**Econ 496, Natural Resource Economics**
**Winter 2012**
**Suggested answers for Assignment 1**

1. (a) For Project 1 we get

| **Year** | **Cost** | **Benefit** | PV of Cost | PV of Benefits |
|---|---|---|---|---|
| **0** | \$15 | \$0 | \$15 | \$0 |
| **1** | - | \$1 | - | $1/(1+0.05) = 0.666\,67$ |
| **2** | - | \$2 | - | $2/(1+0.05)^2 = 1.814\,1$ |
| **3** | - | \$2 | - | $2/(1+0.05)^3 = 1.727\,7$ |
| **4** | - | \$3 | - | $3/(1+0.05)^4 = 2.468\,1$ |
| **5** | - | \$3 | - | $3/(1+0.05)^5 = 2.350\,6$ |
| **6** | - | \$4 | - | $4/(1+0.05)^6 = 2.984\,9$ |
| **7** | - | \$4 | - | $4/(1+0.05)^7 = 2.842\,7$ |
| **8** | - | \$3 | - | $3/(1+0.05)^8 = 2.030\,5$ |
| **9** | - | \$2 | - | $2/(1+0.05)^9 = 1.289\,2$ |

*Table 1: Project 1*

Adding all the benefits we get

$$PVB = 0.666\,67 + 1.8141 + 1.277 + 2.4681 + 2.3506 + 2.9849 + 2.8427 + 2.0305 + 1.2892 = \$17.7238,$$

while the costs are just

$$PVC = \$15$$

Therefore, the Net Value of Project 1 is

$$PVNB_1 = PVB - PVC = 17.7238 - 15 = \$2.7238,$$

while the BCR of Project 1 is

$$BCR_1 = PVB/PVC = 17.7238/15 = 1.181\,6$$

For Project 2 we get:

| **Year** | **Cost** | **Benefit** | PV of Cost | PV of Benefits |
|---|---|---|---|---|
| **0** | \$60 | \$0 | \$60 | \$0 |
| **1** | - | \$10 | - | $10/(1+0.05) = 9.523\,8$ |
| **2** | - | \$15 | - | $15/(1+0.05)^2 = 13.605$ |
| **3** | - | \$20 | - | $20/(1+0.05)^3 = 17.277$ |
| **4** | - | \$20 | - | $20/(1+0.05)^4 = 16.454$ |
| **5** | - | \$15 | - | $15/(1+0.05)^5 = 11.753$ |

*Table 2: Project 2*

Adding all the benefits we get

$$PVB = 9.5238 + 13.605 + 17.277 + 16.454 + 11.753 = \$68.613$$

while the costs are just

$$PVC = \$60$$

Therefore, the Net Value of Project 2 is

$$PVNB_2 = PVB - PVC = 68.613 - 60 = \$8.613,$$

while the BCR of Project 2 is

$$BCR_2 = PVB/PVC = 68.613/60 = 1.1436$$

(b) Since Environment Canada can only undertake one project, the two projects are **mutually exclusive**. Therefore, it will choose the one that gives the highest Net Value. Project 1 has a net value of \$2.7238 while Project 2 has a net value of \$8.613. Therefore, Environment Canada will accept Project 2.

(c) Since Environment Canada can undertake any project or combination of projects (just to spend the entire budget) it should base its decision on BCR instead. By comparing the BCRs we derived in part (a), that should be Project 1. To understand that, we see that given the \$40 million budget, Environment Canada can:

- built $40/15 = 8/3$ of Project 1, and, therefore, receive a net value of $(8/3) \times 2.7238 = 7.2635$.
- built $40/60 = 2/3$ of Project 2, and, therefore, receive a net value of $(2/3) \times 8.613 = 5.742$.

