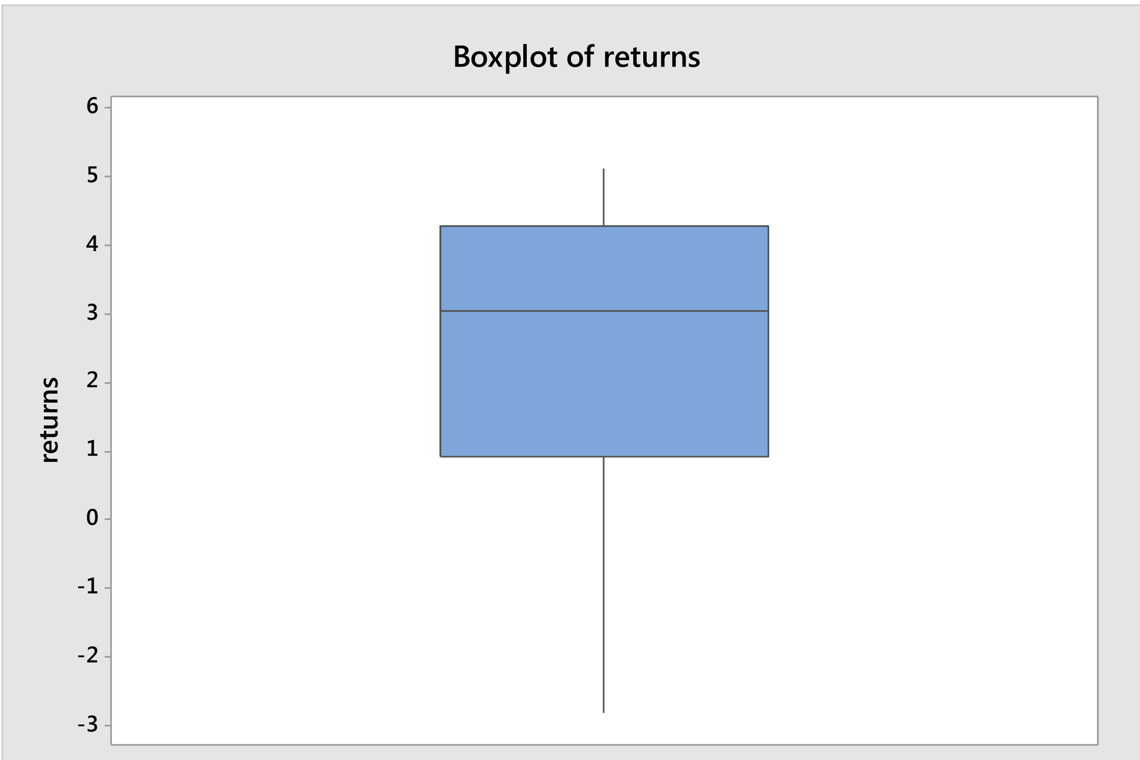
ADM 2304 Assignment 2 Solutions

Question 1. [10 marks]

(a) [2]

This is a small sample which is not symmetric. It suggests that the population is not normally distributed, which assumption is necessary when the CLT does not apply. A non-parametric test is more appropriate.



1 mark for commenting on the skewness in the data which suggests the returns are not normally distributed; 1 mark for saying the non-parametric test is more appropriate.

(b) [4]

Wilcoxon Signed Rank Test: returns

```
Test of median = 0.000000 versus median > 0.000000

              N for   Wilcoxon           Estimated
           N   Test  Statistic        P     Median
returns   10     10       50.0    0.012      2.545
```

The sum of ranks for the positive returns is 50.

The sum of ranks for the negative returns is 10*(10+1)/2 – 50 = 55 – 50 = 5. Note that there are two negative returns, one with rank 1 and the other with rank 4.

-Ho: median = 0; Ha: median > 0 *(1 mark)*

(comment: Under Ha, we expect T- to be small and T+ to be large.)

