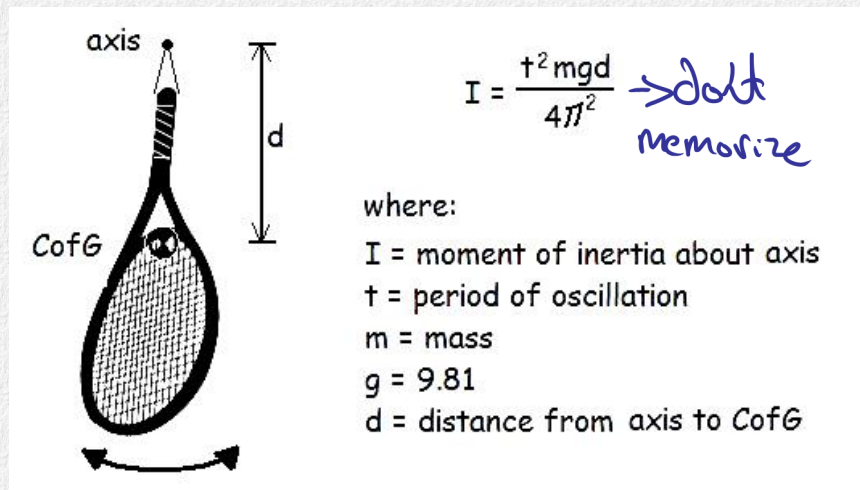


Pendulum Technique

In a previous lecture ([anthropometry](#)) we used a "look-up" table to calculate the segment masses, locations of the center of mass(CM) and moments of inertia of human limb segments. It was explained how mass could be determined with a scale and the CM could be determined by balancing the segment on a knife edge. The moment of inertia (I) determination was not explained. In this lecture we start with an explanation of the Pendulum Technique which is a valuable method for cadaveric segments as well as other objects such as prosthetic limbs, tennis rackets or any other artificial device used in human motion.

We can determine the moment of inertia of any object that is suspended and allowed to swing through a small range (less than 10°). The period of oscillation is dependent on the moment of inertia, the mass of the object, the distance the axis is from the CM and gravity.



Once we know I about the swinging axis, we can use the parallel axis theorem to determine the moment of inertia about any other point and also the radius of gyration (k).

Example: Find the moment of inertia of the tennis racket.

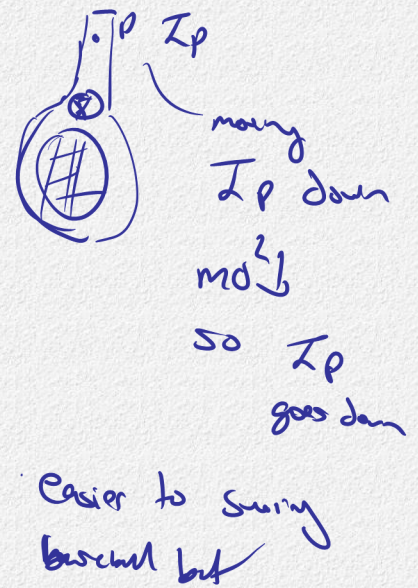
Given: mass = 0.45 kg, t = 0.91 seconds, d = 0.27 m

$$\begin{aligned}
 I &= \frac{t^2 m g d}{4\pi^2} \\
 &= \frac{(0.91)^2 (0.45)(9.81)(0.27)}{4(3.14159)^2} \\
 &= 0.025 \text{ kg.m}^2
 \end{aligned}$$

Solution:

$$I_p = I_{cm} + md^2 \rightarrow m \cdot k^2$$

$$\frac{I_p - md^2}{m} = k^2$$



Now that we know the moment of inertia of the tennis racket is 0.025 kg.m², what is the moment of inertia about an elbow if which is 0.20m further away and the shoulder which is 0.43m away from the axis in the diagram?

