TELFER

VOTRE LIEN AVEC CE QUI COMPTE — CONNECTS YOU TO WHAT MATTERS

Instructions to Students: see your booklet

Statement of Academic Integrity see booklet **Statement to be signed by the student:** I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination. __________

Note: an examination copy or booklet without the above signed statement will not be graded and will receive a exam grade of zero.

Name (LAST, first): __________
Student Number): __________

Section (circle): M, N, or P __________
Instructor's name (circle): Schillo or Brand __________

Question:	1	2	3	4	5	Total
Points:	18	9	5	16	13	61
Score:						

1. 2000 or more individuals per country. Respondents answer "yes" or "no" to this question. From past years, we know that the probability of Canadians responding to this question with "yes" is 10.2%. 9.1 Total for Question 1: 18

   (a) **(1 point) first person called answers this question with "yes"?**

   > **Solution:** 10.2% 9.1 9.9

   (b) **(2 points) first and the second person called answers with "yes"?**

   i. (1 point) probability rule $P(A \cap B) = P(A) * P(B)$
      - A. conditinal probabilities
      - B. simple AND/multiplication rule ,
      - **C. simple OR/addition rule** 0.0104 0.0083 0.0098
      - D. general AND/multiplication rule
      - E. general OR/addition rule

   ii. (1 point) assumption
      - A. the events are disjoint (mutually exclusive)
      - **B. the events are independent**
      - C. the events are disjoint and independent
      - D. the law of averages
      - E. no assumptions are necessary

   (c) (3 points) 11th or 18th person called answers with "no"?

   > **Solution:** As before, define events as answers by individual respondents; Event A is the answer of the 11th respondent; Event B is the answer of the 18th respondent. A and B are not mutually exclusive, therefore you have to use the general OR / addition rule:
   > P(A OR B) = P($\bar{A}$) + P($\bar{B}$) - P($\bar{A}$ AND $\bar{B}$)
   > P(A OR B) = 0.~~102~~ + 0.~~102~~ - 0.~~010404~~ 0.898 + 0.898 − 0.806
   > P(A OR b) = 0.~~193596~~ = 0.989
   > Note: An alternate soluion using the complement is also correct. In this case, P(A OR B) = 1 - NOT (A OR B) = 1 - (P(NOT A) AND P(NOT B)). If this solution is used, the answers to the multiple choice questions should be adjusted accordingly to the simple AND rule and independent events.

   0.990 0.992

   i. (1 point) probability rule ?
      - A. conditinal probabilities
      - B. simple AND/multiplication rule .
      - C. simple OR/addition rule
      - D. general AND/multiplication rule
      - **E. general OR/addition rule**

   ii. (1 point) assumption
      - A. the events are disjoint (mutually exclusive)
      - B. the events are independent
      - C. the events are disjoint and independent
      - D. the law of averages
      - **E. no assumptions are necessary**

(d) **(3 points) 3 out of the first 10 respondents answer with "yes"?**

Solution: Let x be the number of 'yes' answers. As above, the probability p of a 'yes' answer from a single respondent is: 0.102. This means the probability q of a 'no' answer is 0.898
Use the binomial distribution to answer this question:
$P(X=x) = \binom{n}{r} \cdot p^r q^{n-r}$
$P(x=3) = \binom{10}{3} \cdot 0.102^3 0.898^{10-3}$
$P(x=3) = 0.06$ 0.046 0.056
Note: $\binom{n}{r}$ can also be writen as ${}_nC_r$ or $\frac{n!}{r! * (n-r)!}$

(e) **(2 points) first person who answers with yes is person number 15?**

Solution: Use the geometric probability distribution:
$P(X = x) = (1-p)^{(x-1)}p$
$P(x = 15) = (1 - 0.102)^{(15-1)} \times (0.102)$
$P(x = 15) = 0.023$ 0.024 0.023

(f) (1 point) EV

Solution: Use the formula for the binomial distribution: E(X) = np
E(X) = 100 * 0.102 = 10.2 "yes" answers 9.1 9.9

(g) (2 points) Var

Solution: Use the formula for the binomial distribution: $Var(X) = np(1-p)$

$$Var(X) = 100 * 0.102 \times 0.898 = 9.1596 \text{ ("yes" answers)}^2$$

8.27 8.92

2. average life of this type of battery in her toy race car is 11 hours with a standard deviation of 1.8 hours. Total for Question 2: 9

