



**CHG 2314**

**HEAT TRANSFER OPERATIONS**

**MIDTERM EXAM**

**DATE:** Wednesday March 1, 2017, 13:00 – 14:20

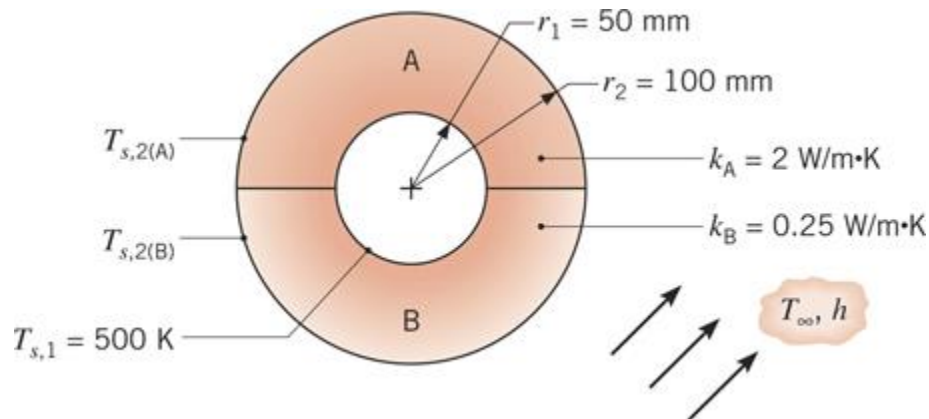
**DURATION:** 80 minutes

**PROFESSOR:** Dr. B. Kruczek

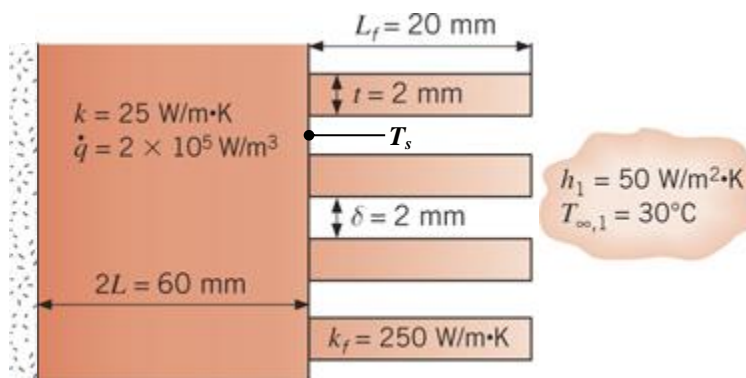
- 1) Closed book examination
- 2) One single-side reference page with your own notes/information is allowed. Your reference page is to be submitted with the exam booklet; it will be returned to you with the marked exam
- 3) Additional useful information is provided in the Appendix.
- 4) Do 2 out of 3 problems; each problem is worth 25 marks.
- 5) Please indicate which problems should be evaluated; if no indication is provided, the first two problems appearing in the exam booklet will be evaluated.
- 6) Calculator allowed: TI-30X or equivalent
- 7) Cell phones and all other electronic devices must be turned off and stored away from the desk
- 8) If you finish the exam before 14:10, you may leave the room. Otherwise, please wait till the end of the exam

***Good luck!!!***

1. Steam is flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature of 500 K. The pipe is covered with an insulation blanket comprised of two different materials, A and B. The interface between the two materials may be assumed to have infinite contact resistance, and the entire outer surface is exposed to air for which  $T_\infty = 300$  K and  $h = 25$  W/m<sup>2</sup> K.