-The Wilcoxon statistic of is T = T- = 5 *(1 mark)*

- rejection region is $T \leq 11$ *(1 mark)*

-We reject Ho and conclude the median return is positive. (*1 mark—0.5 for decision, 0.5 for conclusion)*

(c) [3]

E(T) = n(n+1)/4 = 10(11)/4 = 27.5 *(0.5 mark)*

Var(T) = n(n+1)(2n+1)/24 = 10(11)(21)/24 = 96.25 *(0.5 mark)*

Z = (5 – 27.5)/sqrt(96.25) = -22.5 / 9.81 = -2.29 *(1 mark)*

p-value is P(Z < -2.29) = 0.011 *(1 mark)*

(d) [1]

The approximate p-value (0.011) from the normal approximation is very close to the exact p-value of 0.012 from Minitab.

Question 2. [13]

(a) [6]

Two-Sample T-Test and CI: BMImale, BMIfemale

```
Two-sample T for BMImale vs BMIfemale

            N   Mean  StDev  SE Mean
BMImale    46  26.03   3.23     0.48
BMIfemale  35  24.77   4.60     0.78


Difference = mu (BMImale) - mu (BMIfemale)
Estimate for difference:  1.26186
95% lower bound for difference:  -0.26137
T-Test of difference = 0 (vs >): T-Value = 1.38  P-Value = 0.086  DF = 58
```

Ho: mu(male) – mu(female) = 0; Ha: diff > 0

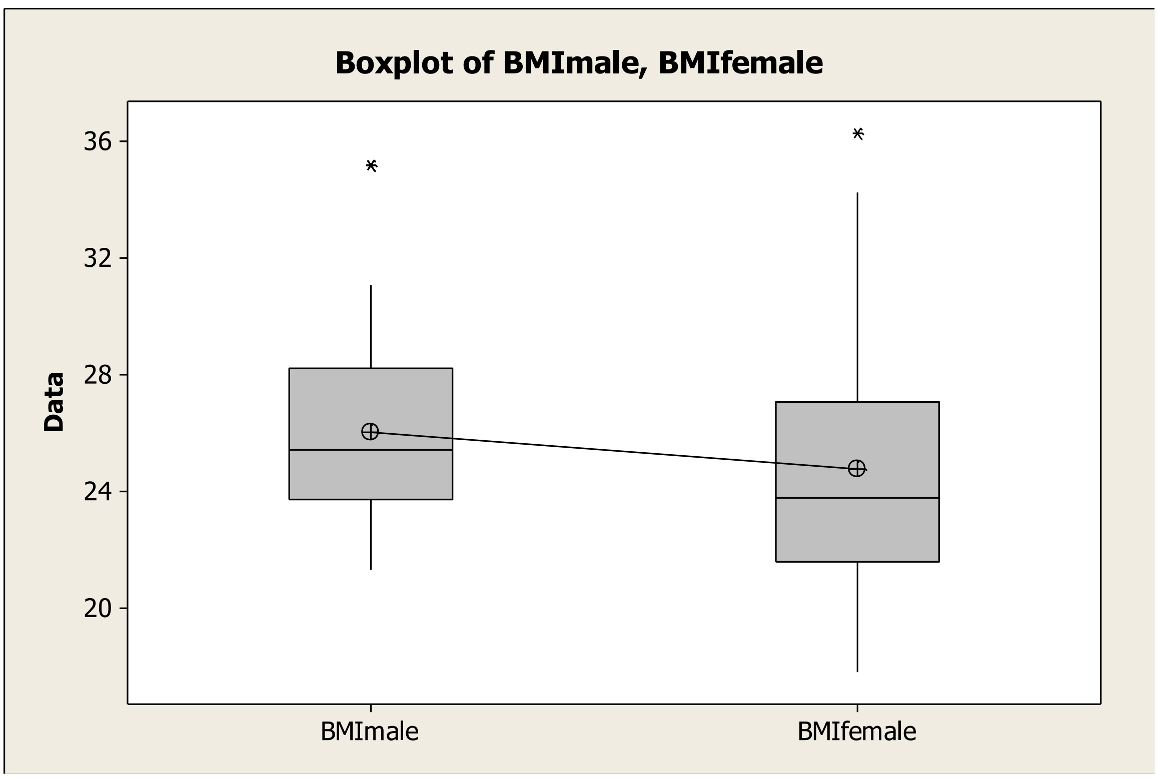
SE is sqrt(3.23^2/46 + 4.6^2/35) = 0.9118.