2. It is always useful to present the benefits and costs in the form of a table. Therefore, for discount rate $r = 0.05$ we get

| Year | Cost | Benefit | PV of Cost | PV of Benefits |
|---|---|---|---|---|
| **0** | \$7 | \$0 | \$7 | \$0 |
| **1** | \$5 | \$0 | $5/(1+0.05) = 4.7619$ | \$0 |
| **2** | \$2 | \$0 | $2/(1+0.05)^2 = 1.8141$ | \$0 |
| **3** | \$0.3 | \$2.8 | $0.3/(1+0.05)^3 = 0.25915$ | $2.8/(1+0.05)^3 = 2.4187$ |
| **4** | \$0.3 | \$2.8 | $0.3/(1+0.05)^4 = 0.24681$ | $2.8/(1+0.05)^4 = 2.3036$ |
| **5** | \$0.3 | \$2.8 | $0.3/(1+0.05)^5 = 0.23506$ | $2.8/(1+0.05)^5 = 2.1939$ |
| **6** | \$0.3 | \$2.8 | $0.3/(1+0.05)^6 = 0.22386$ | $2.8/(1+0.05)^6 = 2.0894$ |
| **7** | \$0.3 | \$2.8 | $0.3/(1+0.05)^7 = 0.2132$ | $2.8/(1+0.05)^7 = 1.9899$ |
| **8** | \$0.3 | \$2.8 | $0.3/(1+0.05)^8 = 0.20305$ | $2.8/(1+0.05)^8 = 1.8952$ |
| **9** | \$0.3 | \$2.8 | $0.3/(1+0.05)^9 = 0.19338$ | $2.8/(1+0.05)^9 = 1.8049$ |
| **10** | \$0.3 | \$2.8 | $0.3/(1+0.05)^{10} = 0.18417$ | $2.8/(1+0.05)^{10} = 1.7190$ |
| **11** | \$0.3 | \$2.8 | $0.3/(1+0.05)^{11} = 0.1754$ | $2.8/(1+0.05)^{11} = 1.6371$ |
| **12** | \$0.3 | \$2.8 | $0.3/(1+0.05)^{12} = 0.16705$ | $2.8/(1+0.05)^{12} = 1.5591$ |

Table A: discount rate $r = 5\%$

while for discount rate $r = 0.10$ we get

| Year | Cost | Benefit | PV of Cost | PV of Benefits |
|---|---|---|---|---|
| **0** | \$7 | \$0 | \$7 | \$0 |
| **1** | \$5 | \$0 | $5/(1+0.1) = 4.5455$ | \$0 |
| **2** | \$2 | \$0 | $2/(1+0.1)^2 = 1.6529$ | \$0 |
| **3** | \$0.3 | \$2.8 | $0.3/(1+0.1)^3 = 0.22539$ | $2.8/(1+0.1)^3 = 2.1037$ |
| **4** | \$0.3 | \$2.8 | $0.3/(1+0.1)^4 = 0.2049$ | $2.8/(1+0.1)^4 = 1.9124$ |
| **5** | \$0.3 | \$2.8 | $0.3/(1+0.1)^5 = 0.18628$ | $2.8/(1+0.1)^5 = 1.7386$ |
| **6** | \$0.3 | \$2.8 | $0.3/(1+0.1)^6 = 0.16934$ | $2.8/(1+0.1)^6 = 1.5805$ |
| **7** | \$0.3 | \$2.8 | $0.3/(1+0.1)^7 = 0.15395$ | $2.8/(1+0.1)^7 = 1.4368$ |
| **8** | \$0.3 | \$2.8 | $0.3/(1+0.1)^8 = 0.13995$ | $2.8/(1+0.1)^8 = 1.3062$ |
| **9** | \$0.3 | \$2.8 | $0.3/(1+0.1)^9 = 0.12723$ | $2.8/(1+0.1)^9 = 1.1875$ |
| **10** | \$0.3 | \$2.8 | $0.3/(1+0.1)^{10} = 0.11566$ | $2.8/(1+0.1)^{10} = 1.0795$ |
| **11** | \$0.3 | \$2.8 | $0.3/(1+0.1)^{11} = 0.10515$ | $2.8/(1+0.1)^{11} = 0.98138$ |
| **12** | \$0.3 | \$2.8 | $0.3/(1+0.1)^{12} = 0.09559$ | $2.8/(1+0.1)^{12} = 0.89217$ |