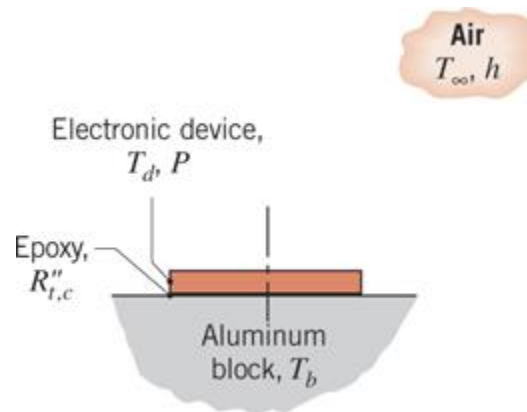


- Sketch the thermal circuit of the system. Label (using the preceding symbols) all pertinent nodes and resistances (10 points).
  - For the prescribed conditions, what is the total heat loss per unit length of the pipe (10 points)?
  - What are the outer surface temperature  $T_{s,2(A)}$  and  $T_{s,2(B)}$  (5 points)?
2. Heat is uniformly generated at the rate of  $2 \times 10^5$  W/m<sup>3</sup> in a wall of thermal conductivity 25 W/m K and thickness 60 mm. The wall is insulated on one side and there are straight rectangular fins on the other side, with dimensions as shown and thermal conductivity of 250 W/m K. The exposed side is in contact with fluid at  $T_{\infty,1} = 30^\circ\text{C}$  with the heat transfer coefficients  $h_1 = 50$  W/m<sup>2</sup> K.



- What is the efficiency of the finned surface (10 points)? *Hint:* as a basis, use unit surface area 1 m (high) x 1 m (wide) = 1 m<sup>2</sup>
- What is the temperature of the prime surface  $T_s$  (7.5 points)?
- What is the maximum temperature in the wall (7.5 points)?

3. An electronic device, in form of a disk 20 mm in diameter and 1 mm thick, is mounted flush on a large aluminum block ( $k = 177 \text{ W/m K}$ ). In the off-mode the electronic device is in thermal equilibrium with the aluminum block and air, which are maintained at  $T_b = T_\infty = 300 \text{ K}$ . The density and specific heat of the device are  $\rho = 2000 \text{ kg/m}^3$  and  $c = 700 \text{ J/kg K}$ , respectively. The mounting arrangement is such that a contact resistance of  $R''_{t,c} = 1 \times 10^{-4} \text{ m}^2 \text{ K/W}$  exists at the interface between the device and the block. For purposes of analysis, the energized electronic device is characterized by uniform volumetric heating of  $9 \times 10^7 \text{ W/m}^3$ . The convective heat transfer coefficient between the exposed surface of the device and air is  $150 \text{ W/m}^2 \text{ K}$ .



- a) Sketch the thermal circuit of the system at steady state when the electronic device is energized **(5 points)**.  
 b) Determine the steady state temperature of the energized electronic device ( $T_d$ ) **(10 points)**.  
 c) Following activation, since  $T_b = T_\infty$  the temperature of the device changes with time according to the following equation:

$$\frac{T - T_\infty - (b/a)}{T_i - T_\infty - (b/a)} = \exp(-at)$$

where:  $T_i = T_b = T_\infty$

Derive expressions for constants  $a$  and  $b$ , and evaluate them using the data provided above **(10 points)**.

## Appendix

**TABLE 3.3** One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall <sup>a</sup>	Spherical Wall <sup>a</sup>
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) = 0$	$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = 0$
Temperature distribution	$T_{s,1} - \Delta T \frac{x}{L}$	$T_{s,2} + \Delta T \frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T \left[ \frac{1 - (r_1/r)}{1 - (r_1/r_2)} \right]$
Heat flux ( $q''$ )	$k \frac{\Delta T}{L}$	$\frac{k \Delta T}{r \ln(r_2/r_1)}$	$\frac{k \Delta T}{r^2 [(1/r_1) - (1/r_2)]}$
Heat rate ( $q$ )	$kA \frac{\Delta T}{L}$	$\frac{2\pi Lk \Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k \Delta T}{(1/r_1) - (1/r_2)}$
Thermal resistance ( $R_{t,cond}$ )	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1) - (1/r_2)}{4\pi k}$

<sup>a</sup>The critical radius of insulation is  $r_{cr} = k/h$  for the cylinder and  $r_{cr} = 2k/h$  for the sphere.

