FINAL EXAM-MAT 1300
FALL TERM, 2016

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First Name:____________________**Last Name:**____________________

I.D. Number______________________________

Instructions-This final examination consists of 12 multiple choice questions worth 4 points each. Your answers to the multiple choice questions must be clearly marked in the squares below. There are also 4 long answer questions worth a total of 52 points. For the long answer questions, you must show your work **on the exam itself**.

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Multiple Choice ANSWERS:

D	D	E	A
#1	#2	#3	#4
B	D	E	C
#5	#6	#7	#8
A	A	C	B
#9	#10	#11	#12

Question 1- Calculate the following limit.

$$\lim_{x \to 2} \frac{\sqrt{x+2}-2}{x^2-4}$$

A) $\frac{1}{4}$ B) $-\frac{1}{4}$ C) 0 D) $\frac{1}{16}$ E) This limit does not exist.

$$\lim_{x \to 2} \frac{\sqrt{x+2}-2}{x^2-4} \cdot \frac{\sqrt{x+2}+2}{\sqrt{x+2}+2} = \lim_{x \to 2} \frac{\overbrace{(x+2)-4}^{(x-2)}}{\cancel{(x-2)}(x+2)(\sqrt{x+2}+2)} =$$

$$\lim_{x \to 2} \frac{1}{(x+2)(\sqrt{x+2}+2)} = \frac{1}{4(4)} = \frac{1}{16}$$

Question 2- Consider the function $f(x) = (x-2)e^{x+3}$. Which of the following statements is correct?

A) There is a local max at $x = 0$.
B) There is a local max at $x = -3$.
C) There is a local min at $x = -3$.
D) There is a local min at $x = 1$.
E) There is a local max at $x = 1$.

$$f'(x) = e^{x+3} + (x-2)e^{x+3}$$

$$= e^{x+3}(x-1) = 0 \Rightarrow x = 1$$

f	↘	$x=1$	↗
f'	$-$		$+$

Question 3- Determine the x values (if any) at which the following function has a horizontal tangent line.

$$f(x) = \sqrt{x^2 + 4x}$$

A) $x = \frac{1}{2}, 0$ B) $x = -\frac{1}{2}$ C) $x = -2$ D) $x = -2,\ 0$ ✓E) For no values of x.

$$f'(x) = \frac{1}{2}(x^2+4x)^{-1/2}(2x+4) = \frac{2x+4}{2\sqrt{x^2+4x}} = 0$$

$\Rightarrow 2x+4=0 \quad x=-2$

$x=-2 \notin D_f$

Therefore, for no values of x

Question 4- Consider the function:

$$f(x) = \frac{2x^2 + 6x + 4}{x^2 - 1}$$

Which of the following statements is correct?

✓A) The asymptotes of f are $y = 2, x = 1$.
B) The asymptotes of f are $y = 2, x = -1$, and $x = 1$.
C) $x = 1$ is the only asymptote of f.
D) $y = 2$ is the only asymptote of f.
E) The asymptotes of f are $y = 2, x = -2$, and $x = 1$.

$$f(x) = \frac{2(x^2+3x+2)}{(x-1)(x+1)} = \frac{2(x+1)(x+2)}{(x-1)(x+1)} = \frac{2(x+2)}{x-1} \quad x \neq -1$$

$x = 1 \rightarrow$ V.A

$\lim_{x \to \infty} f(x) = 2 \quad y=2$ H.A

Question 5- The number of bacteria in a certain culture is 15,000 at 1pm. At 5 pm, the number is doubled. Assuming the population is growing exponentially, how long does it take for the population to be 45,000?

A) $t = \frac{3\ln(2)}{\ln(3)}$ B) $t = \frac{4\ln(3)}{\ln(2)}$ ✓ C) $t = \frac{\ln(4)}{\ln(3)}$ D) $t = \frac{3\ln(3)}{\ln(5)}$ E) $t = \frac{4\ln(5)}{\ln(3)}$

1 pm $\quad P_0 = 15 \times 10^3$

5 m $\quad P(4) = 30 \times 10^3$

$\Rightarrow P(4) = 30\times10^3 = 15\times10^3\, e^{K4}$

$2 = e^{4K} \Rightarrow \ln 2 = 4K$

$\boxed{K = \frac{\ln 2}{4}}$

$P(t) = 45\times10^3 = 15\times10^3\, e^{\frac{\ln 2}{4}t}$

$3 = e^{\frac{\ln 2}{4}t} \qquad \ln 3 = \frac{\ln 2}{4}t \Rightarrow$

$t = \frac{4\ln 3}{\ln 2}.$

Question 6- A function $y = f(x)$ is defined implicitly by the following equation. Use implicit differentiation to find $\frac{dy}{dx}$ at $(-1, 1)$.

