$a_n = a_1 + (n-1)d$ $APY = \left(1 + \frac{r}{m}\right)^m - 1$ Chapt Ber

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics $S = \frac{n}{2}[a_1 - a_n]$

Course	Number		Section(s)
Mathematics	208/4		All
Examination	**Date**	**Time**	**Pages**
Midterm	March 2016	1 Hour 30 minutes	2
Instructors			**Course Examiner**
D. Sen, E. Duma, H. Greenspan, L. Aggoun, M. Padamadan S. Giri, S. White			D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\frac{(1+i)^n - 1}{i}, \quad PV = PMT\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- Answer all questions.
- **Only approved calculators are allowed.**

MARKS

[2+4+4] **1.** A company markets exercise DVDs that sell for \$19.95, including shipping and handling. The monthly fixes costs are \$24,000 and the variable costs are \$7.45 per DVD.

(A) Find the cost equation and the revenue equation.

(B) How many DVDs must be sold each month for the company to break even?

(C) Graph the cost and revenue equations in the same coordinate system and show the break-even point.

[$2\frac{1}{2} \times 4$] **2.** Solve for x in the following equations:

(A) $(81)^{2x} = (9)^{x^2-12}$

(B) $\log_3 x + \log_3(x-3) = \log_3 10$

(C) $\log_b x = 3\log_b 2 + 0.5\log_b 25 - \log_b 20$

(D) $\frac{1}{4}\log_x 256 = 12$

PLEASE TURN OVER

[5+5] **3.** Suppose there are two sequences $a_1, a_2, ..., a_n, ..,$ and $b_1, b_2, ..., b_n, ...$ where $a_n = 5.(1.1)^{n-1}$ for $n \geq 1$ and $b_1 = -4$, $b_n = b_{n-1} + 3$ for $n > 1$.

(A) Find $S_{22} = a_1 + a_2 + ... + a_{22}$.

(B) Find b_{54}.

[10] **4.** A person borrows \$6,000 and agrees to repay the loan in monthly installments over a period of 5 years. The agreement is to pay 1% of the unpaid balance each month for using the money and \$100 each month to reduce the loan. What is the total cost of the loan over the 5 years?

[5+5] **5.** Compubank, an online banking service, offered a money market account with an annual percentage yield(APY) of 1.551%.

(A) If interest is compounded monthly, what is the equivalent nominal rate?

(B) If you wish to have \$10,000 in this account after 4 years, what equal deposit should you make each month?

[5+5] **6.** If you borrow \$500 that you agree to repay in six equal monthly payments at 1% interest per month on the unpaid balance, how much of each monthly payment is used for interest and how much is used to reduce the unpaid balance? Organize your results in a table.

SOLUTION

1. Price DVD = $19.95
FC = 24,000
VC = 7.45

(A) $C(x) = 24{,}000 + 7.45x$, where x is the number of DVD's. $x \geq 0$

$R(x) = 19.95x$, where $x \geq 0$ (# of DVD's sold)

