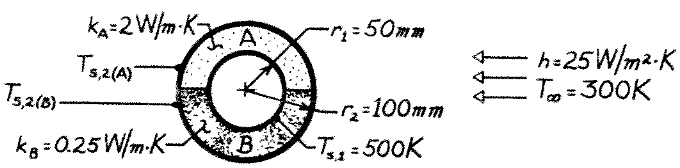
## Problem 1

**Known:** Inner surface temperature of isolation blanket comprised of two semi-cylindrical shells of different materials

**Find:** a) Equivalent thermal circuit ; b) Total heat loss per unit length ; c) outer surface temperatures of the insulation blankets

**Schematic:**



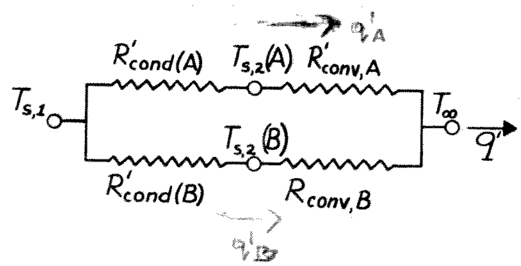

**Assumptions**

1) Steady-state conditions
2) Infinite contact resistance between materials
3) Because of assumption 2, 1-D conduction in radial direction
4) Constant properties

**Analysis:**

a) Equivalent thermal circuit

NB: Since we deal with semi-cylindrical shell all resistances are doubled




$R'_{A,cond} = \dfrac{\ln(r_2/r_1)}{\pi k_A}$ , $R'_{B,cond} = \dfrac{\ln(r_2/r_1)}{2\pi k_B}$ , $R'_{conv,A} = R'_{conv,B} = \dfrac{1}{\pi r_2 h}$

b) Total heat loss per unit length $q'$

$q' = q'_A + q'_B$ $\quad q'_A = \dfrac{T_{s,1} - T_\infty}{R'_{cond,A} + R'_{conv,A}}$ $\quad q'_B = \dfrac{T_{s,1} - T_\infty}{R'_{cond,B} + R'_{conv,B}}$

Evaluation of resistances

$R'_{conv,A} = R'_{conv,B} = \dfrac{1}{\pi \cdot 0.25\,m \cdot 25\frac{W}{m^2K}} = 0.1273\ \dfrac{m\cdot K}{W}$

$R'_{cond,A} = \dfrac{\ln(0.1/0.05)}{\pi \cdot 2\frac{W}{m\cdot K}} = 0.1103\ \dfrac{m\,K}{W}$ ; $R'_{cond,B} = \dfrac{\ln(0.1/0.05)}{\pi \cdot 0.25\frac{W}{m\,K}} = 0.8825\ \dfrac{m\,K}{W}$

Sub-in numerical values:

$q'_A = \dfrac{(500-300)\,K}{(0.1103 + 0.1273)\frac{K\cdot m}{W}} = 842\ \dfrac{W}{m}$ ; $q'_B = \dfrac{(500-300)\,K}{(0.8825 + 0.1273)\frac{K\cdot m}{W}} = 198\ \dfrac{W}{m}$ $\quad \boxed{\therefore q' = 842 + 198 = 1040\ \frac{W}{m}}$

c) outer surface temperatures $T_{s,2(A)}$ and $T_{s,2(B)}$

Refering to thermal circuit, $q'_A$ and $q'_B$ can alternatively be expressed by

$$q'_A = \frac{T_{s,1} - T_{s,2(A)}}{R'_{cond,A}} = \frac{T_{s,2(A)} - T_\infty}{R'_{conv,A}}$$

$$q'_B = \frac{T_{s,1} - T_{s,2(B)}}{R'_{cond,B}} = \frac{T_{s,2(B)} - T_\infty}{R'_{conv,B}}$$

$$\therefore T_{s,2(A)} = T_{s,1} - q'_A R'_{cond,A} = q'_A R'_{conv,A} + T_\infty$$

$$= 500 - 842 \cdot 0.1103 = 407\ K \quad \text{or} \quad 842 \cdot 0.1273 + 300 = 407\ K$$