T = 1.26 / 0.9118 = 1.38

Since the rejection region is t > 1.645, we cannot reject the null hypothesis and conclude that there is insufficient evidence to show the mean male BMI is higher.

Although there is some skewness and one outlier in each sample, it is reasonable to assume that each sample comes from a population that is not extremely skewed. Therefore the 2-sample t-test is appropriate.

-1 for hypotheses, 2 for showing the proper manual calculation of the t-statistic, 1 for rejection region, 1 for decision and conclusion, and 1 for commenting on the skewness/outlier but the reasonableness of the assumption that the population is not extremely skewed.



(b) [1]

Using the normal approximation, the p-value = P(Z > 1.38) = P(Z < -1.38) = 0.0838

(c) [2]

95% 1-sided CI is LB of 1.26 - 1.645 * sqrt(3.23^2/46 + 4.6^2/35)

= 1.26 - 1.645 * 0.9118 = 1.26 - 1.50 = -0.24, using the normal approximation.

The LB is 1.26 – 1.52 = -0.26 using the t-value of 1.67.

-2 marks for finding the correct LB of -0.24 or -0.26—deduct 0.5 for a critical value outside the 1.6 to 1.7 range, deduct 0.5 for wrong SE, deduct 0.5 for wrong mean, deduct 0.5 for 2-sided CI.

(d) [4]

Mann-Whitney Test and CI: BMImale, BMIfemale

```
            N  Median
BMImale    46  25.450
BMIfemale  35  23.800


Point estimate for η1 - η2 is 1.700
95.1 Percent CI for η1 - η2 is (0.200,3.300)
W = 2117.0
Test of η1 = η2 vs η1 > η2 is significant at 0.0140
The test is significant at 0.0140 (adjusted for ties)
```

Mann-Whitney Test and CI: BMIfemale, BMImale

```
            N  Median
BMIfemale  35  23.800
BMImale    46  25.450


Point estimate for η1 - η2 is -1.700
95.1 Percent CI for η1 - η2 is (-3.300,-0.200)
W = 1204.0
Test of η1 = η2 vs η1 < η2 is significant at 0.0140
The test is significant at 0.0140 (adjusted for ties)
```

Ho: no difference in population medians; Ha: median BMI (males) > median BMI (females)

The sum of ranks for the male sample is 2117, and the sum of ranks for the female sample is 1204.

To do the test manually using the M-W table, we would use the sum of ranks for the smaller sample which is 1204. However, since the M-W table does not accommodate the larger sample sizes in this problem, we use the normal approximation to the M-W statistic.

$Z = [T_i – E(T_i)]/\sqrt{[V(T_i)]}$

where $E(T_i) = n_i * (n_1 + n_2 + 1) / 2$

and $Var(T_i) = n_1 * n_2 * (n_1 + n_2 + 1) / 12$.

E(T) = 35(35+46+1)/2 = 1435

Var(T) = 35(46)(35+46+1)/12 = 11001.66667

Z = (1204 – 1435)/sqrt(11001.6667) = -231/104.8888 = -2.20

Here we reject the null hypothesis if Z < -1.645, since the alternative hypothesis favours a smaller sum of ranks for the female BMI sample.

p-value is P(Z < -2.2) = 0.0139.

This is very close to the p-value of 0.0140 from Minitab.

We can also use the other sum of ranks for the larger sample which is 2117.

