_2$

Z_B = 3 (9) + 6 (2) = 39

Z_C = 3 (1) + 6 (6) = 39

Hence **$Z_B = 39 = Z_C = Z_{max}$**

Thus, (9, 2) and (1, 6) both give rise to the new optimal solution with Z^* = 39. This is obviously a multiple optimal solution with {x_1*= 9, x2* = 2} and {x_1*= 1, x2* = 6}.

Question# 2: Model Formulation (18 Marks)

A medium sized manufacturer of motor cycles has to meet the following monthly demand for its most popular model Deluxe.

Month	June	July	August
Demand for Deluxe Model:	1200	1600	1400
Minimum Available Labour Hours:	3200	4100	3400
Maximum Available Labour Hours:	3500	4250	3900

The production cost per unit (including labour and materials) in thousands of dollars is 5, 5.5 and 6 for the three month respectively. It takes 2.5 hrs for building a motor-bike. The inventory cost is 5% of the production cost. The warehouse can store an inventory of 250 units in any given month. The beginning inventory is 100 units and it is desired that the ending inventory be 200 units.

a. Formulate the mathematical model, in the "Solver Compatible" format, for this problem to minimize the total cost.

S1: Let **(1 Marks)**

P_i: Production in units during month "i"

I_i: Inventory in units at the end of month "i"

P_i: Demand in units during month "i"

S2: **Min Z = 5 P_1 + 5.5 P_2 + 6 P_3 + 0.25 I1 + 0.275 I2 + 0.3 I3** **(2 Marks)**

S3: Based the production Identity, $I_n = I_{n-1} + P_n - D_n$, when rewritten as:

$P_n + I_{n-1} - I_n = D_n$

Thus P1 + I0 – I1 = D1 or P1 + 100 – I1 = 1200 ➔ P1 – I1 = 1100 -------------(C1)

P2 + I1 – I2 = 1600 -------------(C2)

P3 + I2 – I3 = 1400 -------------(C3)

Minimum Constraints

2.5 P1 ≥ 3200 ------------(C4)

2.5 P2 ≥ 4100 ------------(C6)

2.5 P3 ≥ 3400 ------------(C8)

Maximum Constraints

2.5 P1 ≤ 3500 ------------(C5)

2.5 P2 ≤ 4250 ------------(C7)

2.5 P3 ≤ 3900 ------------(C9)

I1 ≤ 250 --------------(C10)

I2 ≤ 250 --------------(C11)

I3 ≤ 250 --------------(C12)

I3 = 200 --------------(C13)

(13 marks)

Please note: (C12) is redundant, but (C13) is essential. The 13 constraints should have 1 mark each.

b. How will this model formulation change if it was known that the selling price of this motor-bike is 8 thousand dollars?

Maximize profit with the following objective function, Z.

Max Z = 3 P_1 + 2.5 P_2 + 2 P_3 - 0.25 I1 - 0.275 I2 - 0.3 I3 **(2 Marks)**

Question#3: Sensitivity Analysis (18 Marks)

HiTech Electronics (HTE) produces four consumer electronic products. They are Digital Boxes, PVRs, HD TVs and HD Recorders.

Here is the relevant data in a tabular form.

Item	Unit Cost	X1 Digital Box	X2 PVR	X3 HD TV	X4 HD Recorder	Supply:
Electronic Modules:	$21.00	3	4	4	3	4700
Non-Electronic Modules:	$15.00	2	2	4	3	4500
Assembly Time (Hrs)	$30.00	1	1	3	2	2500
Selling Price		$210.00	$240.00	$450.00	$330.00	

For example, Electronic modules cost $21.00/unit and digital box requires 3 electronic modules. Similarly the digital box also requires 2 non-electronic modules and 1 hr of assembly time with the appropriate cost per unit. The digital box sells for $210.00 producing a profit of $87 per digital box. The number of digital boxes produced is the decision variable X1. Similarly X2, X3, and X4 define the number of PVRs, HD TVs and HD Recorders to be produced.

The following LP formulation specifies the mathematical model to maximize the total profit.

$$\text{Max } Z = 87\,X1 + 96\,X2 + 216\,X3 + 162\,X4$$

s.t.

