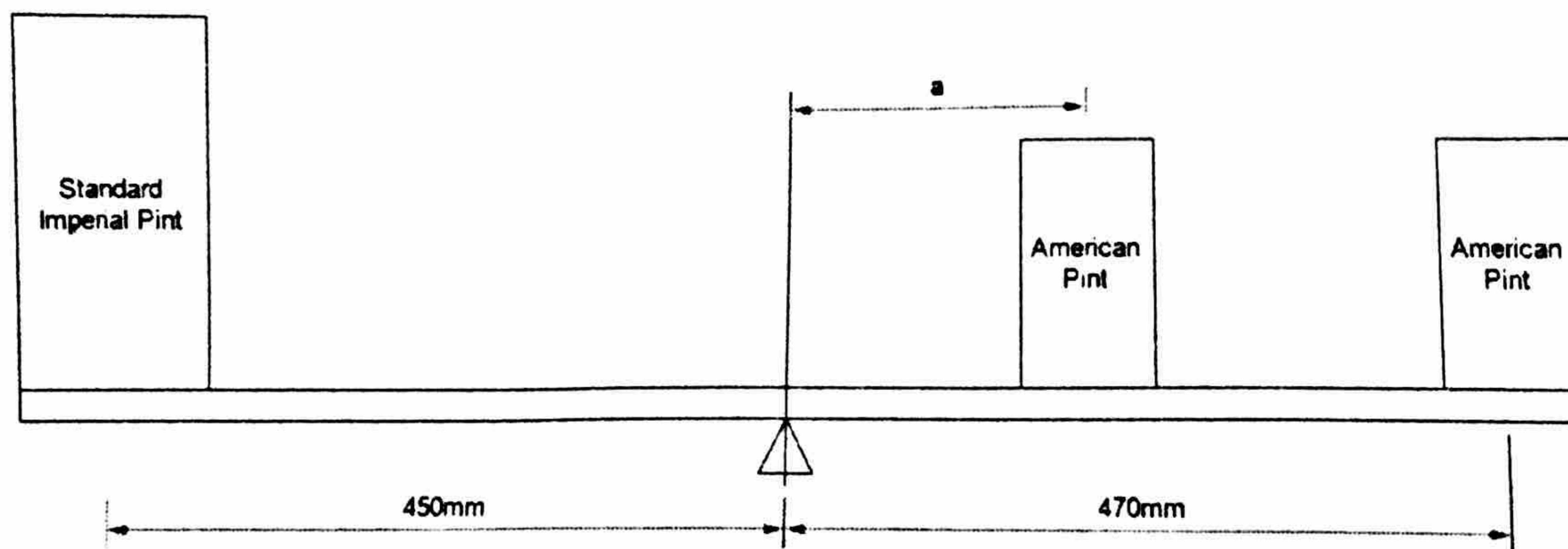


Question 1:

After successfully completing their mechanics midterm, a group of students head to Gatineau to have a beer (legally) to celebrate. At the bar, they order 3 "pints". One of the beers comes in a standard imperial pint glass of 20 imperial fluid ounces (568ml) (800g when filled) and the other two come in American pint glasses of 16 US fluid ounces (473ml) (600g when filled). As is tradition in this pub, the beers must be carried on a metre long plank (2kg), balanced in the middle. The standard imperial pint glass and one of the American pint glasses are placed at the ends of the plank and the third pint is placed somewhere in between.

The problem can be simplified to the following schematic:

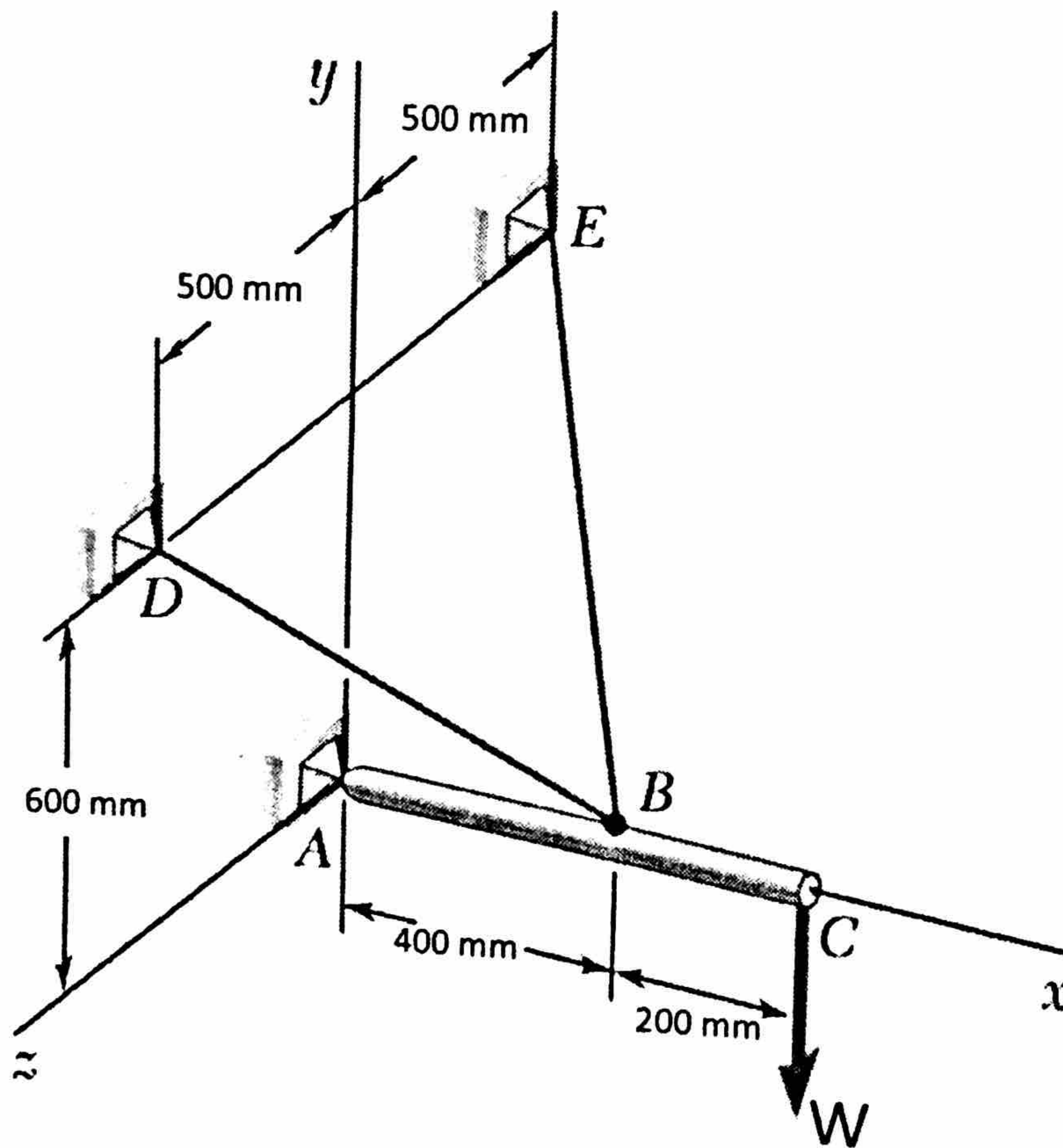


- Draw the free body diagram.
- What is the reaction at the server's hand?
- At what distance "a" will the plank be balanced?

Trivia (Doesn't apply to the question!): In Canada, federal law mandates any drink advertised as a "pint" must contain a standard imperial pint (568 ml) of liquid (not foam); but this is rarely enforced.

Question 2:

After leaving the bar, the group spots a beautiful wooden sign hanging outside of the bar. In a moment of weakness, the group decides to steal the sign. After hauling this sign across town, they weigh the sign and find out it has a mass of 50kg. Since they are up-and-coming engineers, they plan on hanging the sign using the familiar arrangement seen in the figure below.



In this set up, the pole ABC is held by a ball-and-socket joint at A and Cable DBE passes around a frictionless pulley at B .