Living Subject Techniques

Geometric modeling: Hanavan model and Jensen's elliptical models

1) Hanavan Model

The human body has been simplified to a system of standard geometric solids of constant density in order to determine the location of center of mass and moments of inertia. The diagram to the right shows the model created by Hanavan that consisted of cylinders, spheres, and frustra (tapered cylinders). *(NOT CONES)*

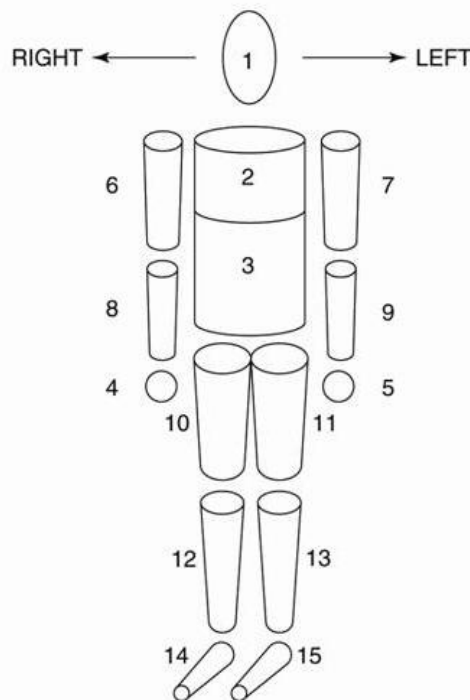


Figure 2.12

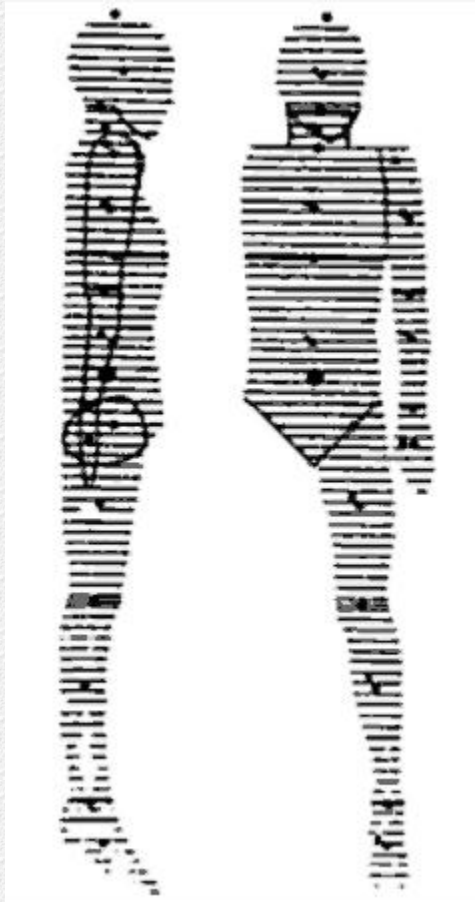
The Mathematical Model From Hanavan, AMRL-TR-64-102, or from Miller and Morrison (1975) *Med and Sci in Sports* 7 (3): 207.

One advantage of these models is that they are mathematically easy to implement and they can be scaled to individuals of different proportions by using segment lengths rather than whole body height and weight.

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His method requires two orthogonal images that will yield major axis of ellipses that can be separated into individual segments. The volume to be multiplied by an assumed density to yield mass similar to the Hannaven model of segments

2) Elliptical Models



Bob Jensen used stereo photogrametry to create elliptical models in order to estimate anthropometric variables of children.

Dempster would work

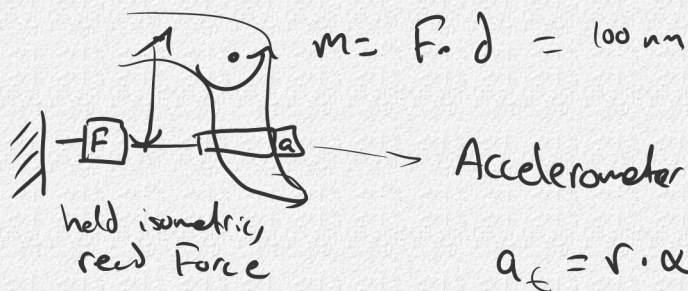
3) Reaction Change Method: This method was covered on the previous page.