$3\,X1 + 4\,X2 + 4\,X3 + 3\,X4 \leq 4700$ ----- Electronic Modules Available

$2\,X1 + 2\,X2 + 4\,X3 + 3\,X4 \leq 4500$ ----- Non-Electronic Modules Available

$1\,X1 + 1\,X2 + 3\,X3 + 2\,X4 \leq 2500$ ----- Assembly Time in hrs Available

$X_i \geq 0$

Microsoft Excel 12.0 Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$I$16	Units Produced: Total Profit:	0	208200

Adjustable Cells

Cell	Name	Original Value	Final Value
$C$16	Units Produced: Digital Box	0.00	0.00
$D$16	Units Produced: PVR	0.00	380.00
$E$16	Units Produced: HD TV	0.00	0.00
$F$16	Units Produced: HD Recorder	0.00	1060.00

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$G$10	Electronic Modules: LHS	4700.00	$G$10<=$I$10	Binding	0
$G$11	Non-Electronic Modules: LHS	3940.00	$G$11<=$I$11	Not Binding	560
$G$12	Assembly Time (Hrs) LHS	2500.00	$G$12<=$I$12	Binding	0

Microsoft Excel 12.0 Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$16	Units Produced: Digital Box	0.00	-3.00	87	3	1E+30
$D$16	Units Produced: PVR	380.00	0.00	96	120	5
$E$16	Units Produced: HD TV	0.00	-24.00	216	24	1E+30
$F$16	Units Produced: HD Recorder	1060.00	0.00	162	30	15

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$G$10	Electronic Modules: LHS	4700.00	6.00	4700	2800	950
$G$11	Non-Electronic Modules: LHS	3940.00	0.00	4500	1E+30	560
$G$12	Assembly Time (Hrs) LHS	2500.00	72.00	2500	466.6666667	1325

(a) Determine optimal production schedule of HTE for the four products. What is the optimal profit of such optimal strategy? **(2 Marks)**

Here (Digital Boxes),X1 = 0, (PVRs) X2 = 380,
(HD TVs) X3 = 0, and (HD Recorders) X4 = 1060

Zmax = $ 208,200

(b) What would be the impact on the production schedule and the optimal profit if the local supplier decreased the purchasing cost of electronic modules by $.75 ? Justify. **(3 Marks)**

When the cost goes down, the Profit Coefficients will go up.

C_{1New} = 87 + 3 x 0.75 = 89.25 C_{2New} = 96 + 4 x 0.75 = 99.00
C_{3New} = 216 + 4 x 0.75 = 219.00 C_{4New} = 162 + 3 x 0.75 = 164.25

All these changed values are **within the allowable ranges**. The product mix will not change but Zmax will change.
ZmaxNew = 89.25 (0) + 99.00 (380) + 219.00 (0) + 164.25 (1060) = $ 211,725

Thus the Profit will increase by $3,525

(c) In order to start producing HD TVs (that is having. X3 > 0), what would need to happen to the unit profit and selling price of HD TV? Justify. **(4 Marks)**

The Reduced Cost for HD Tvs is -24. This means that if the Profit Coefficient for HD TVs was to go up by $24, it would become 216 + 24 = $240 per HD TV. This can happen if the Selling Price became $474. Thus, if the Selling Price became $474 per HD TV, then HD TVs or X3 >0 and HTE should start producing the HD TVs.

(d) What is the shadow price for assembly time? If the assembly time available were to be 2600 hrs instead of the current value, what would be the new profit and what would happen to product mix? **(3 Marks)**

Assembly Time is a “Binding Constraint” with a “Shadow Price” = $72. This means each extra hour of assembly time would result in $72 worth of additional profit. Hence (2600 – 2500) x (72) = $7,200 extra profit will be generated with 2600 hours of assembly time. How ever, the % Increase = Change/Allowable Change = (2600 – 2500)/466.6666 x 100 = 21.4286 %. Thus the Shadow Prices will not change but it is more than likely that the Product Mix or the Solution Values of Decision Variables will change.