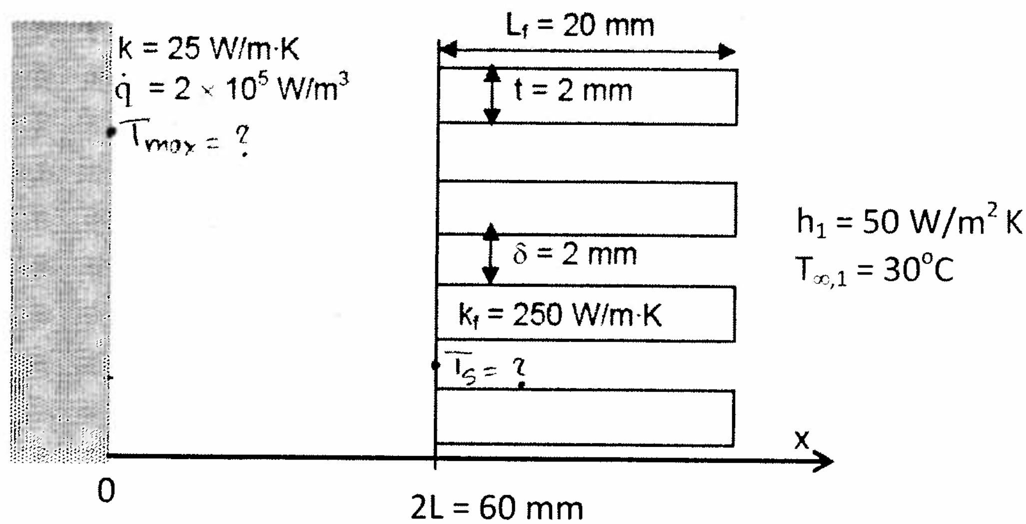
$$\therefore T_{s,2(B)} = 500 - 198 \cdot 0.8825 = 325\ K \quad \text{or} \quad 198 \cdot 0.1273 + 300 = 325\ K$$

## Problem 2

**Known:** Wall with known heat generation rate, thermal conductivity, thickness. Dimensions and thermal conductivity of fins; heat transfer coefficient and fluid temp.

**Find:** a) Efficiency of finned surface; b) temperature of prime surface ($T_s$); c) Maximum temperature within the wall

**Schematic:**




**Assumptions**

1) Steady-state conditions; 2) 1-D conduction in x-direction; 3) no contact resistance between fins and the wall; 5) negligible radiation

**Analysis:**

a) efficiency of finned surface, $\eta_o$. Basis $1\,m^2$ of unfinned surface ($1m \times 1m$)

$$\eta_o = 1 - \frac{N' A_f'}{A_t''}(1-\eta_f) \quad \dots (1)$$

where: $A_t''$ = total surface area of finned surface per unit surface area of the wall

$N'$ = number of fins per unit height of the wall

$A_f'$ = Surface area of fin per unit width of the wall

$$N' = \frac{1}{t+\delta} = \frac{1}{(0.002+0.002)\,m} = 250\ \frac{1}{m}$$

$$A_f' = 2L_c \cdot 1 \quad \text{where} \quad L_c = L_f + \frac{t}{2} = 0.02 + 0.002/2 = 0.021\ \frac{m^2}{m}$$

$$A_f' = 2 \cdot 0.021 \cdot 1 = 0.042\ \frac{m^2}{m}$$

$$A_t'' = 1 \cdot (1 - N't) + N'A_f' = 1 \cdot (1 - 250 \cdot 0.002) + 250 \cdot 0.042 = 0.5 + 10.5 = 11$$

$$\eta_f = \frac{\tanh(mL_c)}{mL_c} \quad \text{where} \quad m = \sqrt{\frac{2h_1}{k_f \cdot t}} = \sqrt{\frac{2\cdot 50}{250\cdot 0.002}} = 14.14\ \frac{1}{m}$$

thus $$\eta_f = \frac{\tanh(14.14\cdot 0.021)}{14.14\cdot 0.021} = 0.9716$$

Sub-in numerical values into eq (1) $$\eta_o = 1 - \frac{250\cdot 0.042}{11}(1-0.9716) = \underline{\underline{0.9729}}$$

b) Determination of $T_s$

At steady state $\dot{E}_g'' = \dot{E}_{out}''$ ... (2)

