Concordia University
Department of Mathematics & Statistics
Course: MATH 202/2 2013-2014

Examination: Midterm Test **Date & Time: October 20th, 2PM**

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Special Instructions

- **Clearly indicate which problem you are solving.**
- **Clearly identify your answers.**
- **Only approved calculators are permitted.**

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1. **(4 points)** Use the remainder theorem and synthetic division to show that $\frac{1}{5}$ and $-\frac{1}{2}$ are solutions of the equation

$$10x^4 - 17x^3 + 3x^2 + 5x - 1 = 0$$

2. **(3 points)** Factor the polynomial

$$10x^4 - 17x^3 + 3x^2 + 5x - 1$$

from the previous problem completely.

3. **(3 points)** Discuss and draw the graph of

$$f(x) = (1 - 9x^2)(x^2 + 3x - 4)$$

4. **(3 points)** Discuss and draw the graph of

$$f(x) = \frac{1 + x}{(2 - x)^2}$$

5. **(3 points)** Rewrite in the trigonometric form $z = |z|(\cos\phi + i\sin\phi)$ the complex number

$$z = \frac{\sqrt{3} + i}{2 - 2i}$$

6. **(4 points)** Use de Moivre's theorem to write

$$\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{11}$$

in the form $a + bi$.

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1. Check $x=\frac{1}{5}$

$$\begin{array}{r|rrrrr} \frac{1}{5} & 10 & -17 & 3 & 5 & -1 \\ & & 2 & -3 & 0 & 1 \\ \hline & 10 & -15 & 0 & 5 & \boxed{0} \end{array}$$

Since Remainder $=0$ $\Rightarrow x=+\frac{1}{5}$ is a zero

AND $(x-\frac{1}{5})$ is a Factor

(i) Note: we now have $\dfrac{10x^4-17x^3+3x^2+5x-1}{(x-\frac{1}{5})} = 10x^3-15x^2+5+\dfrac{0}{x-\frac{1}{5}}$
$=(10x^3-15x^2+5)(x-\frac{1}{5})$

Check $x=-\frac{1}{2}$

4 points

$$\begin{array}{r|rrrr} -\frac{1}{2} & 10 & -15 & 0 & 5 \\ & & -5 & 10 & -5 \\ \hline & 10 & -20 & 10 & \boxed{0} \end{array}$$

Since Rem. $=0$ $\Rightarrow x=-\frac{1}{2}$ is a zero

AND $(x+\frac{1}{2})$ is a Factor

(ii) Note: we now have $\dfrac{10x^3-15x^2+5}{x+\frac{1}{2}} = 10x^2-20x+10+\dfrac{0}{x+\frac{1}{2}}$
$=(10x^2-20x+10)(x+\frac{1}{2})$

2. From Question 1

$10x^4-17x^3+3x^2+5x-1 = (10x^3-15x^2+5)(x-\frac{1}{5})$ — From 1(i)

$= (10x^2-20x+10)(x+\frac{1}{2})(x-\frac{1}{5})$ — From 1(ii)

3 points

$= 10(x^2-2x+1)(x+\frac{1}{2})(x-\frac{1}{5})$ — Common Factor 10

$= 10(x-1)(x-1)(x+\frac{1}{2})(x-\frac{1}{5})$ — Factor $x^2-2x+1 = (x-1)(x-1)$

3. $f(x) = (1-9x^2)(x^2+3x-4)$

(i) ZEROS: $(1-9x^2)(x^2+3x-4)=0$

$1-9x^2=0$ | $x^2+3x-4=0$

$(1-3x)(1+3x)=0$ | $(x-1)(x+4)=0$

$1-3x=0$, $-3x=-1$, $x=\frac{1}{3}$ | $1+3x=0$, $3x=-1$, $x=-\frac{1}{3}$ | $x-1=0$, $x=1$ | $x+4=0$, $x=-4$

(ii) y intercept: let $x=0$

$y = (1-0)(0+0-4)$

$y=-4$

(iii) test sign of y between $x=-1$, $x=-4$

(use $x=-3$) $y=[1-9(-3)^2][(-3)^2+3(-3)-4]$

$y = (-)(-) \Rightarrow$ y is pos (Graph above x Axis)

