CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	202	All
Examination	**Date**	**Pages**
Final	December 2009	2
Instructors		**Course Examiner**
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Special Instructions:

- ▷ Only approved calculators are allowed.
- ▷ Justify and explain all your answers.

MARKS

[7] **1.** (a) Use synthetic division to find the quotient and the remainder when $f(x) = x^4 - 6x^3 + 5x^2 + 4x - 24$ is divided by $g(x) = x - 3$.

[7] (b) Find an upper integral bound and a lower integral bound for the real zeros of the polynomial $2x^3 - 5x^2 - 10x + 25$, using the Theorem on Bounds.

[7] **2.** (a) Apply Descartes' Rule of Signs to discuss the zeros of

$$-x^4 + 2x^2 - x + 7$$

[7] (b) Show by Mathematical Induction that

$$(1 \cdot 2) + (2 \cdot 3) + (3 \cdot 4) + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

[7] **3.** (a) Write the series $4+(-2)+(-8)+(-14)+(-20)+(-26)$ using the summation notation $(\sum)$.

[7] (b) Find the sum of the arithmetic series $\sum_{k=1}^{500} (5 - 6k)$.

[7] **4.** (a) Find all horizontal and vertical asymptotes of $f(x) = \dfrac{2x}{x-2}$ and sketch its graph.

[7] (b) Use De Moivre's theorem to express $\left(\dfrac{\sqrt{2}}{2} - i\dfrac{\sqrt{2}}{2}\right)^{200}$ in the form $a + bi$.

[7] **5.** (a) Compute the 5 5^{th}-roots of $\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}$.

[7] (b) Find a real polynomial of least degree having $3 - 2i$,$-i$,2 as. zeros.

[8] **6.** (a) Express the number 3.141514151415... in the form $\frac{a}{b}$, where a, b are integers and $b \neq 0$.

[8] (b) Find and simplify the coefficient of $a^{10}b^{7}$ in the expression $(a+b)^{17}$.

[7] **7.** (a) How many hands of 13 cards,none of which is a heart or a spade, can be dealt from a 52 card bridge deck?

[7] (b) There are 10 good computers and 3 defective computers. In how many ways can 6 computers be chosen so that 3 are defective?

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1a)

3	1	−6	5	4	−24
		3	−9	−12	−24
	1	−3	−4	−8	$\boxed{-48}$

$\Rightarrow$ Quotient: x^3-3x^2-4x-8

Remainder -48

b)

1	2	−5	−10	25
		2	−3	−13
	2	−3	−13	$\boxed{12}$

2	2	−5	−10	25
		4	−2	−24
	2	−1	−12	$\boxed{1}$

3	2	−5	−10	25
		6	3	−21
	2	1	−7	$\boxed{4}$

4	2	−5	−10	25
		8	12	8
	2	3	2	$\boxed{33}$

$\Rightarrow$ 4 is an upper bound

−1	2	−5	−10	25
		−2	7	3
	2	−7	−3	$\boxed{28}$

−2	2	−5	−10	25
		−4	18	−16
	2	−9	8	$\boxed{9}$

−3	2	−5	−10	25
		−6	33	−69
	2	−11	23	$\boxed{-44}$

$\Rightarrow$ −3 is a lower bound

2a) $f(x) = \underbrace{-x^4+2x^2}_{1sc}\underbrace{\;-\;}_{1sc}\underbrace{x+7}_{1sc}$ $\Rightarrow$ 3 sign changes $\Rightarrow$ 3 or 1 positive zeros

$f(-x) = -(-x)^4+2(-x)^2-(-x)+7$

$= \underbrace{-x^4+2x^2}_{1sc}\underbrace{+x}_{NSC}\underbrace{+7}_{NSC}$ $\Rightarrow$ 1 sign change $\Rightarrow$ 1 negative zero

Summary

Neg zeros	Pos zeros	Non Real zeros
1	3	0
1	1	2

2b) Step 1 P(1) true? $(1\cdot 2) \stackrel{?}{=} \frac{1(1+1)(1+2)}{3}$

$2 \stackrel{?}{=} \frac{(2)(3)}{(3)}$

Step 2 Assume P(k) true for $2 \stackrel{\checkmark}{=} 2$ $\Rightarrow$ P(1)'s true.

Step 3 $k \geq 1$, Is P(k+1) true?

Consider p(k+1)

$(1\cdot 2)+(2\cdot 3)+ \dots + k(k+1) = \frac{k(k+1)(k+2)}{3}$

$(1\cdot 2)+(2\cdot 3)+ \dots k(k+1)+(k+1)(k+2) = \frac{k(k+1)(k+2)}{3}+(k+1)(k+2)$

$= \frac{k(k+1)(k+2)+3(k+1)(k+2)}{3}$

$= \frac{(k+1)(k+2)[k+3]}{3}$

$\Rightarrow$ p(k+1) true if p(k) Assumed true

$\Rightarrow$ by Math. Induction P(n) true for all $n \geq 1$.