**TABLE 3.5** Efficiency of common fin shapes

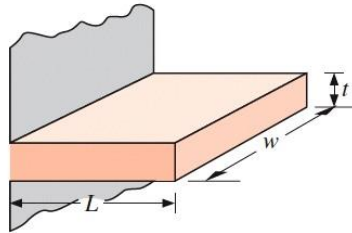
### Straight Fins

*Rectangular<sup>a</sup>*

$$A_f = 2wL_c$$

$$L_c = L + (t/2)$$

$$A_p = tL$$

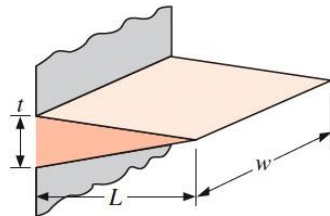


$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

*Triangular<sup>a</sup>*

$$A_f = 2w[L^2 + (t/2)^2]^{1/2}$$

$$A_p = (t/2)L$$



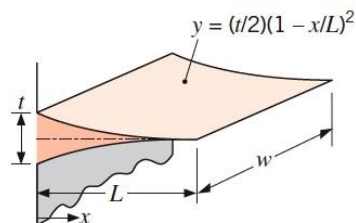
$$\eta_f = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)}$$

*Parabolic<sup>a</sup>*

$$A_f = w[C_1L + (L^2/t)\ln(t/L + C_1)]$$

$$C_1 = [1 + (t/L)^2]^{1/2}$$

$$A_p = (t/3)L$$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$$

$${}^a m = (2h/kt)^{1/2}$$

$${}^b m = (4h/kD)^{1/2}$$

**TABLE C.3 One-Dimensional, Steady-State Solutions to the Heat Equation for Uniform Generation in a Plane Wall with One Adiabatic Surface, a Solid Cylinder, and a Solid Sphere**

<b>Temperature Distribution</b>		
<b>Plane Wall</b>	$T(x) = \frac{\dot{q}L^2}{2k} \left( 1 - \frac{x^2}{L^2} \right) + T_s$	(C.22)
<b>Circular Rod</b>	$T(r) = \frac{\dot{q}r_o^2}{4k} \left( 1 - \frac{r^2}{r_o^2} \right) + T_s$	(C.23)
<b>Sphere</b>	$T(r) = \frac{\dot{q}r_o^2}{6k} \left( 1 - \frac{r^2}{r_o^2} \right) + T_s$	(C.24)
<b>Heat Flux</b>		
<b>Plane Wall</b>	$q''(x) = \dot{q}x$	(C.25)
<b>Circular Rod</b>	$q''(r) = \frac{\dot{q}r}{2}$	(C.26)
<b>Sphere</b>	$q''(r) = \frac{\dot{q}r}{3}$	(C.27)
<b>Heat Rate</b>		
<b>Plane Wall</b>	$q(x) = \dot{q}xA_x$	(C.28)
<b>Circular Rod</b>	$q(r) = \dot{q}\pi Lr^2$	(C.29)
<b>Sphere</b>	$q(r) = \frac{\dot{q}4\pi r^3}{3}$	(C.30)

**TABLE 4.1** Conduction shape factors and dimensionless conduction heat rates for selected systems.

(a) Shape factors [ $q = Sk(T_1 - T_2)$ ]

System	Schematic	Restrictions	Shape Factor
<b>Case 1</b> Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\frac{2\pi D}{1 - D/4z}$
<b>Case 2</b> Horizontal isothermal cylinder of length $L$ buried in a semi-infinite medium		$L \gg D$  $L \gg D$ $z > 3D/2$	$\frac{2\pi L}{\cosh^{-1}(2z/D)}$  $\frac{2\pi L}{\ln(4z/D)}$
<b>Case 3</b> Vertical cylinder in a semi-infinite medium		$L \gg D$	$\frac{2\pi L}{\ln(4L/D)}$
<b>Case 4</b> Conduction between two cylinders of length $L$ in infinite medium		$L \gg D_1, D_2$ $L \gg w$	$\frac{2\pi L}{\cosh^{-1}\left(\frac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$
<b>Case 8</b> Conduction through the edge of adjoining walls		$D > 5L$	$0.54D$
<b>Case 9</b> Conduction through corner of three walls with a temperature difference $\Delta T_{1-2}$ across the walls		$L \ll \text{length and width of wall}$	$0.15L$
<b>Case 10</b> Disk of diameter $D$ and temperature $T_1$ on a semi-infinite medium of thermal conductivity $k$ and temperature $T_2$		None	$2D$
<b>Case 11</b> Square channel of length $L$		$\frac{W}{w} < 1.4$  $\frac{W}{w} > 1.4$  $L \gg W$	$\frac{2\pi L}{0.785 \ln(W/w)}$  $\frac{2\pi L}{0.930 \ln(W/w) - 0.050}$