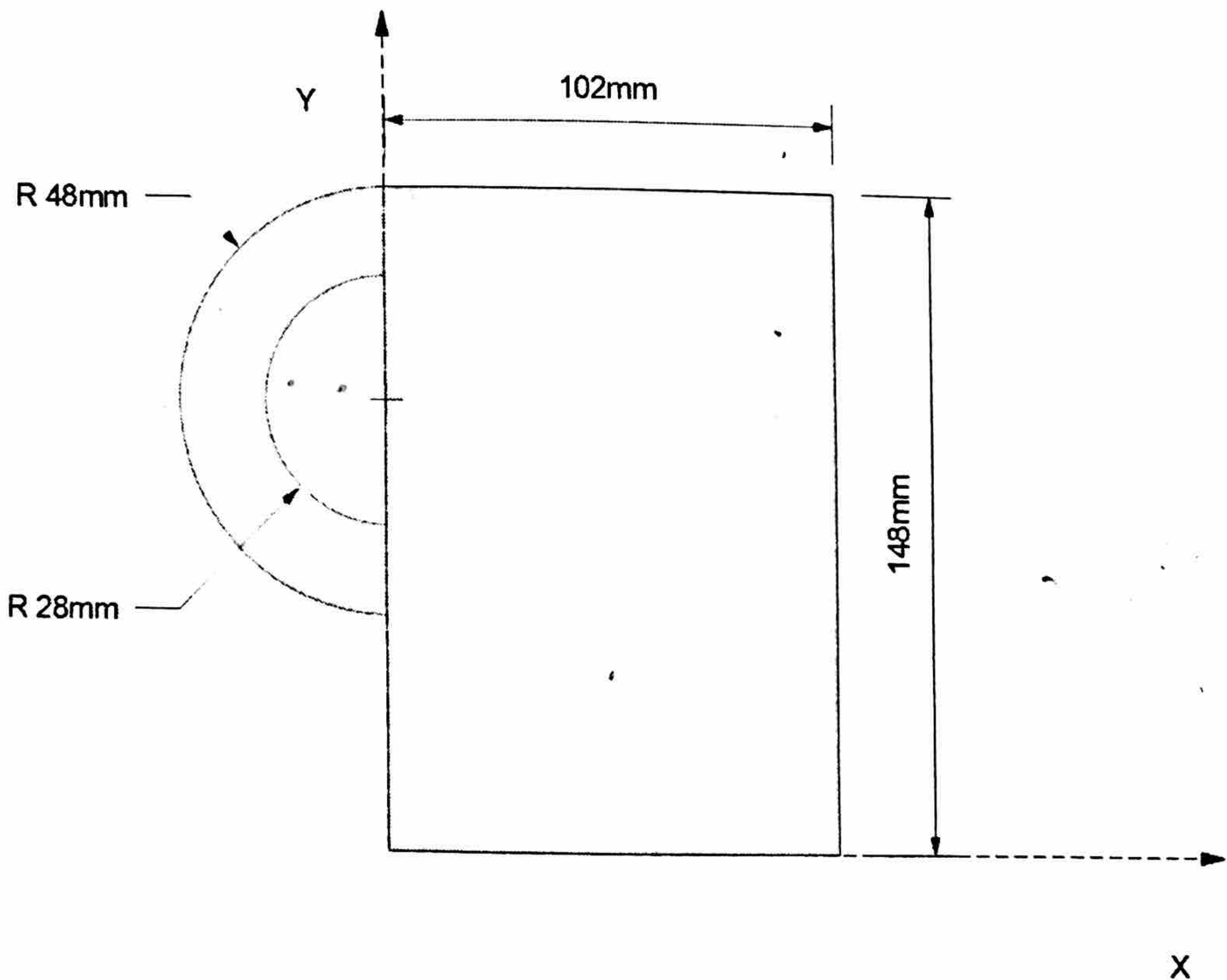
- Draw the FBD of the sign hanging pole (AC).
- Define the vectors \vec{T}_{BD} and \vec{T}_{BE} in components (\hat{i} , \hat{j} , \hat{k}).
- Solve for the tension in the ropes (T_{BD} and T_{BE}).

Question 3:

After waking to see the unsuccessfully mounted sign lying on the dirty floor - the group is overwhelmed with guilt. They decide to return the sign to the bar. The bar elects to hang the sign flat along the ceiling to prevent future theft. However, the bar staff is struggling to find a point on the sign from which it can be hung from a single wire and still be balanced. Since the group is so versed in engineering mechanics, they know that this balancing point would be the centroid!