$$x^2y + e^{x+1} = y^2 + 1$$

A) $-\frac{1}{2}$ B) $\frac{1}{2}$ C) 1 D) -1 ✓ E) e^2+1

$2xy + x^2y' + e^{x+1} = 2yy'$

@ $(-1,1) \rightarrow -2 + (-1)^2y' + \underbrace{e^0}_{1} = 2y'$

$-1 + y' = 2y' \Rightarrow \boxed{y' = -1}$

Question 7- Calculate the following definite integral

$$\int_0^1 \frac{4x+6}{x^2+3x+2}dx = \int_0^1 \frac{2(2x+3)}{x^2+3x+2}dx$$

A) $\ln(6)-2$ B) $\ln(3)+2$ C) $\frac{1}{e}$ D) $\ln(6)+e^3$ E) $\ln(9)$ ✓

$u = x^2+3x+2$

$du = (2x+3)dx$

$x=0 \rightarrow u=2$

$x=1 \rightarrow u=6$

$$2\int_0^1 \frac{(2x+3)\,dx}{x^2+3x+2} = 2\int_2^6 \frac{du}{u} =$$

$$2\ln u\Big|_2^6 = 2(\ln 6 - \ln 2)$$

$$= 2\ln\left(\frac{6}{2}\right) = 2\ln 3$$

$$= \ln 9$$

Question 8- Suppose $f(x,y) = \ln(xy^2) + 2y$. Find $f_{yy}(1,1)$.

A) $\ln(2)$ B) 0 C) -2 ✓ D) 4 E) 2

$$f_y = \frac{2xy}{xy^2} + 2 = \frac{2}{y} + 2$$

$$f_{yy} = \frac{-2}{y^2}\Big|_{(1,1)} = -2$$

Question 9- Find $f(0)$, when $f'(x) = \frac{x-\sqrt{x}}{\sqrt{x}}$ and $f(1) = \frac{2}{3}$.

A) $f(0) = 1$ ✓

B) $f(0) = -1$

C) $f(0) = \frac{1}{3}$

D) $f(0) = \frac{2}{3}$

E) $f(0) = \frac{4}{3}$

$f'(x) = \frac{x}{\sqrt{x}} - 1$ (split the fraction

$= \sqrt{x} - 1$

$f(x) = \int (\sqrt{x} - 1)\,dx =$

$f(x) = \frac{2}{3} x^{3/2} - x + C$

$f(1) = \frac{2}{3} - 1 + C = \frac{2}{3} \Rightarrow C = 1$

$f(0) = 1$

Question 10- How many critical points does the following function of 2 variables have?

$$g(x, y) = y^2 - x^2 + 2y + x^3 + 8$$

A) 2 ✓ B) 3 C) 4 D) 1 E) 0

$\begin{cases} g_x = -2x + 3x^2 = 0 \\ g_y = 2y + 2 = 0 \end{cases}$ $\quad x(-2 + 3x) = 0 \quad x = 2/3, \; x = 0$

$y = -1$

$(0, -1) \quad (2/3, -1)$

Question 11- Suppose that for a certain product, the demand function is given by $p = D(q) = 8 - q^2$ and the supply function is given by $S(q) = 2q$, where p is the price and q is the quantity. Calculate the consumer surplus.

A) 3 B) $\frac{11}{3}$ ✓C) $\frac{16}{3}$ D) 1 E) $\frac{25}{7}$

$S(q) = D(q)$ $\quad 2q = 8 - q^2$ $\quad q^2 + 2q - 8 = 0$

$(q+4)(q-2) = 0$

$q \neq -4 \quad , \quad q = 2$

$p = 8 - 4 = 4$

$$CS = \int_0^2 \left((8-q^2) - 4\right) dq = \int_0^2 (4-q^2)\, dq = 4q - \frac{q^3}{3}\Big|_0^2$$

$$= 8 - \frac{8}{3} = \frac{24-8}{3} = \frac{16}{3}$$

Question 12- The demand function of a product relating the price per unit, p, in dollars to the demand, q, is given by

$$p = -q + 6.$$

The cost in dollars of producing one unit of the product is \$2. How many units do we want to produce to maximize our total profit?

