

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

MATH 209/4 all sections except EC: - Fundamental Mathematics II
Midterm - Saturday, March 5, 2016 (1h30min)

Only approved calculators are permitted.
Justify all your answers.

1. (a) [7 marks] Find

$$\lim_{x \rightarrow -\infty} \frac{4x^2 - 5}{-3x^3 + 7x - 2}$$

- (b) [8 marks] Give an example of a function $a(x)$ and a function $b(x)$ with the following properties:

$$(i) \lim_{x \rightarrow 3} a(x) = 0 \quad (ii) \lim_{x \rightarrow 3} b(x) = 0 \quad (iii) \lim_{x \rightarrow 3} \frac{[b(x)]^2}{a(x)} = -3$$

2. [7 marks] Let $h(x) = -x^3 + x$. Work out the following in detail:

$$\lim_{s \rightarrow 0} \frac{h(x+s) - h(x)}{s}$$

3. [12 marks]

- (a) If $f(x) = 7x^{4/3} - x^{-8} + 2$, find $f'(x)$. You don't have to simplify the answer.
(b) If $g(x) = [4x^3 - 7][3 - \ln(x^3)]$, find $g'(x)$. You don't have to simplify the answer.
(c) Find $h'(x)$ if $h(x) = \frac{x^3 - \frac{1}{x}}{x^3 - e^x}$. You don't have to simplify the answer.
(d) Find the value of dy if $y = \ln(x+2)$, $x = 3$, and the change in x is 0.2.

4. [6 marks] A population grew from 12 million to 34 million in 50 years. Assuming continuous compounding, what is the associated annual rate of growth?

5. [10 marks] A small machine shop manufactures drill bits used in the petroleum industry. The manager estimates that the total daily cost (in dollars) of producing x bits is

$$C(x) = 1,000 + 25x - 0.1x^2$$

- (a) Find $\overline{C}(x)$ and $\overline{C}'(x)$.
(b) Find $\overline{C}(10)$ and $\overline{C}'(10)$. Interpret these quantities.
(c) Use the results in part (b) to estimate the average cost per bit at a production level of 11 bits per day.

PLEASE TURN OVER

6. [10 marks] Find x' for $x = x(t)$ defined implicitly by

$$1 + x \ln t = te^x$$

and evaluate x' at $(t, x) = (1, 0)$.

7. [10 marks] The price p (in dollars) and demand x for a product are related by

$$x^2 + 2xp + 25p^2 = 74,500.$$

If the demand is decreasing at a rate of 6 per month when the demand is 150, find the rate of change of the price.