Here E(T) = 46(35+46+1)/2 = 1886

Z = (2117 – 1886) / 104.8888 = 231 / 104.8888 = 2.20

Here we reject the null hypothesis if Z > 1.645, since the alternative hypothesis favours a larger value for the sum of ranks for the male BMi sample.

p-value is P(Z > 2.2) = 0.0139.

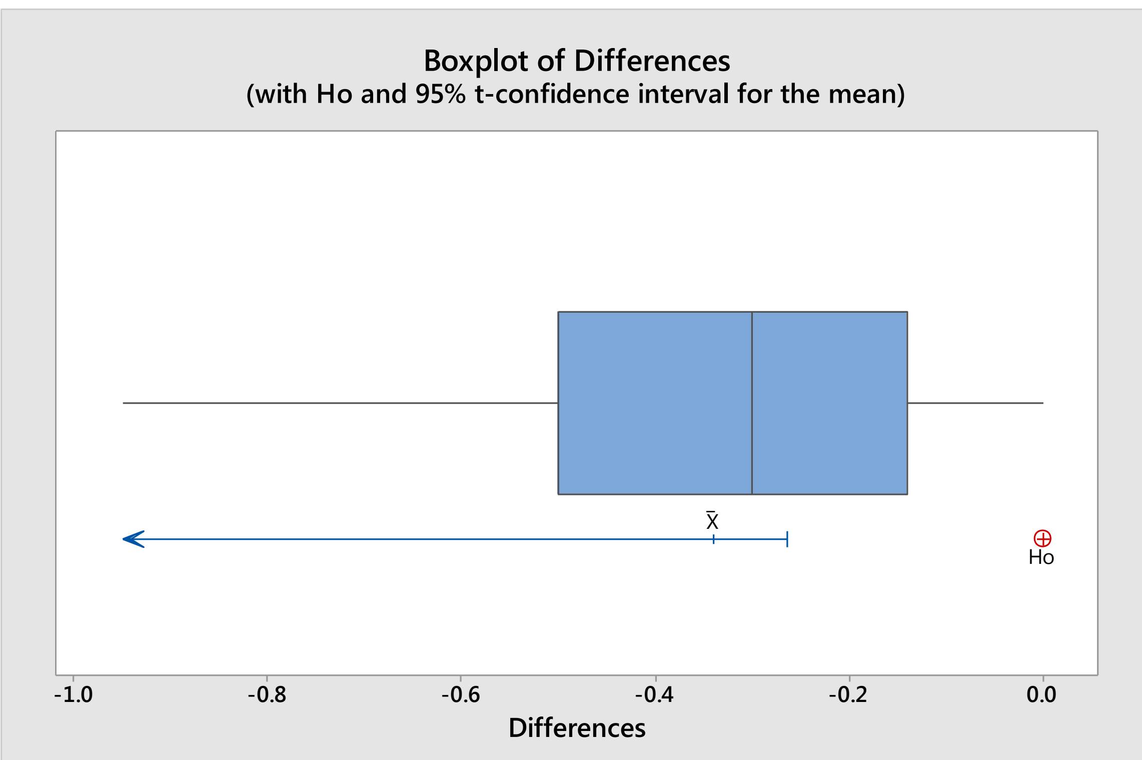
-1 mark for hypotheses, 1 mark for z = ± 2.2, 1 for appropriate rejection region Z > 1.645 or Z < -1.645, *depending on whether Z is positive or negative, 1 for decision and conclusion.*

Question 3. [11]

(a) [1]

The two samples are matched because each pair of prices correspond to the same item.

(b) [2]



Since the sample is large (>30), we can use the parametric paired sample t-test since the CLT assures us that the sample mean difference is normally distributed, provided that the population of differences is not extremely skewed. This proviso is met because while the sample of differences appears to be negatively skewed, it does not suggest a population that is extremely skewed.

-1 mark for comment on need for population of differences to be not extremely skewed, 1 for comment on a graph that shows the assumption is reasonable.

