Math 204 April 2012 Final Exam

1.

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \quad \begin{array}{l} R2 = R2 + (1)R1 \\ R3 = R3 + (-3)R1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right] \quad \begin{array}{l} R1 = R1 + R2 \\ R3 = R3 + (-10)R2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{array}\right] \quad \begin{array}{l} R2 = (-1)R2 \\ R3 = -\frac{1}{52}R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} R1 = R1 + (-7)R3 \\ R2 = R2 + (5)R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

(3) $\Rightarrow x_1 = 2$

(2) $\Rightarrow x_2 = 1$

(1) $\Rightarrow x_1 = 3$

2.

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & (a^2-14) & a+2 \end{array}\right] \quad \begin{array}{l} R2 = R2 + (-3)R1 \\ R3 = R3 + (-4)R1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & \underset{a^2-2}{(a^2-14+12)} & \underset{a-14}{a+2-16} \end{array}\right] \quad R3 = R3 + (-1)R2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & (a^2-2-14) & a-14+10 \end{array}\right]$$

OR

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2-16 & a-4 \end{array}\right] \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

In eq (3) if

$a = 4$

we have $0 \quad 0 \quad 0 \quad 0 \Rightarrow$ Inf. # sol.

$a = -4$

we have $0 \quad 0 \quad 0 \quad -8 \Rightarrow$ No sol.

If $a \neq 4$ and $a \neq -4$ then we can divide by a^2-16 to get

$0 \quad 0 \quad 0 \quad 1 \quad \frac{1}{a+4}$

and we will get a unique sol.

3.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \quad R2 = R2 + (-1)R1$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -3 & 2 & -2 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R1 = R1 + (-1)R3 \\ R2 = R2 + (1)R3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -1 \\ 0 & -1 & 3 & -2 & 1 & 1 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \quad R2 = (-1)R2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -1 \\ 0 & 1 & -3 & 2 & -1 & -1 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \quad R3 = R3 + (-2)R2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -1 \\ 0 & 1 & -3 & 2 & -1 & -1 \\ 0 & 0 & 7 & -4 & 2 & 3 \end{array}\right] \quad R1 = 7R1$$

$$\left[\begin{array}{ccc|ccc} 7 & 0 & -7 & 7 & 0 & -7 \\ 0 & 1 & -3 & 2 & -1 & -1 \\ 0 & 0 & 7 & -4 & 2 & 3 \end{array}\right] \quad R1 = R1 + (1)R3$$

$$\left[\begin{array}{ccc|ccc} 7 & 0 & 0 & 3 & 2 & -4 \\ 0 & 1 & -3 & 2 & -1 & -1 \\ 0 & 0 & 7 & -4 & 2 & 3 \end{array}\right] \quad \begin{array}{l} R2 = 7R2 \\ R3 = 3R3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 7 & 0 & 0 & 3 & 2 & -4 \\ 0 & 7 & -21 & -14 & -7 & -7 \\ 0 & 0 & 21 & -12 & 6 & 9 \end{array}\right] \quad R2 = R2 + 1R3$$

$$\left[\begin{array}{ccc|ccc} 7 & 0 & 0 & 3 & 2 & -4 \\ 0 & 7 & 0 & 2 & -1 & 2 \\ 0 & 0 & 21 & -12 & 6 & 9 \end{array}\right] \quad \begin{array}{l} R1 = \frac{1}{7}R1 \\ R2 = \frac{1}{7}R2 \\ R3 = \frac{1}{21}R3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{7} & \frac{2}{7} & -\frac{4}{7} \\ 0 & 1 & 0 & \frac{2}{7} & -\frac{1}{7} & \frac{2}{7} \\ 0 & 0 & 1 & -\frac{4}{7} & \frac{2}{7} & \frac{3}{7} \end{array}\right]$$