A) 4 units

✓B) 2 units

C) 6 units

D) 12 units

E) None of the answers above

$p \geq 0 \rightarrow q \leq 6$

$q \geq 0 \qquad 0 \leq q \leq 6$

$C(q) = 2q$

$R(q) = -q^2 + 6q$

$P(q) = (-q^2 + 6q) - 2q$

$= -q^2 + 4q$

$P'(q) = -2q + 4 = 0$

$\boxed{q = 2}$

$P(0) = 0$

$P(2) = 4$ ✓

$P(6) = -36 + 24 = -12$

Long Answer Question 1 (14 points)
Calcualte the following integrals:

[a] **(8 points)**

Integration by parts

$$\int_1^e \frac{\ln x}{x^2}\,dx = \int_1^e \ln x \; x^{-2}\,dx$$

$$\boxed{\begin{aligned} u &= \ln x \\ u' &= \frac{1}{x} \\ v' &= x^{-2} \\ v &= \frac{-1}{x} \end{aligned}}$$

$$uv \quad - u'v$$

$$= \left(\ln x \cdot \frac{-1}{x}\Big|_1^e\right) - \int_1^e \frac{1}{x}\left(\frac{-1}{x}\right)dx$$

$$= \left(\frac{-\ln e}{e} - 0\right) + \int_1^e \frac{1}{x^2}\,dx = \left(\frac{-1}{e}\right) + \left(\frac{-1}{x}\Big|_1^e\right)$$

$$= \frac{-1}{e} - \frac{1}{e} + 1 = \frac{-2}{e} + 1$$

$$= \frac{-2+e}{e}$$

[b] **(6 points)**

$$\int \frac{x}{\sqrt{x^2+3}}\,dx = \int x\,(x^2+3)^{-1/2}\,dx$$

$$u = (x^2+3)$$

$$du = 2x\,dx$$

$$\frac{du}{2} = x\,dx$$

$$= \int u^{-1/2}\frac{du}{2} = \frac{1}{2}\left(2\sqrt{u}\right) + C$$

$$\sqrt{u} + C$$

$$= \sqrt{x^2+3} + C$$

Long Answer Question 2 (8 points) Explain why the following integral is an improper integral. Find out whether or not the following improper integral converges. If it does converge, compute its value.

$$\int_4^8 \frac{1}{\sqrt{x-4}}dx$$

$f(x) = \dfrac{1}{\sqrt{x-4}}$

$D_f = \{ x \in \mathbb{R} \mid x > 4 \}$ 2

$f(x)$ is not continuous @ $x=4$

($f(4)$ is not defined)

$$\int_4^8 (x-4)^{-1/2}dx = \lim_{b \to 4^+} \int_b^8 (x-4)^{-1/2}dx$$ 3 poins

$$= \lim_{b\to 4^+}\left(2\sqrt{x-4} \Big|_b^8 \right)$$

$$= \lim_{b\to 4^+}\left(2(\sqrt{4}) - 2\sqrt{b-4} \right)$$

2

$$= 4 - 2\lim_{b\to 4^+} \sqrt{b-4}$$

$$= 4 - 2(0^+) = 4$$

The improper integral is convergent 1

Long Answer Question 3 (14 points)

Consider the two functions:

$f(x) = x^2 + 2x - 3$ and $g(x) = 2x + 1$

(a) **(2 points)** Find the intersection points of the graphs of the two functions.

(b) **(6 points)** On the next page, graph these functions, and shade the region between the graphs of f and g for x such that $1 \leq x \leq 3$. (Select appropriate units on the coordinate system)

(c) **(6 points)** Find the area of the shaded region.