$\dot{E}_g'' = \dot{q}\,2L = 2\cdot 10^5\ \frac{W}{m^3}\cdot 0.06\ m = 1.2\cdot 10^4\ \frac{W}{m^2}$

$\dot{E}_{out}'' = h_1 A_t'' \eta_o (T_s - T_{\infty,1}) \Rightarrow T_s = \frac{\dot{E}_{out}''}{h_1 A_t'' \eta_o} + T_{\infty,1}$ ... (3)

Sub-in numerical values into Eq 3

$$T_s = \frac{1.2\cdot 10^4\ \frac{W}{m^2}}{50\ \frac{W}{m^2K}\cdot 11\ \frac{m^2}{m^2}\cdot 0.9729} + 30^\circ C = \underline{\underline{52.4^\circ C}}$$

c) Determination of $T_{max}$

$T_{max}$ will occur at the insulated surface

Temperature distribution in plane wall with one adiabatic surface Eq (C.22) from Appendix (Table C.3)

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s \qquad \text{where } L = 0.06\ m$$

$$T_{max} = T(x=0) = \frac{\dot{q}L^2}{2k} + T_s = \frac{2\cdot 10^5\ \frac{W}{m^3}\cdot(0.06m)^2}{2\cdot 25\ \frac{W}{mK}} + 52.4^\circ C$$

$\therefore \underline{\underline{T_{max} = 66.8^\circ C}}$

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if no fins $\quad T_s = \frac{1.2\cdot 10^4}{50\cdot 1} + 30^\circ C = 270^\circ C$

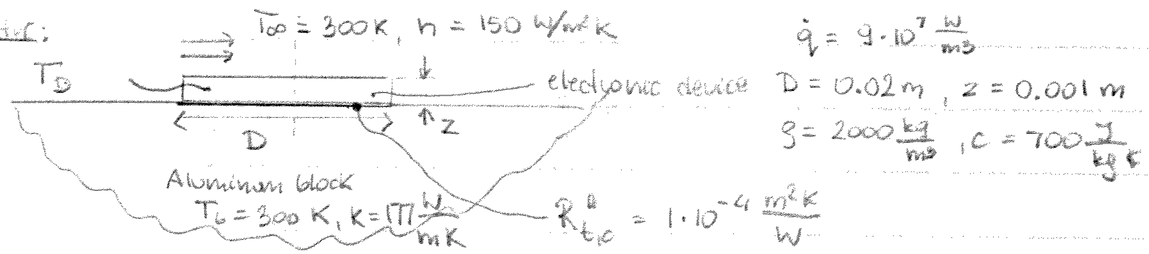
$T_{max} = 14.4 + 270 = 284.4^\circ C$

## Problem 3

**Known:** Dimensions and operating conditions of electronic device. Properties of the device, contact resistance and cooling conditions.

**Find:** a) Thermal circuit; b) steady-state temp of device $T_d$; c) Evaluation of constants a and b in equation governing T as a function of t

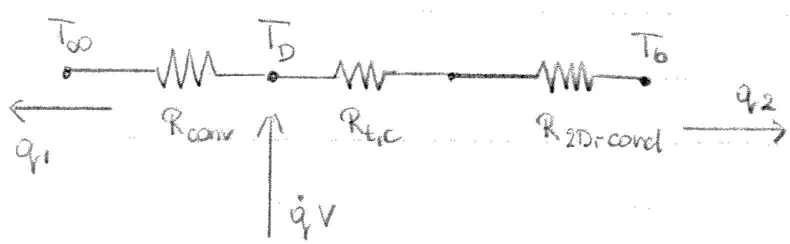
**Schematic:**




**Assumptions:**
1) Lumped thermal capacitance model is applicable
2) Negligible radiation
3) Constant properties
4) Aluminum block can be treated as semi-infinite medium at constant temperature away from the disk
5) Negligible heat transfer from the side of device