(use $x=\frac{2}{3}$) test sign of y bet. $x=\frac{1}{3}$, $x=1$

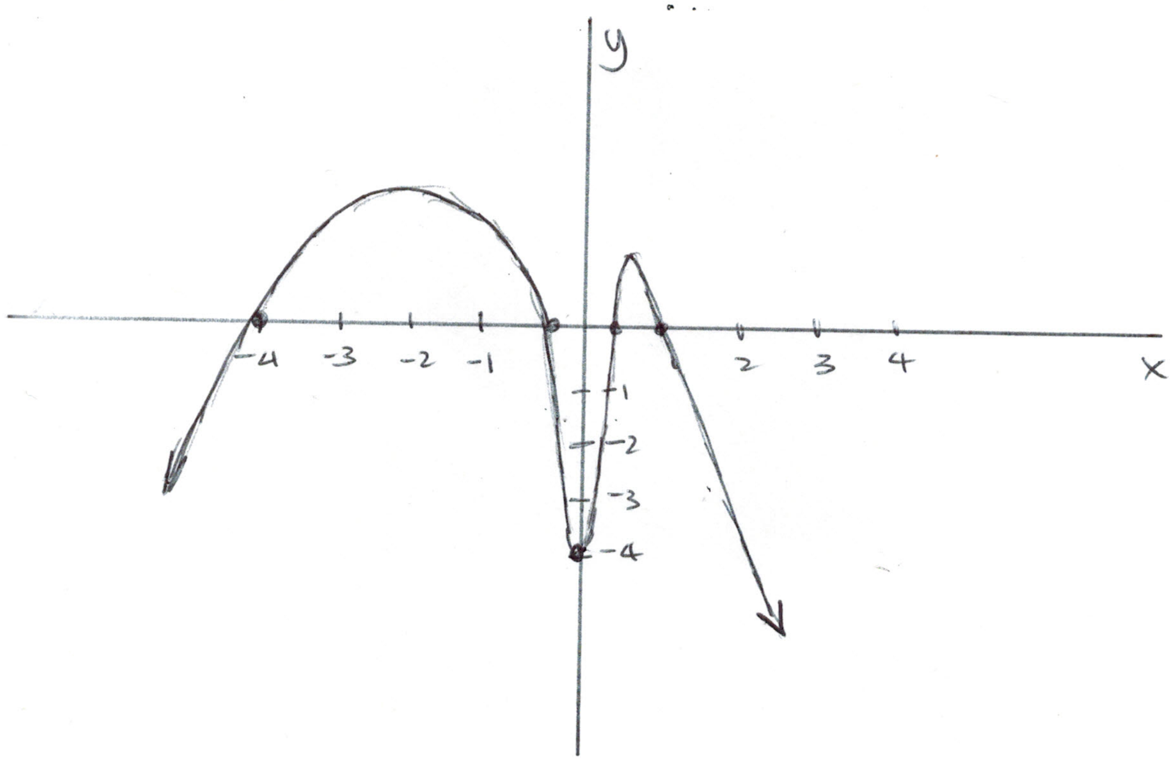
$y=[1-9(2/3)^2][(2/3)^2+3(2/3)-4]$

$y= (-)(-) \Rightarrow$ y is pos. (Graph above x Axis)

(iv) If you multiply out we would have highest power term as $-9x^4$ - - - - -

So if $x \to +\infty \Rightarrow y \to -\infty$ (Graph goes down)

if $x \to -\infty \Rightarrow y \to -\infty$ (Graph goes down)



4. (i) Vertical Asymptote:

$(2-x)^2=0$

$2-x=0$

$x=2$

Since the numerator $\neq 0$ when we substitute $x=2$

$\Rightarrow x=2$ is a V.A.

(ii) Horizontal Asymptote.

