

MAT 2379, Introduction to biostatistics

Solution to Assignment 2

Due date: Wednesday October 5, 2011

Total = 16 marks

Problem 6.2. (2 marks) (a)

X	0	1	2	3	4
$P(X = x)$	0.65	0.216	0.07	0.042	0.022

(b) $P(X \geq 3) = P(X = 3) + P(X = 4) = 0.042 + 0.022 = 0.064.$

(c) $E(X) = 0(0.65) + 1(0.216) + 2(0.07) + 3(0.042) + 4(0.022) = 0.57.$

Problem 6.3 (2 marks) Let X be the number of children with blue eyes. X has a binomial distribution with $n = 3$ and $p = 0.25$.

(a) $P(X = 3) = (0.25)^3 = 0.0156$

(b) $P(X = 0) = (0.75)^3 = 0.4219$

(c) $P(X \geq 1) = 1 - P(X = 0) = 1 - (0.75)^3 = 0.5781.$

Problem 6.6 (2 marks) Let X be the number of sturgeons large enough to swallow a salmon whole among 10 sturgeons. X has a binomial distribution with $n = 10$ and $p = 0.45$.

a) $P(X = 0) = \binom{10}{0} (0.45)^0 (0.55)^{10} = 0.0025.$

b) $P(X \leq 1) = 1 - P(X = 0) = 1 - 0.0025 = 0.9975$

c)

$$\begin{aligned}
P(2 \leq X \leq 5) &= \binom{10}{2} (0.45)^2 (0.55)^8 + \binom{10}{2} (0.45)^2 (0.55)^8 \\
&\quad + \binom{10}{2} (0.45)^2 (0.55)^8 + \binom{10}{2} (0.45)^2 (0.55)^8 \\
&= 0.7152
\end{aligned}$$

Problem 7.1 Let X be the height of a randomly chosen boy. Then $X \sim N(125, 8^2)$.

(a)

$$P(X < 122) = P\left(\frac{X - 125}{8} < \frac{122 - 125}{8}\right) = P(Z < -0.375) = 0.3538$$

(b)

$$\begin{aligned}
P(122 < X < 130) &= P\left(\frac{122 - 125}{8} < \frac{X - 125}{8} < \frac{130 - 125}{8}\right) = \\
&P(-0.375 < Z < 0.625) = 0.7340 - 0.3538 = 0.3802
\end{aligned}$$

(c)

$$0.95 = P(X < h) = P\left(\frac{X - 125}{8} < \frac{h - 125}{8}\right)$$

Hence $(h - 125)/8 = 1.645$ and $h = 125 + 8(1.645) = 138.16$.**Problem 7.3** (a)

$$P(X \leq 0) = P\left(\frac{X - 2}{0.7} \leq \frac{0 - 2}{0.7}\right) = P(Z < -2.86) = 0.0021$$

(b) Let Y be the number of seedlings which did not germinate in a tray of 25. Y has a binomial distribution with $n = 25$ trials and $p = 0.0021$ probability of success. The desired probability is $P(Y = 0) = (1 - 0.0021)^{25} = 0.9488$.

Problem 8.10

(a) We denote by f a female sample and m a male sample. Using a tree diagram, we see that there are 6 possible outcomes. We list them below with the associated probabilities:

- fff (with probability $1/8$);

- ffm (with probability $1/8$);

- fm (with probability $1/4$);

- mf (with probability $1/4$);

- mmf (with probability $1/8$);

- mmm (with probability $1/8$).

X takes the values 2 and 3 with the following probabilities: $P(X = 2) = 1/4 + 1/4 = 1/2$ and $P(X = 3) = 1/8 + 1/8 + 1/8 + 1/8 = 1/2$. Hence

$$E(X) = 2(1/2) + 3(1/2) = 2.5.$$

(b) Y takes the values 0, 1, 2 and 3 with the following probabilities: $P(Y = 0) = 1/8$, $P(Y = 1) = 1/4 + 1/4 + 1/8 = 5/8$, $P(Y = 2) = 1/8$ and $P(Y = 3) = 1/8$. Hence

$$E(Y) = 0(1/8) + 1(5/8) + 2(1/8) + 3(1/8) = 10/8 = 1.25.$$