Table B: discount rate $r = 10\%$

(a) Adding all the benefits in table A we get

$$PVB = 19.611$$

while adding the costs in the same table yields

$$PVC = 15.677$$

Therefore, the Net Value of the project is

$$PVNB = PVB - PVC = 19.611 - 15.677 = \$3.934\ ,$$

while the BCR of the project is

$$BCR = PVB/PVC = 19.611/15.677 = 1.2509$$

(b) Adding all the benefits in table B we get

$$PVB = 14.2188$$

while adding the costs in the same table yields

$$PVC = 14.72119$$

Therefore, the Net Value of the project is

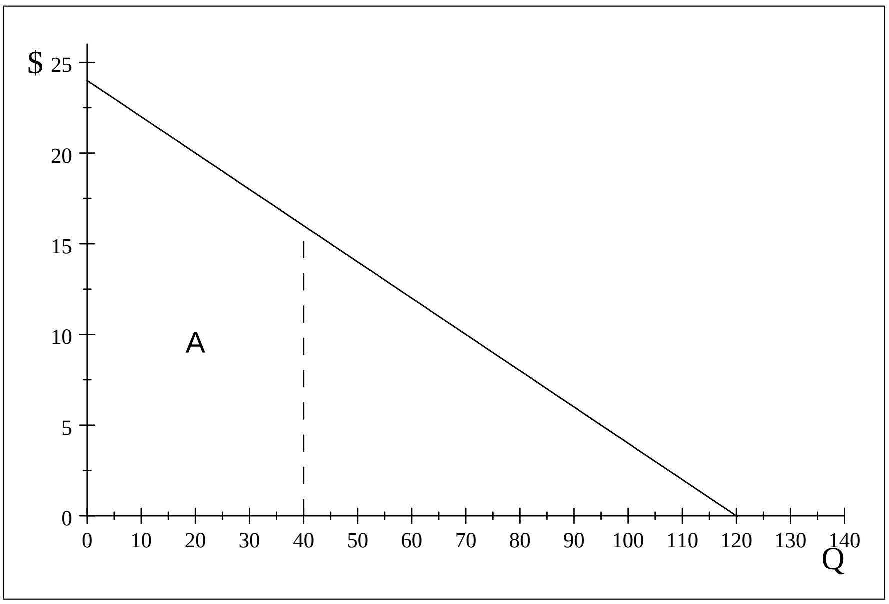
$$PVNB = PVB - PVC = 14.2188 - 14.72119 = -\$0.50296,$$

while the BCR of the project is

$$BCR = PVB/PVC = 14.2188/14.72119 = 0.965\,87$$

We observe that the result of our analysis is completely different (with respect to accepting or rejecting the project). More specifcally, we accept the project for $r = 5\%$ while we clearly rejected for $r = 10\%$. That kind of sensitivity of the result to the discount rate calls for extra caution when to make a decision (one should analyze furthermore the possible benefits and costs, and perhaps should wait before making any decision).

3. (a) Graphically, the marginal $WTP$ for the 40th unit is the length of the vertical line at $q = 40$, while the total $WTP$ is the area $A$.




When $q = 40$ the marginal willingness to pay is represented by the demand price, i.e.,

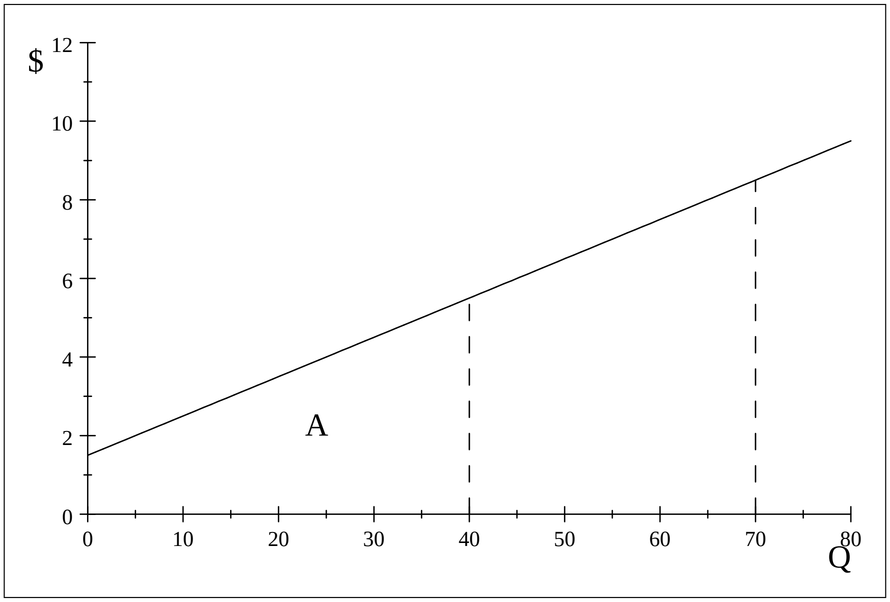
$$40 = 120 - 5P \Rightarrow P = 16 = \text{marginal } WTP.$$