**Analysis**

a) thermal circuit at steady state when the device is energized




where: $R_{conv} = \frac{1}{hA}$ $\qquad R_{t,c} = \frac{R''_{t,c}}{A}$ $\qquad R_{2D-cond} = \frac{1}{kS}$

b) Determination of steady-state temp. of the device when energized ($T_d$)

Energy balance: $\dot{E}_{st} = E_{in} - \dot{E}_{out} + \dot{E}_g \qquad (1)$

At steady state $\quad -\dot{E}_{out} + \dot{E}_g = 0 \qquad (2)$

where $\dot{E}_{out} = q_1 + q_2 = \frac{T_D - T_\infty}{R_{conv}} + \frac{T_D - T_b}{R_{t,c} + R_{2D-cond}} \qquad (3)$

$\dot{E}_g = \dot{q}V = \dot{q} \cdot \frac{\pi}{4}D^2 \cdot z = 9 \cdot 10^7\ \frac{W}{m^3} \cdot \frac{\pi}{4}(0.02m)^2 \cdot 0.001m = 28.3W$

• Evaluation of resistances

$$R_{conv} = \frac{1}{150 \frac{\pi}{4} \cdot 0.02^2} = 21.23 \left[\frac{K}{W}\right] \qquad R_{t,c} = \frac{10^{-4}}{\frac{\pi}{4} 0.02^2} = 0.318 \left[\frac{K}{W}\right]$$

$$R_{2D\text{-}cond} = \frac{1}{k\,S} \quad \text{where } S = 2D = 2 \cdot 0.02 = 0.04\,m$$

$$= \frac{1}{177 \cdot 0.04} = 0.141 \left[\frac{K}{W}\right]$$

Rearranging Eq (3)

$$\dot{E}_g = (T_D - T_\infty)\left(\frac{1}{R_{conv}} + \frac{1}{R_{t,c} + R_{2D\text{-}cond}}\right) \Rightarrow T_D = \frac{\dot{E}_g}{U} + T_\infty$$

$$\text{where } U = \frac{1}{R_{conv}} + \frac{1}{R_{t,c} + R_{2D\text{-}cond}} = \frac{1}{21.23} + \frac{1}{0.318 + 0.141} = 2.22 \left[\frac{W}{K}\right]$$

$$\therefore \; T_D = \frac{28.3}{2.22} + 300 = 312.7 [K]$$

c) Evaluation of constants a and b

At transient state $\dot{E}_{st} \neq 0$, thus Eq (1) becomes:

$$\rho c V \frac{dT}{dt} = \dot{E}_g - U (T - T_\infty) \quad |: \rho c V$$

$$\frac{dT}{dt} = \frac{\dot{E}_g}{\rho c V} - \frac{U}{\rho c V}(T - T_\infty) \iff \frac{dT}{dt} = b - a(T - T_\infty)$$

$$\text{where } b = \frac{\dot{E}_g}{\rho c V} = \frac{28.3 (W)}{2000\left(\frac{kg}{m^3}\right) \cdot 700 \left(\frac{J}{kg\,K}\right) \cdot \frac{\pi}{4}(0.02\,m)^2 \cdot 0.001\,m} = 643 \frac{J\,K}{s\,J} = 64.3 \left[\frac{K}{s}\right]$$

$$a = \frac{U}{\rho c V} = \frac{2.22 \left(\frac{W}{K}\right)}{2000 \left(\frac{kg}{m^3}\right) \cdot 700 \frac{J}{kg\,K} \cdot \frac{\pi}{4}(0.02\,m)^2\, 0.001\,m} = 5.06 \left[\frac{1}{s}\right]$$