$\lim_{x\to+\infty} \frac{1+x}{(2-x)^2} = \frac{\infty}{\infty}$

$\lim_{x\to+\infty} \frac{\frac{1}{x^2}+\frac{x}{x^2}}{\left(\frac{2}{x}-\frac{x}{x}\right)\left(\frac{2}{x}-\frac{x}{x}\right)}$

$\lim_{x\to+\infty} \frac{\frac{1}{x^2}+\frac{1}{x}}{\left(\frac{2}{x}-1\right)\left(\frac{2}{x}-1\right)}$

$= \frac{0+0}{(0-1)(0-1)} = 0$

$\Rightarrow$ HA is $y=0$ (x axis) AT Right

If $x\to-\infty$ $y\to 0$ also

$\Rightarrow$ HA is $y=0$ (x axis) AT LEFT

(iii) y intercept (let $x=0$) $\Rightarrow y=\frac{1}{4}$

(iv) Zero (let $y=0$) $\Rightarrow x=-1$

(v) VA behaviour

① test sign as $x\to 2^-$

(use $x=1.9$) $\Rightarrow y=\frac{1+1.9}{(2-1.9)^2}=+$

② test sign as $x\to 2^+$

(use $x=2.1$) $\Rightarrow y=\frac{1+2.1}{(2-2.1)^2}=+$

③ test as $x\to-\infty$

(use $x=-10$)

$\Rightarrow y=\frac{1-10}{(2-[-10])^2}=-$

④ test as $x\to\infty$

use $x=10$

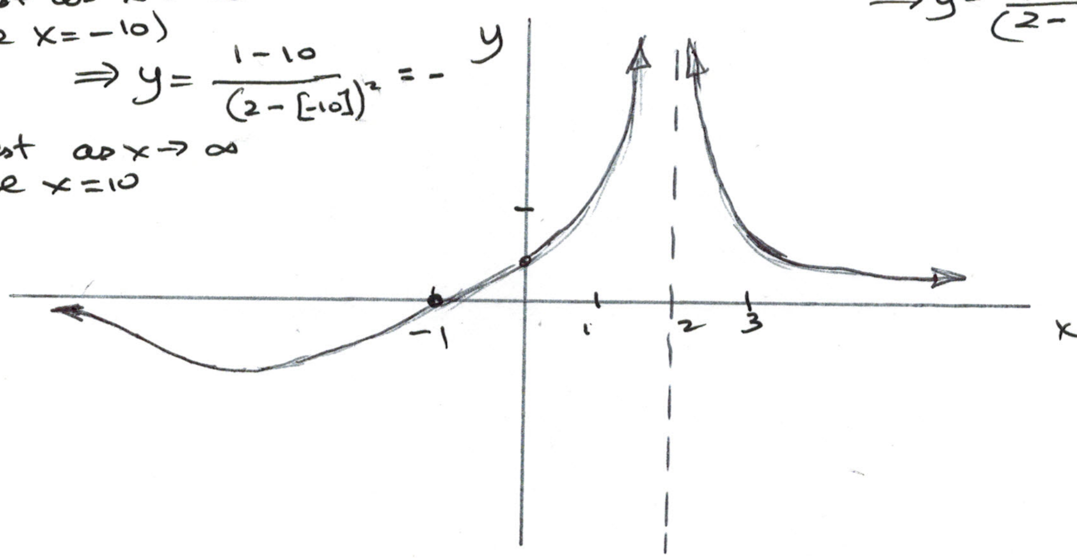
(vi) HA behaviour

③ test as $x\to-\infty$ (use $x=-10$)

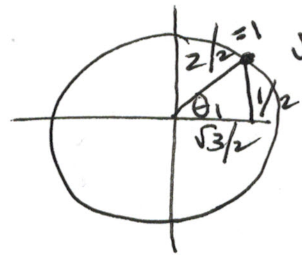
$\Rightarrow y=\frac{1-10}{(2-[-10])^2}=-$

④ test as $x\to+\infty$ (use $x=10$)

$\Rightarrow y=\frac{1+10}{(2-[10])^2}=+$



5. Step 1 Convert Numerator to polar



$r = \sqrt{a^2+b^2}$

$r = \sqrt{(\sqrt{3})^2+1^2}$

$r = 2$

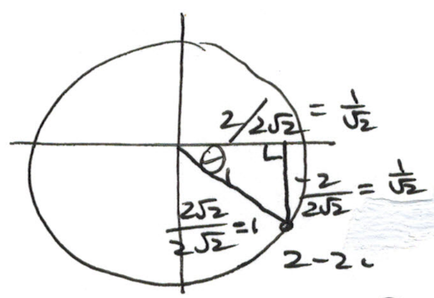
If you use unit circle to get θ_1 Make $r=1$ then we have