The sign can be simplified as the following shape with a uniform thickness and density:

- (a) Find the location of the centroid. Use a table (as shown in class) and the coordinate system shown for full points.

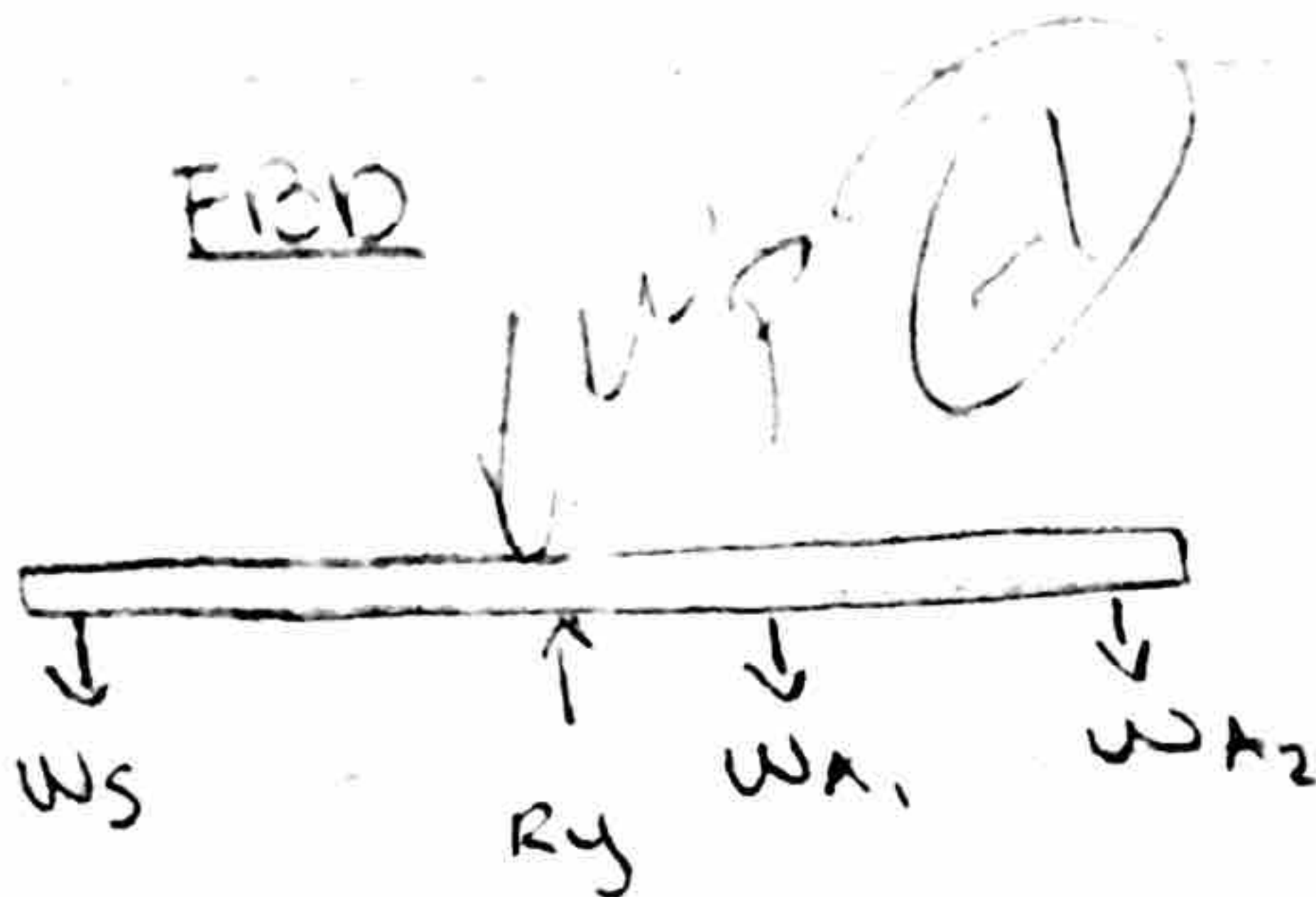


Question 1:

12/14

a)

FBD



b)

$$W = mg$$

$$W_S = (800g)(-9.81)$$

$$W_S = -7848 \text{ N}$$

$$W_A = (600g)(-9.81)$$

$$W_A = -5886 \text{ N}$$

$$\sum F_y = R_y - 7848 \text{ N} - 5886 \text{ N} - 5886 \text{ N}$$

$$R_y = 19,620 \text{ N}$$

$$c) \sum M = 0 = (7848 \text{ N})(450 \text{ mm}) - (5886)(470 \text{ mm}) - 5886 a$$

$$a = 130 \text{ mm}$$

130

Question 2.

30/30

a) FBD



$$W = mg$$

$$W = (50 \text{ kg})(-9.81)$$

$$W = -490.5 \text{ N}$$

$$b) \vec{T}_{BE} = T_{BE} \vec{m}_{BE}$$

$$= T_{BE} \frac{(-400 \text{ mm} \hat{i} + 600 \text{ mm} \hat{j} - 500 \text{ mm} \hat{k})}{\sqrt{400^2 + 600^2 + 500^2}}$$

$$= T_{BE} (-0.4558 \hat{i} + 0.6838 \hat{j} - 0.5698 \hat{k})$$

$$\vec{T}_{CD} = T_{CD} \vec{m}_{CD}$$

$$= T_{CD} \frac{(-400 \text{ mm} \hat{i} + 600 \text{ mm} \hat{j} + 500 \text{ mm} \hat{k})}{\sqrt{400^2 + 600^2 + 500^2}}$$

$$= T_{CD} (-0.4558 \hat{i} + 0.6838 \hat{j} + 0.5698 \hat{k})$$

c) $\sum M_A = 0$

$r_C \times 490.5$	$\left \begin{array}{ccc} \uparrow & \uparrow & \hat{k} \\ 600 & 0 & 0 \\ 0 & 490.5 & 0 \end{array} \right $	$0 \hat{i} - 0 \hat{j} + (600)(490.5) \hat{k}$ $294,300 \hat{k}$
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$r_B \times T_{BE}$	$\left \begin{array}{ccc} \uparrow & \uparrow & \hat{k} \\ -400 & 0 & 0 \\ -0.4558 & 0.6838 & -0.5698 \end{array} \right $	$0 \hat{i} - (-400)(-0.5698) \hat{j} + (-400)(0.6838) \hat{k}$ $T_{BE} (-227.92 \hat{j} - 273.52 \hat{k})$
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continued →

Problem 2 (continued)

$$r_B \times T_{BO} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -400 & 0 & 0 \\ -0.558 & 0.6838 & 0.5698 \end{vmatrix} = (-400)(0.5698)\hat{j} + (-400)(0.6838)\hat{k}$$
$$T_{BO}(227.92\hat{j} - 273.52\hat{k})$$

$$\sum M_y = 0 = 227.92T_{BO} - 273.52T_{BE}$$
$$T_{BO} = T_{BE} = T$$

$$\sum M_z = 0 = -273.52T - 273.52T + 294,300$$
$$-294,300 = -547.04T$$

$$T = 537.99 \text{ N} = 540 \text{ N}$$

Question 3:

24/15

a)		A (mm ²)	\bar{x} (mm)	\bar{y} (mm)	$\bar{x}A$ (mm ³)	$\bar{y}A$ (mm ³)
+	1	15006 ✓	51 ✓	74 ✓	769,896	1,117,104
+	2	3619,115 ✓	-20,372 ✓	100 ✓	-73,728,005	361,911.5
-	3	231,504 ✓	-11,884 ✓	100 ✓	-14,634,663	123,105.4
	Σ	17,483,611			710,802.658	1,355,910.1

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{1,355,910.1}{17,483,611} = 77.55 \text{ mm} \checkmark$$

$$\bar{x} = \frac{\sum \bar{x}A}{\sum A} = \frac{710,802.658}{17,483,611} = 40.66 \text{ mm} \checkmark$$

$$\bar{x} = \frac{4r}{3\pi} = \frac{4(48)}{3\pi}$$

$$\frac{4r}{3\pi} = \frac{4(28)}{3\pi}$$