(e) If the availability of non-electronic modules increases by 500 units, what effect will it have on the product mix? Why is its shadow price zero? **(3 Marks)**

The Non-Electronic Components constraint, C2, is “non-binding”. The resources are currently not being fully utilized. Purchasing extra units, whether 500 or infinite number, will add nothing to the total profit. Extra units of non-electronic components are therefore worthless giving rise to “Zero” “Shadow Price”.

(f) If the available electronic modules were only 4000, how would it impact on the total profit. What can you say about the product mix ? **(3 Marks)**

Electronic Modules constraint, C1, is “binding” with a “Shadow Price”=$6. If the RHS is reduced by 700 units, it is in the allowable range, and the profit will decrease by $6 x 700 = $4,200 and the shadow price will not change but the product mix will most likely change.

Question#4: Multiple Choice, Filling in the Blanks (12 Marks)

Circle the proper answer to each question

Consider the following linear program shown below and entered into Excel.
Questions 1 – 4 make reference to this problem.

x = # of lift-and-cut (LAC) electric shavers manufactured
y = # of standard (STD) electric shavers manufactured

	Unit Profit	**Production time Department A**	**Production time Department B**
LAC Shavers:	**$35**	**18 minutes**	**15 minutes**
STD Shavers:	**$25**	**3 minutes**	**1.5 minutes**
	Hours available/Dept	**240**	**30**

Maximize 35 x + 25 y
subject to:

$$.3x + .25y \leq 240 \text{ production hours in Department A}$$
$$.05x + .025y \leq 30 \text{ production hours in Department B}$$
$$y \leq 120 \text{ maximum market for product y}$$
$$x \geq 0,\ y \geq 0$$

	A	B	C	D	E	F	G	H	I
1									
2		Ultra	Stand			Total Profit			
3	Units to Make								
4	Unit Profit	35	25						
5				LHS		RHS			
6	Dept A	.3	.25		≤	240			
7	Dept B	.05	.025		≤	30			
8	Demand	0	1		≤	120			
9									

1. The formula behind the target cell is

   a. SumProduct (B3:B4,C3:C4)
   b. SumProduct (B3:C3,B6:C6)
   c. SumProduct (B3:C3,B4:C4) Or "c"
   d. There is no formula.

2. The formula behind the left-hand side of the Department A constraint is

   a. SumProduct (B3:B4,$C$3:$C$4)
   b. SumProduct ($B$3:$C$3,B6:C6) Or "b"
   c. SumProduct (B3:C3,$B$4:$C$4)
   d. There is no formula.

3. The formula behind the changing cell for x, the number of ultrasonic toothbrushes is

   a. SumProduct (B3:B4,$C$3:$C$4)
   b. SumProduct (B3:C3,$B$6:$C$6)
   c. SumProduct ($B$3:$C$3,B4:C4)
   d. There is no formula. Or "d"

4. To add the Department A constraint, the following formula is entered into the *Add Constraint* dialog box:

   a. .3B3 + .25C3 $\leq$ 240
   b. .3x + .25y $\leq$ 240
   c. D6 $\leq$ F6 — Or " c"
   d. B3*B6+C3*C6 $\leq$ F6

5. The following types of constraints are ones that might be found in linear programming formulations:
   1. $\leq$
   2. =
   3. >
   4. <
   5. $\geq$

   a. 2, 3 and 4
   b. 1, 2 and 5 — Or "b"
   c. 1, 2 and 3
   d. all of the above

6. If a change is made in only one of the objective function coefficients:

   a. the slope of the objective function line always will change. — Or "a"
   b. the optimal solution always will change.
   c. one or more of the decision variables always will change.
   d. All of the above.

   1.5 marks each for the first six correct answers.

Fill in the Blank or Blanks for question (7) and (8)

7. A linear programming maximizing problem for two variables has a closed polygonal FSR. For the problem to have a solution, this polygon must be ___ Convex._____. **(1 Mark)**

8. The mathematical method (algorithm) used to solve linear programming problem is called _Simplex _ and was first proposed by ___G. Dantzig__. **(2 Marks)**