24/30 = 80 %

School of Mathematics and Statistics
Carleton University
Math. 2004A, Fall 2016
TEST 3

Only calculators are permitted, 1 or more blank sheets permitted for roughs

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Tutorial Section (A1, A4, ...): A4

PART I: Multiple Choice Questions

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(Choose and CIRCLE only ONE answer - No part marks here.)

2 1. **[2 marks]** Let $f(x,y) = e^{2x+y^2}$. Find the mixed partial derivative $f_{xyy}(0,0)$
(a) 4e, (b) 4, (c) 1, (d) 2 (e) None of these

2 2. **[2 marks]** Find the directional derivative of $f(x,y,z) = \sqrt{xyz}$ at $(2,4,2)$ in the direction of $(4,2,-4)$.
(a) $\frac{1}{6}$, (b) $-\frac{1}{4}$, (c) $-\frac{1}{2}$, (d) 0. (e) None of these.

→ 3. **[2 marks]** A surface S in $\mathbf{R}^3$ has the equation $z \sin z + xy^3 = 0$. Assume that z may be written as a function of (x,y) near the point $P(x,y,z) = (0,1,\pi)$. Evaluate the partial derivative $\frac{\partial z}{\partial x}(P)$.

2 (a) π, (b) 0, (c) -1, (d) $\frac{1}{\pi}$ (e) None of these.

4. **[2 marks]** Find the equation of the **normal plane** to the curve C defined parametrically by $\mathbf{r}(t) = (t - \sin t, 1 - \cos t, 4\sin(t/2))$ at the value of t corresponding to $P(\pi/2 - 1, 1, 2\sqrt{2})$ in rectangular coordinates.

2 (a) $x + y + \sqrt{2}\,z = 4$ (b) $x + \sqrt{2}y + z = \pi/2$, (c) $x + y + \sqrt{2}\,z = 4 + \pi/2$, (d) $2x + y + z = 4$
(e) None of these

5. **[2 marks]** Which of the following vector fields is conservative?
(a) $\mathbf{F}(x,y) = (x\cos y, x\sin y)$, (b) $\mathbf{F}(x,y) = (x\sin y, -x\cos y)$, (c) $\mathbf{F}(x,y) = (x\sin y, y\cos x)$,
(d) $\mathbf{F}(x,y) = (y\sin x, x\cos y)$, (e) None of these are conservative.

6. **[2 marks]** The divergence of a vector field is a scalar quantity:

2 (a) TRUE (b) FALSE. Dot product $\nabla \cdot \vec{F}$ → ∴ scalar

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PART II: Show all work ...

6. [2 marks] The divergence of a vector field is a scalar ... $\nabla\cdot$ of gradient ... scalar

2 (a) TRUE (b) FALSE.

PART II: Show all work here and give details.
No additional pages will be accepted

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8 6. [5 + 3 marks] a) Let $f(x,y) = x^2y^3$, $x = 2s+t$, $y = 2s - t$. Evaluate $\frac{\partial f}{\partial s}$ at the point $(s,t) = (0,1)$.

b) Let $\mathbf{F}(x,y) = (3x^2y^4, 4x^3y^3)$. Find a scalar function f such that $\nabla f = \mathbf{F}$.

P Q

(a) 5

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\cdot\frac{dx}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{dy}{ds}$$

$$\frac{\partial f}{\partial s} = (2xy^3)(2) ✓ + (3x^2y^2)(2) ✓$$

$$\frac{\partial f}{\partial s} = (2(2s+t)(2s-t)^3)(2) + (3(2s+t)^2(2s-t)^2)(2)$$

At $(s,t)=(0,1)$

$$\frac{\partial f}{\partial s}(0,1) = (2(2(0)+1)(2(0)-1)^3)(2) + (3(2(0)+1)^2(2(0)-1)^2)(2)$$

$$\frac{\partial f}{\partial s}(0,1) = (2)(1)(-1)(2) + (3)(1)(1)(2)$$

$$\frac{\partial f}{\partial s}(0,1) = -4+6 \boxed{=2} ✓$$

3 (b) $\overline{F}(x,y) = (P,Q)$ $P = 3x^2y^4$ ✓, $Q = 4x^3y^3$ ✓

$f_x = P \rightarrow f(x,y) = \int P\,dx$

$= \int 3x^2y^4\,dx$

$f(x,y) = x^3y^4 + g(y)$ ✓ ①

↓

$Q = 4x^3y^3$

$f(x,y) = x^3y^4 + g(y)$

Finding $g(y)$:

$f_y = Q$

$\Rightarrow 4x^3y^3 + g'(y) = 4x^3y^3$

$\Rightarrow g'(y) = 0 \Rightarrow g(y) = 0$ (2)

2

(2) $\rightarrow$ (1): $\boxed{f(x,y) = x^3y^4 + C}$ ✓

7. [5+5 marks] a) Let $f(x,y,z) = xyz$ and let C be the line segment joining $(0,0,0)$ to $(1,1,1)$. Evaluate the line integral $\int_C xyz\, ds$.

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b) By evaluating the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the line segment from $(0,0,0)$ to (x,y,z) find a potential function f such that $\nabla f = \mathbf{F}$ where $\mathbf{F}(x,y,z) = (2xy, z+x^2, y)$.

(a) Parametrize the line segment: $P = (0,0,0)$ $Q = (1,1,1)$ $\to \overline{OP} = (0,0,0)$ $\to \overline{OQ} = (1,1,1)$

3 $r: (1-s)\overline{OP} + \overline{OQ}\, s$

$= (1-s)(0,0,0) + (1,1,1)s$

$= (s,s,s) = (x,y,z)$ $\quad \underline{0 \le s \le 1}$

$x = s, \quad y = s, \quad z = s$ $\quad \times\sqrt{1^2+1^2+1^2}$

$\Rightarrow \int_C xyz\, ds = \int_0^1 (s)(s)(s)\, ds = \int_0^1 s^3 ds$

$= \frac{s^4}{4}\Big|_0^1 = \frac{1}{4} \Rightarrow \boxed{\int_C xyz\, ds = \frac{1}{4}}$

(b) $F(x,y,z) = (2xy, z+x^2, y) = (P,Q,R)$ $\quad P = 2xy,\ Q = z+x^2,\ R = y$

3 $f_x = P \Rightarrow f(x,y) = \int P\, dx$

$f(x,y) = \int 2xy\, dx = x^2y + g(y)$ ①

Finding $g(y)$:

$f_y = Q$

$x^2 + g'(y) = z + x^2$

$g'(y) = z$

$g(y) = yz$ ②

-2 use the line integral method

② → ①: $\boxed{f(x,y) = x^2y + yz + C}$ ✓