Please answer the following questions:

(a) (3 points) lasts greater than 9 hours

Solution: The battery life is assumed to be normally distributed with N(11, 1.8). The probability to be calculated in this question is $P(x \geq 9)$.
Approach: Transform x to z; then look up probabiliy and subtract from 1.
$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{9-11}{1.8} = -1.111$ -0.909 -0.769
Looking this up in the table, we read $p(z < -1.111) = 0.133$, which is equal to $P(x \leq 9)$. As we were looking for $P(x \geq 9)$, we need to substract this probabiliy from 1 to obtain the final solution:

$$P(x \geq 9) = 1 - 0.133 = 0.867.$$

0.818 0.779

(b) (3 points) top 5% of all batteries (in relation to battery duration, namely X).

> **Solution:** Appoach: Use the table to look up the z value for p=0.05. The z value is 1.645. Use the formula for z to transform z back to x:
> $z = \frac{x-\mu}{\sigma} \Rightarrow x = z * \sigma + \mu \Rightarrow x = 1.645 * 1.8 + 11 \Rightarrow x = 13.961$

17.619 12.138

(c) (3 points) 68/95/99.7 rule

3. he finds out that there are around 1.7 errors per 100 pages. new book with 225 pages. What is the probability that there are more than 3 errors in the book?
.
Total for Question 3: 5

(a) (1 point) Plan:

> **Solution:** This question should be solved using the Poisson distribution. (1Point).
>
> $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$
> To calculate lambda: $\frac{1.7 errors}{100 pages} * \frac{200 pages}{1 book}$ =3.825 errors per book 3.25
>
> $$P(x > 3) = 1 - P(x \leq 3)$$
> $$= 1 - (P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3))$$

3

(b) (3 points) Do:

Solution:

x	P(x)
0	$\frac{e^{-3.825}3.825^0}{0!} = 0.022$
1	$\frac{e^{-3.825}3.825^1}{1!} = 0.083$
2	$\frac{e^{-3.825}3.825^2}{2!} = 0.16$
3	$\frac{e^{-3.825}3.825^3}{3!} = 0.204$
$P(x \leq 3)$	0.468

0.591 0.647

Therefore:

$$P(x > 3) = 1 - P(x \leq 3) = 1 - 0.468 = 0.532$$

0.409 0.353

(c) (1 point) Report:

> **Solution:** The probability that there are more than 3 errors in the book is 0.532.

4. Total for Question 4: 16

X \$	Pr[X]
200	0.50
11000	0.45
40000	0.05

Table 1: Probability Model for Health Expenditures X (in Canadian\$)

(a) (1 point) expenditures of \$10000 or more in any one year?

Solution: For discrete probability models the $\Pr\{X \geq x\}$ are determined obtaining the probability for all events that satisfy that inequality. Here just one event satisfies the inequality.

$$\Pr\{X \geq x\} = \Pr\{X = 10000\} = 0.05$$

0.05 0.05

(b) (1 point) Which of the following best describes the relationship between events **A** and **B**?

A. Independent
B. Independent and disjoint
C. Disjoint
D. Complementary
E. Complementary and disjoint
F. Complementary and independent

(c) (5 points) , calculate the following summaries for the random variable X.

i. **expected value**

Solution: (1.5 point)

$$E[X] = \sum_{1}^{3} X_i \times \Pr[X = x_i]$$
$$= 7050 \text{ dollars}$$

\$ 7850 \$ 7750

where $\mu_x = E[X]$.

ii. **variance**

Solution: (2.5 point)

$$Var[X] = \sum_{1}^{3} (X_i - \mu_x)^2 \times \Pr[X = x_i]$$
$$= 84767500 \text{ dollars}^2$$

$\$^2$ 118382500

$\$^2$ 106007500

where $\sigma_x^2 = Var[X]$.

iii. **coefficient of variation**

Solution: (1 point) Let ν_x denote the coefficient of variation for RV X.