Start with Step 2 Assump. Add (k+1)(k+2) to both sides, CD, Factor

this is correct pattern for RHS.

3. a) The series is Arithmetic: $-2-4=-6$

$-8-(-2)=-8+2=-6$

$-14-(-8)=-14+8=-6$

$\Rightarrow d=-6,\ a=4$

Now Formula for Arithmetic Series term $a_n=a_1+(n-1)d$

In our case $a_n=4+(n-1)(-6)$

$\Rightarrow$ Series: $\sum_{n=1}^{6} 4+(n-1)(-6)$ OR $\sum_{n=1}^{6}(10-6n)$

3b) $\sum_{k=1}^{500}(5-6k) = 5-6[1]+5-6[2]+5-6[3]\ldots$

$= -1\ -7\ -13\ \ldots$

this is Arithmetic: $-7-(-1)=-7+1=-6$

$-13-(-7)=-13+7=-6$

$\Rightarrow a_1=-1,\ d=-6$

for Arithmetic Series $S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{500}=\frac{500}{2}[2(-1)+(500-1)(-6)]$

$S_{500}=250[-2+(499)(-6)]=-749\ 000$

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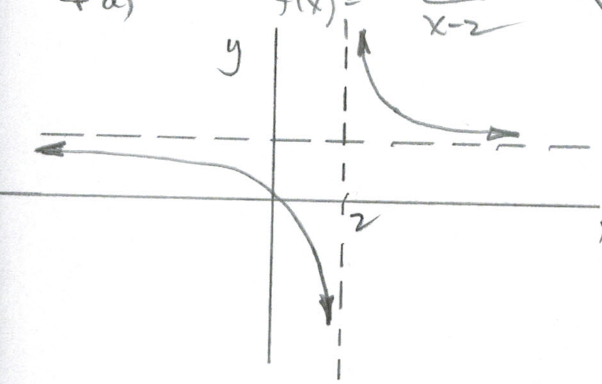
4 a) $f(x)=\frac{2x}{x-2}$



VA: $x-2=0 \Rightarrow x=2$, Num $\neq 0$ when $x=2 \Rightarrow$ $x=2$ is a VA.

HA: $\lim_{x\to\infty}\frac{2x}{x-2}=\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{\frac{2x}{x}}{\frac{x}{x}-\frac{2}{x}} = \lim_{x\to\infty}\frac{2}{1-\frac{2}{x}} = \frac{2}{1-0}=2$

$\Rightarrow y=2$ is HA

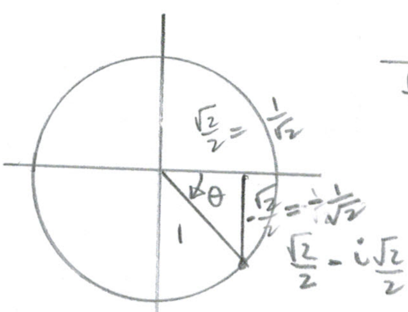
x	1.9	1.99	1.999	$\to 2$	2.001	2.01	2.1
$f(x)$	-38	-398	-3998		4002	402	42

$\Rightarrow \begin{cases} \text{as } x\to 2^-,\ f(x)\Rightarrow -\infty \\ \text{as } x\to 2^+,\ f(x)\Rightarrow +\infty \end{cases}$

x	10	100	$\to +\infty$
$f(x)$	2.5	2.04	$\to 2^+$ (above 2)

x	-10	-100	$\to -\infty$
$f(x)$	1.66	1.96	$\to 2^-$ (below 2)

4b) Step 1

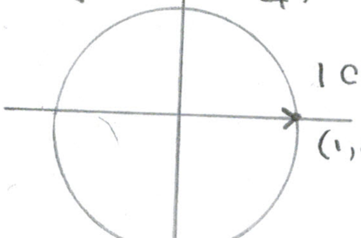


$r=\sqrt{\left(\frac{\sqrt2}{2}\right)^2+\left(-\frac{\sqrt2}{2}\right)^2}=\sqrt{\frac{2}{4}+\frac{2}{4}}=1$

$\theta=-45^\circ$ or $\theta=315^\circ$ or $\theta=-\frac{\pi}{4}^R$ or $\theta=\frac{7\pi}{4}^R$

Step 2 $\left(1\ \text{cis}\ -\frac{\pi}{4}\right)^{200}=1^{200}\ \text{cis}\left(-\frac{\pi}{4}^R\right)*200 = 1\ \text{cis}(-50\pi^R)$

Step 3



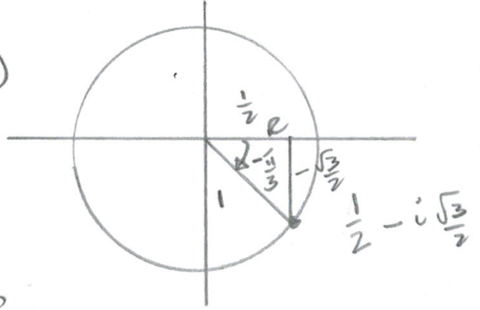
$1\ \text{cis}-50\pi^R = \cos(-50\pi^R)+i\sin(-50\pi^R) = \cos 0^R + i\sin 0^R$