The total willingness to pay is represented by the area under the demand (marginal willingness to pay, or marginal benefit). Therefore,

$$\text{total } WTP = (16 + 24) \times 40 \times 1/2 = 800$$

*Note that we have used the formula for the area of a trapezoid (so, the value of the long base is the marginal WTP=16).*

(b) Graphically, the marginal $WTA$ for the 70th unit is the length of the vertical line at $q = 70$, while the total $WTP$ is the area $A$.




When $q = 70$ the marginal willingness to accept is represented by the supply price, i.e.,

$$70 = 10P - 15 \Rightarrow P = 8.5 = \text{marginal } WTA.$$

The total willingness to accept is represented by the area under the demand for $q = 40$. Therefore,

$$\text{total } WTP = (1.5 + 5.5) \times 40 \times 1/2 = 140$$

*Note that we have used the formula for the area of a trapezoid (so, we needed to find the marginal WTA for $q = 40$).*

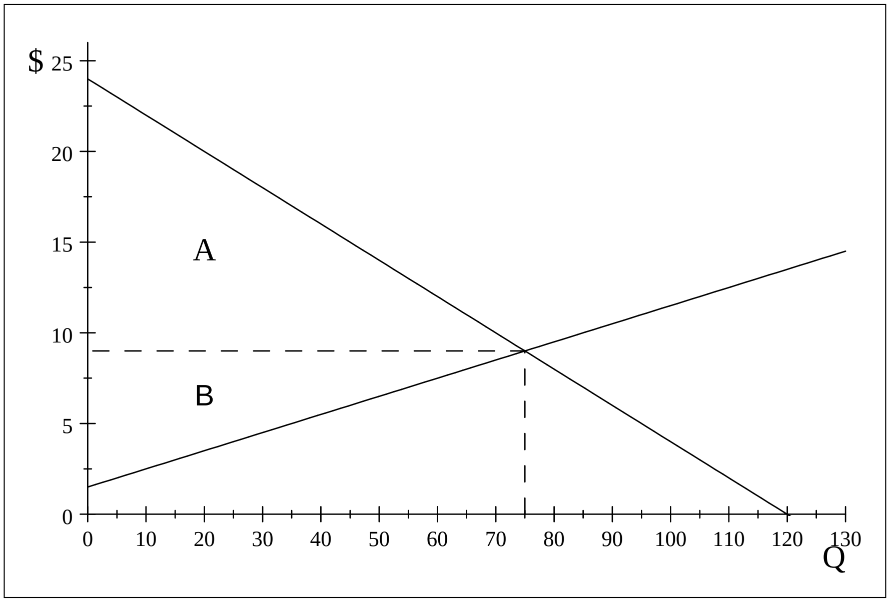
(c) Equating demand (marginal benefit) and supply (marginal cost) yields

$$120 - 5P = 10P - 15 \Rightarrow P = \$9.$$

Therefore, the optimal quantity will be

$$q = 120 - 5(9) = 75.$$

In the graph below the consumer surplus is represented by area $A$ while the producer rsurplus is area $B$.




Therefore, we have

$$CS = (24 - 9) \times 75 \times 1/2 = 562.5,$$

and

$$PS = (9 - 1.5) \times 75 \times 1/2 = 281.25.$$

This solution is efficient according to the *first equimarginal principal: "to maximize the difference between total benefits and total costs, find the quantity that equates marginal benefit to marginal costs."*