Math 209 Mid TERM (Alternate) Oct 2014. /60 MARKS (UN EDITED)

1. a) $\lim_{x\to 2} \frac{x^2-x-6}{x^2+3x-4} = \frac{2^2-2-6}{2^2+3(2)-4} = \frac{-4}{6} = -\frac{2}{3}$

b) $\lim_{x\to 4} \frac{\frac{1}{x+2} - \frac{1}{x-10}}{x-4} = \frac{\frac{1}{6} - \frac{1}{-6}}{4-4} = \frac{\frac{1}{3}}{0}$ (UNDEFINED)

$\boxed{3@3\text{ MARKS}}$

c) $\lim_{x\to -\infty} \frac{2x^2-5}{x^2+4x+4} = \frac{\infty}{\infty}$

$\lim_{x\to -\infty} \frac{\frac{2x^2}{x^2} - \frac{5}{x^2}}{\frac{x^2}{x^2} + \frac{4x}{x^2} + \frac{4}{x^2}} = \lim_{x\to -\infty} \frac{2 - \frac{5}{x^2}}{1 + \frac{4}{x} + \frac{4}{x^2}} = \frac{2-0}{1+0+0} = 2$

2. $g'(x) = \lim_{t\to 0} \frac{g(x+t)-g(x)}{t}$

$g(x) = \sqrt{x} + 2$

$g(x+t) = \sqrt{x+t} + 2$

$= \lim_{t\to 0} \frac{\sqrt{x+t} \cancel{+2} - (\sqrt{x} \cancel{+2})}{t} = \frac{0}{0}$

Indeterminant

$= \lim_{t\to 0} \frac{(\sqrt{x+t} - \sqrt{x})(\sqrt{x+t} + \sqrt{x})}{t(\sqrt{x+t} + \sqrt{x})}$

Rationalize

$\boxed{6\text{ MARKS}}$

$= \lim_{t\to 0} \frac{\cancel{x} + \cancel{t} + \cancel{\sqrt{x}\sqrt{x+t}} - \cancel{\sqrt{x}\sqrt{x+t}} - \cancel{x}}{\cancel{t}(\sqrt{x+t} + \sqrt{x})}$

$= \lim_{t\to 0} \frac{1}{\sqrt{x+t} + \sqrt{x}}$

$= \frac{1}{\sqrt{x+0} + \sqrt{x}}$

$= \frac{1}{2\sqrt{x}}$

3 a) $f(x) = 2x^{23} - 3e^x + x^3 - \pi^4$

$\sqrt{\frac{x^9}{x^3}} = \sqrt{x^6} = (x^6)^{\frac{1}{2}} = x^3$

$f'(x) = 46x^{22} - 3e^x + 3x^2 - 0$

$f'(1) = 46(1)^{22} - 3e^1 + 3(1)^2$

3 b) $g(x) = (x-1)\ln(x^2+x-2)$

$g'(x) = (x-1)\frac{d}{dx}\ln(x^2+x-2) + \ln(x^2+x-2)\frac{d}{dx}(x-1)$

$\boxed{4@3}$

$g'(x) = (x-1)\frac{1}{x^2+x-2}\frac{d}{dx}(x^2+x-2) + \ln(x^2+x-2)(1)$

$g'(x) = \frac{(x-1)(2x+1)}{(x^2+2x-2)} + \ln(x^2+x-2)$

$g'(2) = \frac{(2-1)(2[2]+1)}{2^2+2[2]-2} + \ln(2^2+2-2)$

3c) $h(x) = \dfrac{x^3 - 8x^2}{x-6}$

$$h'(x) = \frac{(x-6)\frac{d}{dx}(x^3-8x^2) - (x^3-8x^2)\frac{d}{dx}(x-6)}{(x-6)^2}$$

$$= \frac{(x-6)(3x^2-16x) - (x^3-8x^2)(1)}{(x-6)^2}$$

3d) $y = (4x - e^x)^{-49}$

$$\frac{dy}{dx} = -49(4x-e^x)^{-50}\frac{d}{dx}(4x-e^x)$$

$$\frac{dy}{dx} = -49(4x-e^x)^{-50}(4-e^x)$$

$$dy = \frac{-49(4-e^x)}{(4x-e^x)^{50}}\,dx$$

$$dy\Big|_{\substack{x=2\\ dx=.1}} = \frac{-49(4-e^2)}{[4(2)-e^2]^{50}} * (0.1) \approx 8.32 \times 10^{11}$$