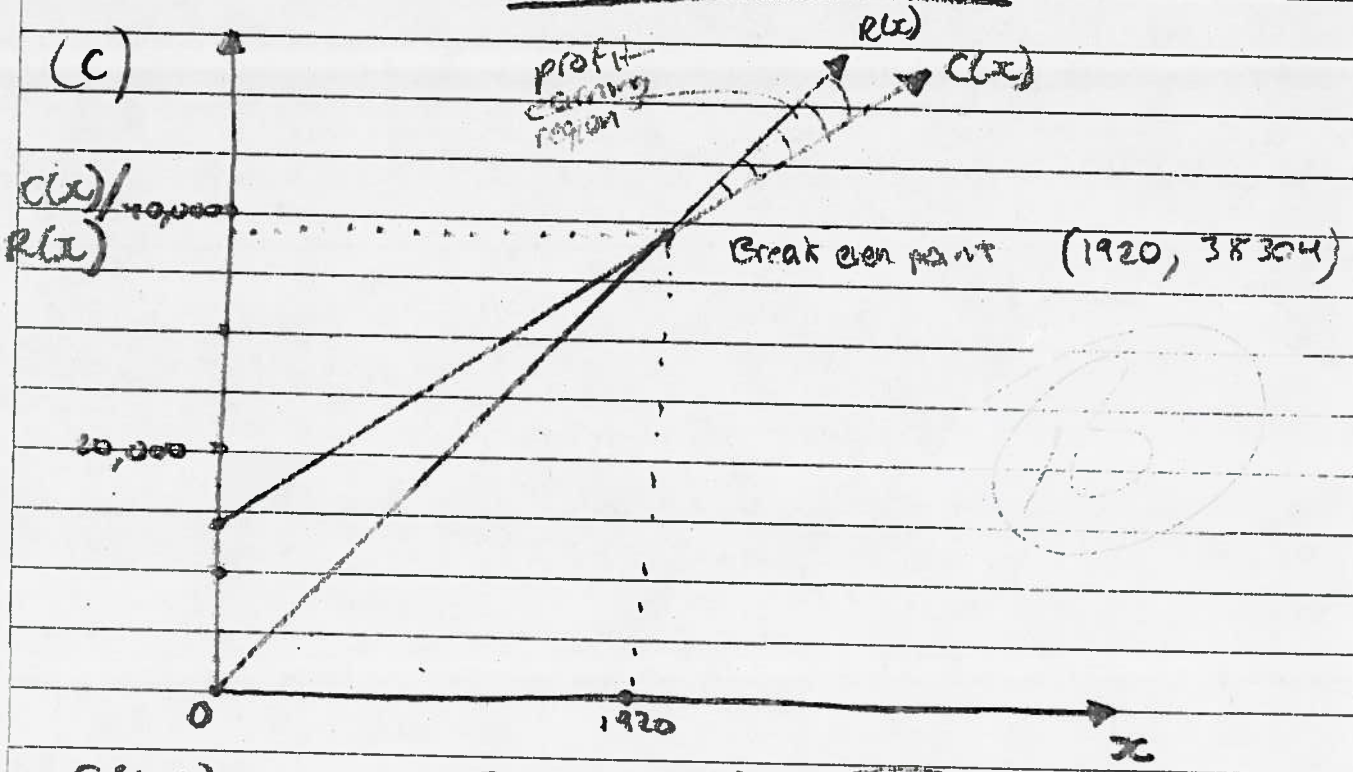
(B) where does $C(x) = R(x)$?

$19.95x = 24{,}000 + 7.45x$

$12.5x = 24{,}000$

$x = 1920$ DVD's

(C)



- $C(1920) = 24000 + 7.45(1920) = 38304$ Break even point

$C(0) = 24{,}000$

$R(0) = 0$ $R(1920) = 38304$.

(2)

Arith $\begin{cases} S_n = \frac{n}{2}(2a_1 + (n-1)d \\ a_n = a_1 + (n-1)d \end{cases}$ geo $\begin{cases} a_n = ar^{n-1} \\ S_n = \frac{a(r^n-1)}{r-1} \end{cases}$