$= 1+i(0)$

$= 1+0i$

$= 1$

5 a)



Step 1

$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\theta = -\frac{\pi}{3}^R$ or -60° or $\frac{5\pi}{3}^R$ or 300°

$\frac{1}{2} - i\frac{\sqrt{3}}{2} = 1 \text{Cis} -\frac{\pi}{3}^R$

Step 2

$1^{\frac{1}{5}}\text{Cis}(-\frac{\pi}{3}) * \frac{1}{5}$ $\quad 1^{\frac{1}{5}}\text{Cis}(-\frac{\pi}{3} + 2\pi) * \frac{1}{5}$ $\quad 1^{\frac{1}{5}}\text{Cis}(-\frac{\pi}{3} + 4\pi) * \frac{1}{5}$ $\quad 1^{\frac{1}{5}}\text{Cis}(-\frac{\pi}{3} + 6\pi) * \frac{1}{5}$ $\quad 1^{\frac{1}{5}}\text{Cis}(-\frac{\pi}{3} + 8\pi) * \frac{1}{5}$

$1\text{Cis} -\frac{\pi}{15}^R$ $\quad 1\text{Cis} \frac{5\pi}{15}^R$ $\quad 1\text{Cis} \frac{11\pi}{15}^R$ $\quad 1\text{Cis} \frac{17\pi}{15}^R$ $\quad 1\text{Cis} \frac{23\pi}{15}^R$

5b) WORD roots is better translated as zeros

$3-2i$ a zero $\Rightarrow [x-(3-2i)]$ is a Factor

$p(x)$ Real $\Rightarrow 3+2i$ is a zero $\Rightarrow [x-(3+2i)]$ is a Factor

$-i$ a zero $\Rightarrow [x+i]$ is a Factor

$p(x)$ Real $\Rightarrow i$ is a zero $\Rightarrow [x-i]$ is a Factor

2 a zero $\Rightarrow [x-2]$ is a Factor

$$\Rightarrow \quad p(x) = [(x-3)+2i][(x-3)-2i][x+i][x-i][x-2]$$
$$= [(x-3)^2+4][x^2+1][x-2]$$
$$= [x^2-6x+9+4][x^2+1][x-2]$$
$$= [x^2-6x+13][x^2+1][x-2]$$
$$= [x^2-6x+13][x^3-2x^2+x-2]$$
$$= x^5 - 2x^4 + x^3 - 2x^2 - 6x^4 + 12x^3 - 6x^2 + 12x + 13x^3 - 26x^2 + 13x - 26$$
$$= x^5 - 8x^4 + 26x^3 - 34x^2 + 25x - 26$$

(6a) $3.14151415\ldots = 3 + .1415 + .00001415 + \ldots$

$= 3 + \underbrace{.1415 + .1415 * 10^{-4} \ldots}$

Geom Series $a_1 = .1415 = \frac{1415}{10000}$

$r = 10^{-4} = \frac{1}{10^4} = \frac{1}{10000}$

$$S_\infty = \frac{a_1}{1-r} = \frac{\frac{1415}{10000}}{1 - \frac{1}{10000}} = \frac{1415}{\cancel{10000}} * \frac{\cancel{10000}}{9999}$$

$$\Rightarrow 3 + .\overline{1415} = 3 + \frac{1415}{9999} = \frac{3(9999)+1415}{9999} = \frac{31412}{9999}$$

6b) $(a+b)^{17} = \frac{17!}{17!\,0!}a^{17}b^0 + \frac{17!}{16!\,2!}a^{16}b^2 + \frac{17!}{15!\,3!}a^{15}b^3 + \ldots \frac{17!}{10!\,7!}a^{10}b^7 + \ldots$

Coefficient : $\frac{17!}{10!\,7!} = \frac{17 * \cancel{16}^{4} * \cancel{15}^{3} * \cancel{14}^{2} * 13 * \cancel{12}^{2} * 11 * \cancel{10!}}{\cancel{7 * 6 * 5 * 4 * 3 * 2} * 1 \quad \cancel{10!}} = 19448$

7a) 1 op. DRAW 13 CARDS FROM a deck with no spades, no hearts

$$= {}_{26}C_{13} = \frac{26!}{13!(26-13)!}$$

b) 2 ops. $\left\{ \begin{array}{l} \text{choose} \quad \text{3 defectives from 3 defective computers} \\ \quad \text{"} \quad \text{3 good ones from 10 good computers} \end{array} \right.$

$$\underset{\text{(defects)}}{{}_{3}C_{3}} * {}_{10}C_{3}$$