$1, \frac{1}{2}, \frac{\sqrt{3}}{2}$ and opposite $1/2$

is $\theta = 30^\circ$ or $\frac{\pi}{6}^R$

$\Rightarrow \sqrt{3}+i = 2 \text{ cis } \frac{\pi}{6}$ or $2 \text{ cis } 30^\circ$

Step 2 Convert Denominator to polar



$r = \sqrt{2^2+(-2)^2}$

$r = \sqrt{8}$

$r = \sqrt{4}\sqrt{2}$

$r = 2\sqrt{2}$

If you use unit circle to get θ make $r = 1$ then we have

$1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ and opposite $\frac{1}{\sqrt{2}}$

is $\theta_1 = 45^\circ$ or $\theta_1 = \frac{\pi}{4}$

But because of position

Final $\theta = -45^\circ$ or 315°, $\theta = -\frac{\pi}{4}$ or $\frac{7\pi}{4}^R$

Step 3 do Division

$$z = \frac{\sqrt{3}+i}{2-2i} = \frac{2 \text{ cis } 30^\circ}{2\sqrt{2} \text{ cis } 315^\circ} = \frac{1}{\sqrt{2}} \text{ cis}(30-315)^\circ$$

$$= \frac{1}{\sqrt{2}} \text{ cis } (-285^\circ)$$

OR

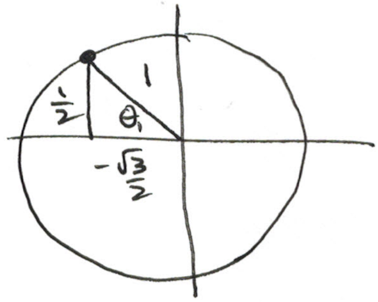
$$= \frac{1}{\sqrt{2}} \text{ cis } (30 - [-45])$$

$$= \frac{1}{\sqrt{2}} \text{ cis } 75^\circ$$

OR use Radian measure

6. Step 1 Change $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$ to polar

$-\frac{\sqrt{3}}{2} + \frac{1}{2}i$



$r = \sqrt{(-\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2}$

$r = \sqrt{\frac{3}{4} + \frac{1}{4}}$

$r = 1$

we have numbers $1, \frac{1}{2}, \frac{\sqrt{3}}{2}$

$\Rightarrow \theta_1 = 30^\circ$ or $\frac{\pi}{6}^R$

because of position the Final $\theta = 180^\circ - 30^\circ$
$= 150^\circ$

$\Rightarrow 1 \text{ cis } 150^\circ$

Step 2 Do power

$\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^{11} = (1 \text{ cis } 150^\circ)^{11}$

$= 1^{11} \text{ cis } (11 * 150)^\circ$

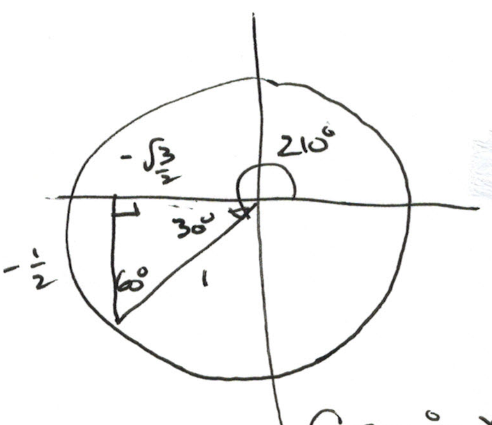
$= 1 \text{ cis } 1650^\circ$

Step 3 Change to Rectangular

$= 1 \text{ cis } (360*4 + 210)^\circ$

$= 1 \text{ cis } 210^\circ$ (*)

$= 1(\cos 210^\circ + i \sin 210^\circ)$



$= 1\left(-\frac{\sqrt{3}}{2} + (-\frac{1}{2})i\right)$

$= -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

$\cos 210^\circ = \frac{x}{r} = \frac{-\frac{\sqrt{3}}{2}}{1} = -\frac{\sqrt{3}}{2}$

$\sin 210^\circ = \frac{y}{r} = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$

(*) note angle
$1650^\circ \neq 210^\circ$
but sin, cos of 1650° =
Sin, Cos of 210°.