APV: $(1+\frac{r}{n})^{nt} - 1$

APV: $e^{rt} - 1$

$-b \pm \frac{\sqrt{b^2-4ac}}{2a}$

A: P(1+

2 A) $81^{2x} = 9^{x^2-12}$

$9^{2(2x)} = 9^{x^2-12}$

$4x = x^2 - 12$

$0 = x^2 - 4x - 12$

$(x-6)(x+2)$

$x = 6$ and $x = -2$

B) $\log_3 x + \log_3(x-3) = \log_3 10$

$x(x-3) = 10$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2)$

$x = 5$, $x = -2$

C) $\log_b x = 3\log_b 2 + 0.5\log_b 25 - \log_b 20$

$\frac{2^3 \cdot 25^{0.5}}{20} = \frac{8 \cdot 5}{20} = 2$

D) $\frac{1}{4}\log_x 256 = 12$

$256^{1/4} = x^{12}$

$\sqrt[12]{4} = \sqrt[n]{x}$

$x = \sqrt[12]{4}$

3) A) $a_n = 5 \cdot (1.1)^{n-1}$ for $n \geq 1$

$a_1 = 5 \cdot (1.1)^0 = 5$

$a_2 = 5 \cdot (1.1)^1 = 5.5$

$a_3 = 5 \cdot (1.1)^2 = 6.05$

$\frac{a_2}{a_1} = \frac{5.5}{5} = 1.1$

$\frac{a_3}{a_2} = \frac{6.05}{5.5} = 1.1$

} Geometric series

$S_n = \frac{a_1(r^n-1)}{r-1} \rightarrow S_{22} = \frac{5(1.1^{22}-1)}{1.1-1}$

$= 357.01$

B) $b_1 = -4$, $b_n = b_{n-1} + 3$ for $n > 1$ → d

$b_2 = -4 + 3 = -1$ $\quad b_2 - b_1 = -1 - (-4) = 3$ } Arithmatic

$b_3 = -1 + 3 = 2$ $\quad b_3 - b_2 = 2 - (-1) = 3$ }

$a_n = a_1 + (n-1)\,d$

$b_{54} = -4 + (53)\,3 = 155$

4) $a_1 = 6000 \cdot 0.01 = 60$ $\quad a_2 - a_1 = 59 - 60 = -1$ } arithmatic series

$a_2 = (6000 - 100) \cdot 0.01 = 59$ $\quad a_3 - a_2 = -1$ }

$a_3 = (5900 - 100) \cdot 0.01 = 58$

$S_{(5-12)} = S_{60} = \frac{n}{2}(2a_1 + (n-1)d)$

$S_{60} = \frac{60}{2}(2(60) + 59(-1)) = 1830$

Total cost = $6000 + 1830 = 7830$

PV PMT

142.58

3% $\left(1+\frac{3}{12}\right)^{12}-1 = APY$

APY = 13.55

(4)

5. APY = 0.01551 r = ?

(a) $0.01551 = \left(1+\frac{r}{m}\right)^{m}-1$

$0.01551 = \left(1+\frac{r}{12}\right)^{12}-1$

$-12\sqrt{1.01551}$ is ommitted

$\left(1+\frac{r}{12}\right) = \sqrt[12]{1.01551}$

can't use negative rate ...

$\frac{r}{12} = 0.001283401\ldots$ ← compound rate per month

$\Rightarrow r = 0.015400823$ monthly nominal rate

∴ equivalent nominal rate is equal to 0.015400823 or ≈ 1.54 % compounded monthly. ✓

(B) $PMT = FV\ \frac{i}{(1+i)^{n}-1}$

$= 10{,}000\ \frac{\left(\frac{0.0154}{12}\right)}{\left(1+\frac{0.0154}{12}\right)^{12\times 4}-1}$

PMT = 202.11615

PMT ≈ 202.12

∴ To have 10,000 in account after 4 years you will deposit $202.12 at nominal rate of ≈ 1.54 % compounded monthly.

6. 12(c)

PMT	PMT AMT	interest	balpaid	newbalance	unpaid balance
0	0	—	—	—	500
1	86.27	5.00	81.27	418.73	
2	86.27	4.19	82.08	336.65	
3	86.27	3.37	82.9	253.75	
4	86.27	2.54	83.73	170.02	
5	86.27	1.70	84.57	85.45	
6	86.3	0.85	85.45	0	
	✓	17.65			

payment = ?

↑ What actually reduces balance.

$$PMT = PV \frac{c}{1-(1+c)^{-n}}$$

$$= 500 \frac{0.01}{1-(1.01)^{-6}}$$

$$= 86.27418336$$

$$= 86.27 \text{ / month}$$

∴ Interest paid = $17.65 and

b/c money to pay off balance at first = $517.65