$$\nu_x = \frac{\sigma_x}{\mu_x}$$
$$= 1.31 \text{ unitless}$$

1.39 1.33

(d) (4 points) to represent US dollars (assume that 1 Canadian dollar is worth 0.75 US dollars)

i. **expected value**

Solution: (1 point) Recall rules for operating on RV's ($E[aX] = aE[X]$)

$$E[0.75X] = 0.75E[X] = 5288 \text{ US \$}$$

5495 US\$ 5038 US\$

ii. **variance**

Solution: (2 point) Recall rules for operating on RV's ($Var[aX] = a^2Var[X]$)

$$Var[0.75X] = 0.562Var[X] = 17681720 \text{ US \$}^2$$

58007420 US\$^{2} 44788170 US\$^{2}

iii. **coefficient of variation**

Solution: (1 point) Note that ν_x can be shown to be invariant to any scalar multiple of X. Thus, ν_x will be unchanged whether expressed in CN\$ or US\$.

$$\nu_x = 1.31 \text{ unitless}$$

1.39 1.33

(e) (5 points) just 255 patients.

i. **expected value**

Solution: (1 point)

$$\begin{aligned} E[Y] &= n \times E[X] \\ &= 255 \times 7050 \\ &= 1797750 \text{ \$} \end{aligned}$$

2512000 \$ 3177500 \$

ii. **variance**

Solution: (2 points) Note that X for each individual in the Kapo population is thought to be independent (like the insurance problem)

$$\begin{aligned} Var[Y] &= n \times Var[X] \\ &= 255 \times 84767500 \\ &= 21615712000 \text{ \$}^2 \end{aligned}$$

37882400000 \$^{2} 43463075000 \$

iii. **coefficient of variation**

Solution: (1 point) The coefficent of variation should change (just as it did in the insurance problem)

$$\nu_y = 0.08 \text{ unitless}$$

0.08 0.07

5. low probability, Pr [succeed] = 0.1. that 85 out of every 100 successful ventures would have scored *positive* (at start-up), whereas only 10 of every 100 failed companies would have scored *positive* (at start-up).

	positive	negative	Total
Succeed	850	150	1000
Fail	900	8100	9000
Total	1750	8250	10000

(a) (4 points) **Solution:** Must give some explanation for how the filled in the table for full marks. We note that 85 out of every 100 ventures that succeed would test positive, and thus 850 of every 1000 successful ventures will be positive. This fills in the top-left cell of the contingency table. Similar logic can be used to obtain one other joint-event count, and then simple deduction (tantamount to the complement rule) can help fill in the remainder of the contingency table.

(b) (3 points) Consider a company that has scored *positive* (predicted to succeed). Calculate the probability that it will …

i. …**succeed**

Solution: (2 points) Take expression for conditional probability and use it to obtain appropriate conditional probability

$$\begin{aligned} \Pr[\text{succeed} \mid \text{positive}] &= \frac{\Pr[\text{succeed} \cap \text{positive}]}{\Pr[\text{positive}]} \\ &= \frac{0.085}{0.175} \\ &= 0.486 \end{aligned}$$

0.321 0.088

ii. …**fail**

Solution: (1 point) Exploit the complement rule to obtain answer based on the conditional probability for success.

$$\begin{aligned} \Pr[\text{failure} \mid \text{positive}] &= 1 - \Pr[\text{succeed} \mid \text{positive}] \\ &= 1 - 0.486 \\ &= 0.514 \end{aligned}$$

0.679 0.912

(c) (4 points) What is the probability that at least one of the 10 will prove sucessfull?

Solution: Look at A2 part II question 16. Similar steps are used here. (1 mark can be allocated to each of the four lines of this solution that follows)

$$\begin{aligned} \Pr[\text{At least one S}] &= 1 - \Pr[10 \text{ F's}] \\ &= 1 - (1 - \Pr[S \mid \text{positive}])^{10} \\ &= 1 - 0.001 \\ &= 0.999 \end{aligned}$$

0.979 0.6

(d) (2 points) The probability that they will get an award?

Solution: Make use of the general addition rule and a frequency version of the contingency table to count (but not double count) all events that are compatible with getting an award. An alternative solution is to take the complement of the only joint event (fail and negative) that would not earn an venture an award (namely failing and scoring negative).

$$\begin{aligned} \Pr[\text{award}] &= 1 - \Pr[\text{fail and negative}] \\ &= 1 - 0.81 \\ &= 0.19 \end{aligned}$$

0.145

0.109