$$f(x) = g(x)$$

$$x^2 + 2x - 3 = 2x + 1$$

$$x^2 - 4 = 0 \qquad (x-2)(x+2) = 0 \qquad x = 2$$
$$x = -2 .$$

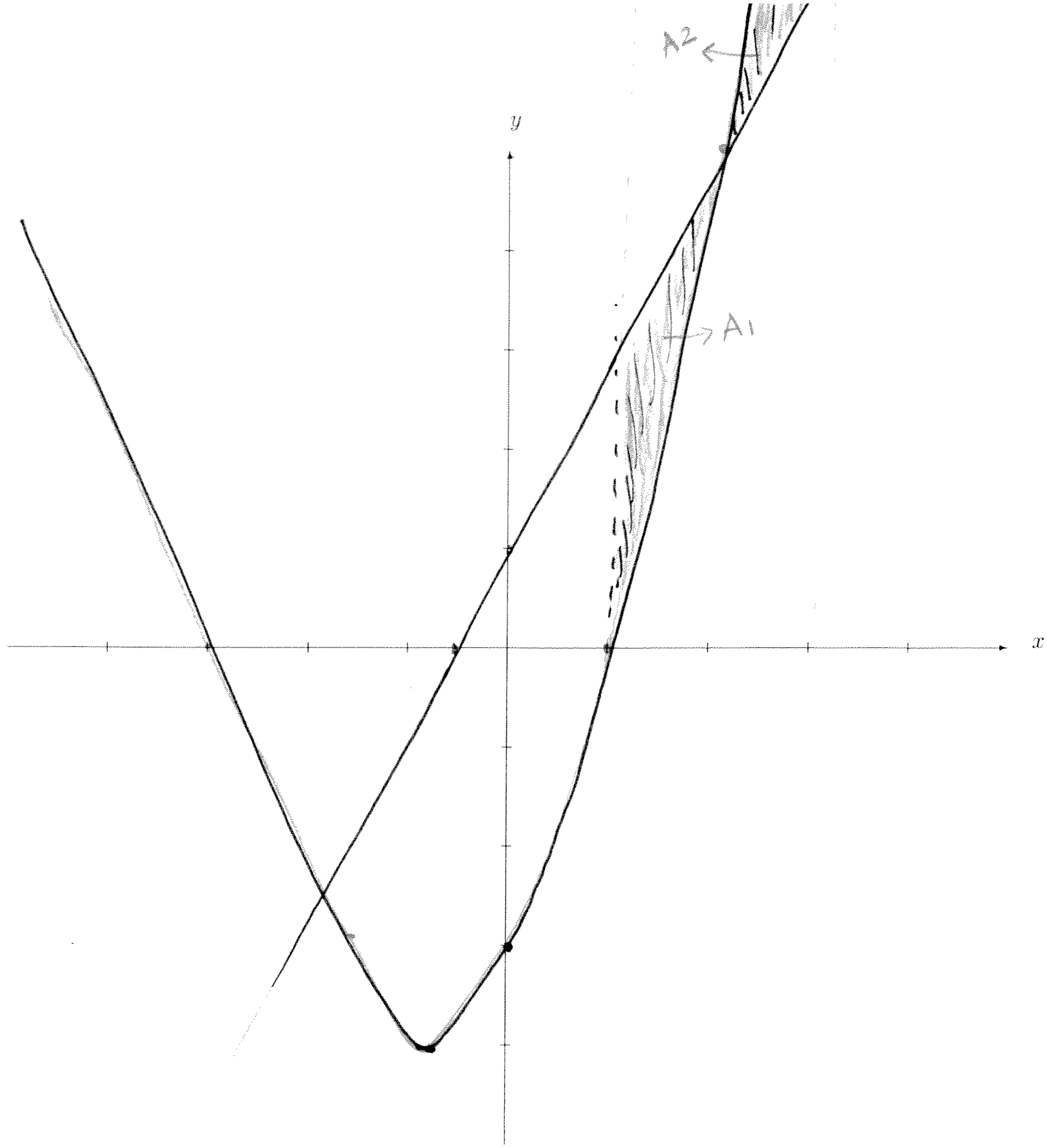
$$A_1 = \int_1^2 \left((2x+1) - (x^2 + 2x - 3) \right) dx = \int_1^2 (4 - x^2)\, dx$$

$$= \left(4x - \frac{x^3}{3}\right)\Big|_1^2 = 5/3$$

$$A_2 = \int_2^3 \left((x^2 + 2x - 3) - (2x+1) \right) dx = \int_2^3 (x^2 - 4)\, dx$$

$$= \left(\frac{x^3}{3} - 4x\right)\Big|_2^3 = 7/3$$

$$\text{Total Area} = \frac{5}{3} + \frac{7}{3} = 4.$$



$f(x) = x^2+2x-3$

$f'(x) = 2x+2=0$

$x=-1 \quad (-1,-4)$

$f''(x) = 2 > 0 \rightarrow$ local min

$g(x) = 2x+1$

$g(0) = 1$

$g(x)=0 \rightarrow 2x+1=0$

$x = -1/2$

$(-1/2, 0)$

Long Answer Question 4 (16 points)
Consider the function of two variables

$$f(x,y) = \frac{x^3}{3} - x^2 + y^2 - 2xy + 2$$

(a) **(4 points)** Calculate the first-order partial derivatives.

(b) **(6 points)** Find all critical points.

(c) **(6 points)** Identify the type (local maximum,local minimum or saddle point) of each of the critical points.

$$\begin{cases} f_x = x^2 - 2x - 2y = 0 \\ f_y = 2y - 2x = 0 \end{cases} \Rightarrow x = y$$

$$x^2 - 2x - 2x = x^2 - 4x = 0 \qquad x(x-4) = 0$$

$$x = 0 \qquad x = 4$$

$(0,0)$ $(4,4)$ critical point.

$$f_{xx} = 2x - 2 \qquad f_{yy} = 2$$

$$f_{xy} = -2 \rightarrow = \leftarrow f_{yx} = -2$$

$$D(x,y) = (2x-2)2 - (-2)^2 = (2x-2)2 - 4$$

$D(0,0) = -8 < 0 \rightarrow$ saddle point

$D(4,4) = 8 > 0 \rightarrow f_{xx} = 6 > 0 \rightarrow (4,4)$ is a local min.

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Submission Time:	06:06 PM
Images Scanned:	0
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