4) Water Immersion Method: The volume of human limb segments can be measured by immersing them in water and measuring the amount of water displaced. This volume can be converted to mass

by multiplying the volume by a density estimate.



This method was first used by Harlis in the 1850's and can also yield the location of the center of gravity if density can be assumed to be evenly distributed throughout the segment.

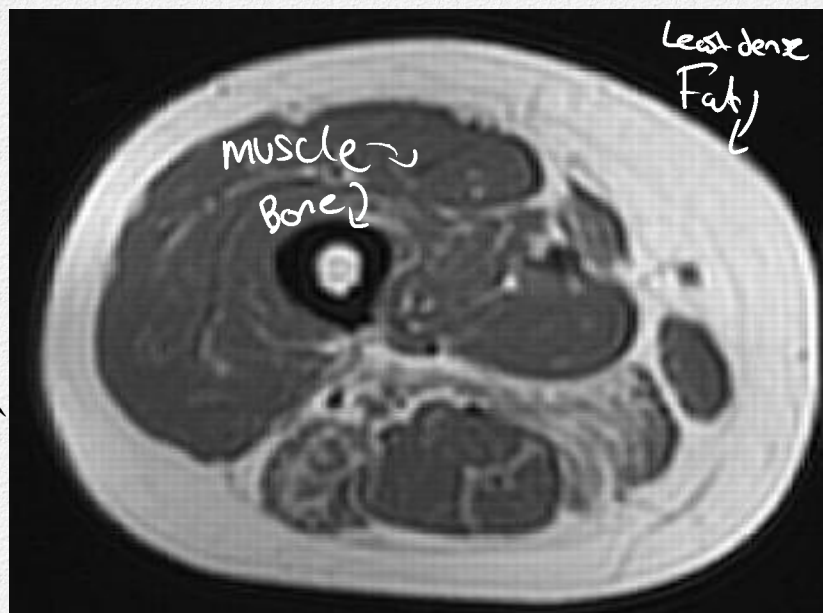


$$M_{\text{limb}} = I \cdot \alpha$$

5) Quick Release Method: The moment of inertia of a human limb segment can be calculated by placing an accelerometer on the limb and securing the limb with a cable attached to a force transducer. The subject contracts the muscles against the force transducer which records the applied moment of force. The limb is then released which causes it to accelerate due to the muscle forces. The force transducer is multiplied by the moment arm to get the applied moment and it is then divided by the tangential acceleration to yield the moment of inertia.

6) Medical Imaging

CT scan:



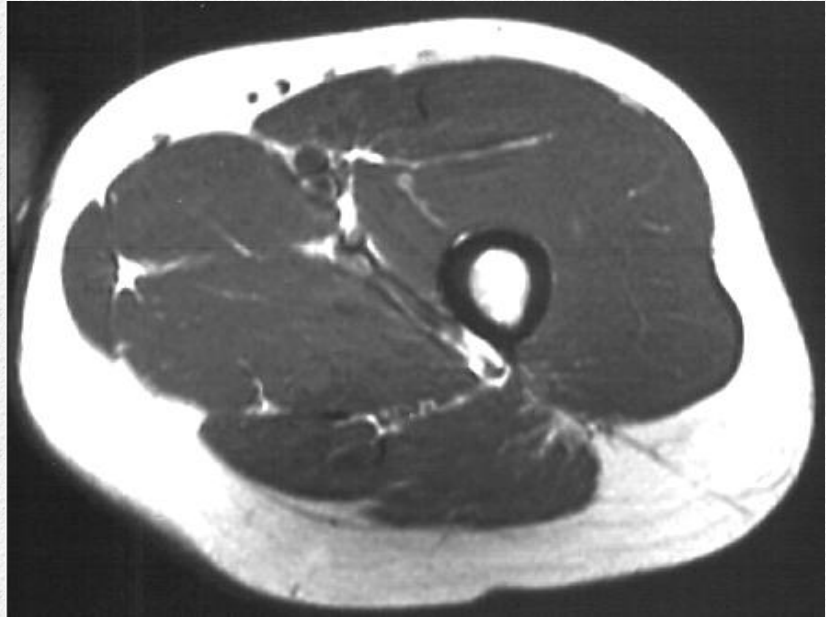
Water immersion Δ

Jensen's ellipsoid would have to make assumption of constant composition

density

here we get a better idea of distribution of densities

MRI scan:



Dual X-ray
Absorptiometry (DXA):

- most practical to use on
living subjects

DXA machines are
traditionally used to
estimate bone density.

Patients are scanned by
two photon beams of
different power. As the
beams pass through the tissues, their energies are absorbed and a
calculation can be made regarding the amount (mass) of tissue as well as
the composition of bone, fat and lean tissue. Dowling, Durkin, et al. have

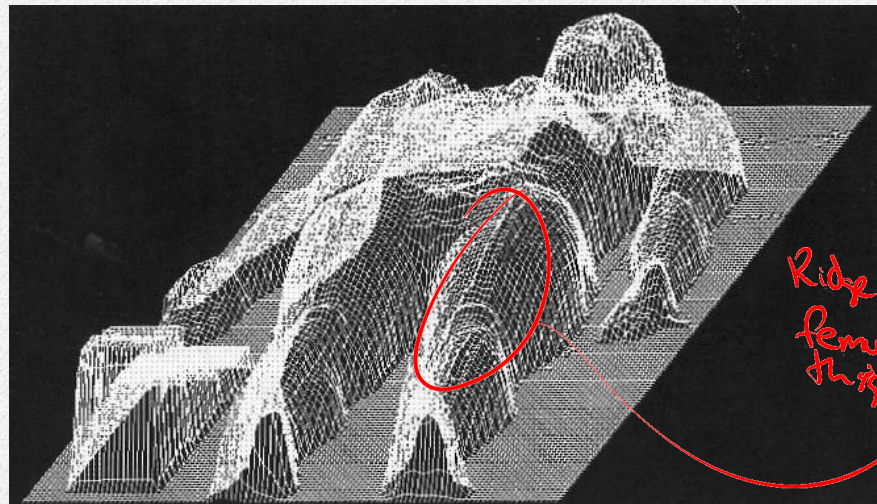


for ppl w/
osteoporosis

shown that the scanned elements can be plotted in a "mass map", in which the height of each element is proportional to the amount of mass (see lower figure).

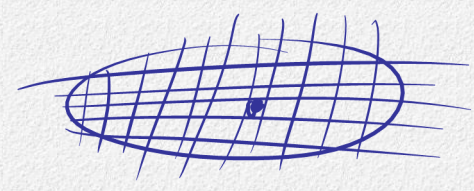
The mass map can be divided digitally in a similar way to a cadaver dissection and the mass, center of mass and moment of inertia can be determined for each body segment.

The procedure exposes the subject to a low dose of radiation and is very accurate. However, the machines are expensive and the process is quite time consuming. Currently, this method is mostly been used in research to build more accurate elliptical models.



Ridge showing femur or tibia

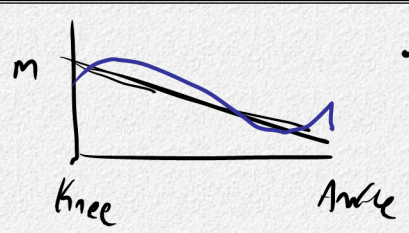
You can digitally cut the body into segments




She looked for data that made Dempster's data was off, but Dempster's tables weren't too far off in actuality.

The DXA scan is easier to use but hard to book, is it worth it to do all that for data that is a little bit better than Dempster's tables?

We can just use Hanavan model
used tapered cones



• Hanavan - assumes constant density → 
 • DXA
 ↳ reality density is not constant
 ex: ankle joint