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4. (i) $\dfrac{d}{dt}R = \dfrac{d}{dt}\left(400x - \dfrac{x^2}{30}\right)$

$$\frac{dR}{dt} = 400\frac{dx}{dt} - \frac{1}{30}\frac{d}{dt}x^2$$

$$\frac{dR}{dt} = 400\frac{dx}{dt} - \frac{1}{30}(2)x\frac{dx}{dt}$$

$$\frac{dR}{dt}\Big|_{\substack{x=9000\\ \frac{dx}{dt}=800}} = 400(800) - \frac{1}{15}(9000)(800)$$

$$= -160\ 000\ \$/\text{week} \quad (\text{decrease in } R)$$

$\boxed{2@4}$

(ii) $\underset{\text{PROFIT}}{P} = \underset{\text{Rev}}{R} - \underset{\text{cost}}{C}$

$$= \left(400x - \frac{x^2}{30}\right) - (100\ 000 + 20x)$$

$$P = 380x - \frac{x^2}{30} - 100\ 000$$

$$\frac{dP}{dt} = \frac{d}{dt}380x - \frac{d}{dt}\frac{1}{30}x^2 - \frac{d}{dt}100\ 000$$

$$= 380\frac{dx}{dt} - \frac{1}{30}(2)x\frac{dx}{dt} - 0$$

$$\frac{dP}{dt} = 380\frac{dx}{dt} - \frac{x}{15}\frac{dx}{dt}$$

$$\frac{dP}{dt}\Big|_{\substack{x=9000\\ \frac{dx}{dt}=800}} = 380(800) - \frac{9000}{15}(800)$$

$$= -176\ 000\ \$/\text{week} \quad (\text{decrease in PROFIT } P)$$

5. a) (i) $C(100) = 5000 + 40(100) + .05(100)^2 = 9500\$$

(ii) $C'(x) = 40 + .05(2)x$

(ii) $C'(100) = 40 + .10(100) = 50\$/\text{Article}$

b) (i) $\bar{C}(x) = \frac{C(x)}{x} = \frac{5000 + 40x + .05x^2}{x}$

$\boxed{3@3}$

$\bar{C}(x) = \frac{5000}{x} + 40 + .05x$

$\bar{C}(100) = \frac{5000}{100} + 40 + .05(100) = 95\$$

(ii) $\bar{C}'(x) = 5000(-1)x^{-2} + 0 + .05$

$\bar{C}'(x) = -\frac{5000}{x^2} + .05$

$\bar{C}'(100) = -\frac{5000}{100^2} + .05 = -0.\cancel{45}\$/\text{Article}$

NOTE: Error in question WORDING.

c) $C(101) = C(100) + C'(100)$
$= 9500 + 50$
$= 9550\$$

OR

$\bar{C}(101) = \bar{C}(100) + \bar{C}'(100)$
$= 95 + (-.45)$
$= 94.55\$$

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6. Note Find y' for $f(x)$ means do $\frac{d}{dx}y$

$\Rightarrow$ Find x' for $x(t)$ means do $\frac{d}{dt}x$

$\boxed{8 \text{ MARKS}}$

$\frac{d}{dt}x^3 - \frac{d}{dt}tx^2 - \frac{d}{dt}4 = \frac{d}{dt}0$

$3x^2\frac{d}{dt}x - \left[t\frac{d}{dt}x^2 + x^2\frac{d}{dt}t\right] - 0 = 0$

$3x^2\frac{dx}{dt} - t(2)x\frac{d}{dt}x + x^2(1) = 0$

$\frac{dx}{dt}\left[3x^2 - 2xt\right] = -x^2$

$\frac{dx}{dt} = \frac{-x^2}{3x^2 - 2xt}$

OR $\frac{dx}{dt} = \frac{-x^2}{3x^2 - 2tx}$

$\left.\frac{dx}{dt}\right|_{t=-3,\ x=-2} = \frac{-(-2)^2}{3(-2)^2 - 2(-2)(-3)} = \frac{-4}{12 - 12}$

UNDEFINED

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7. Do $\frac{d}{dt}$ of given Equation (Compare to time)

$\boxed{8 \text{ MARKS}}$

$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}25$

$2x\frac{d}{dt}x + 2y\frac{d}{dt}y = 0$

$2(-3)(.4) + 2(4)\frac{dy}{dt} = 0$

$8\frac{dy}{dt} = -2(-3)(.4)$

$\frac{dy}{dt} = \frac{6(.4)}{8}$

$\frac{dy}{dt} = 0.3 \text{ UNIT/sec}$

Note: $\frac{dx}{dt} = .4 \text{ UNIT/sec}$

$x = -3$
$y = 4$