(c) [4]

Paired T-Test and CI: WinCo, Wal Mart

```
Paired T for WinCo - Wal Mart

              N      Mean   StDev  SE Mean
WinCo        31     2.346   2.434    0.437
Wal Mart     31     2.686   2.480    0.445
Difference   31   -0.3396  0.2499   0.0449


95% upper bound for mean difference: -0.2634
T-Test of mean difference = 0 (vs < 0): T-Value = -7.57  P-Value = 0.000
```

Ho: mu(WinCo) – mu(Walmart) = 0; Ha: difference > 0

Ho: mu(Walmart) – mu(WinCo) = 0; Ha: difference < 0

Rejection region is > 2.32 to > 2.46 (30 df) or < -2.32 to < -2.46 (for 1% level of significance)

T = *± 7.5 to 7.6*

At the 1% level of significance, we reject the null hypothesis and conclude the prices at the local discount grocer are lower than at Walmart.

-1 for hypotheses, 1 for t-statistic, 1 for p-value or rejection region, and 1 for decision and conclusion.

(d) [3]

For the Wilcoxon test, we calculated the differences as the Wal Mart minus the WinCo price and a “greater than” alternative hypothesis:

Wilcoxon Signed Rank Test: diff

```
Test of median = 0.000000 versus median > 0.000000

         N for   Wilcoxon         Estimated
      N   Test  Statistic      P     Median
diff  31    28      406.0  0.000     0.3300
```

Ho: median diff = 0, Ha: median diff > 0 for diff (Walmart minus WinCo)
Ho: median diff = 0, Ha: median diff < 0 for diff (WinCo minus Walmart)

-1 for hypotheses, 1 for p-value (from Minitab), 1 for decision/conclusion.

(e) [1]

If the sample of grocery items is not a random sample, then the test regarding the population means or medians is not valid.

-1 mark for comment that sample not randomly chosen, therefore test is not valid.

Question 4. [6]

(a) [3]

Chi-Square Test for Association: Worksheet rows, Worksheet columns

```
Rows: Worksheet rows   Columns: Worksheet columns

       Business                 Govt.
          Execs  Economists  Officials  All

1            10          20         42   72
          21.82       26.18      24.00

2            37          42         35  114
          34.55       41.45      38.00

3            24          39         18   81
          24.55       29.45      27.00

4            29          19         15   63
          19.09       22.91      21.00

All         100         120        110  330

Cell Contents:      Count
                    Expected count


Pearson Chi-Square = 35.410, DF = 6, P-Value = 0.000
Likelihood Ratio Chi-Square = 34.317, DF = 6, P-Value = 0.000
```

For the cells in the first row, the chi-square contributions are:

(10-21.82)^2/21.82 + (20-26.18)^2/26.18 + (42-24)^2/24.

Ho: Opinions are independent of the respondent's background

Ha: Opinions depend on the respondent's background

With a p-value of 0.000 < 0.05, we conclude that the opinions expressed are dependent on the background of the respondent.

-1 mark for hypotheses, 1 mark for chi-square statistic, 1 mark for decision/conclusion.

(b)

Chi-Square Goodness-of-Fit Test for Observed Counts in Variable: Observed

Category	Observed	Test Proportion	Expected	Contribution to Chi-Sq
1	72	0.2	66	0.54545
2	114	0.4	132	2.45455
3	81	0.2	66	3.40909
4	63	0.2	66	0.13636

N	DF	Chi-Sq	P-Value
330	3	6.54545	0.088

Ho: p1 = p3 = p4 = 0.2, p2 = 0.4

Ha: one of the probabilities above is different

With a p-value of 0.088 > 0.05, we cannot reject the null hypothesis, and conclude that there is insufficient evidence to show that the probabilities in Ho are not as stated.

-1 mark for hypotheses, 1 mark for chi-square statistic, 1 mark for decision/conclusion.