



2013 ENGG317 Final Examination, Wed 24 Apr, 15:30-18:30, Gold Gym

Instructions:

- **Most Important:** Indicate name, last name, student ID, and course section on the answer booklet
 - 09:00 – 09:50 MWF (L01) – Epstein
 - 09:00 – 09:50 MWF (L02) – Sudak
 - 15:00 – 15:50 MWF (L03) – Federico
 - 16:00 – 16:50 MWF (L04) – Federico
- **Write clearly: bad writing may cause the markers not to understand what you mean**
- Clearly indicate your answer (e.g., underline or draw a box)
- Answer all questions; maximum marks for each question are indicated
- This is an open textbook and closed notes exam
- Calculators are permitted (Schulich calculators only)
- Exam duration is 180 minutes

Question 1 [10 marks]

A rod of length L has hollow circular cross-section with external radius R and internal radius $R/2$, is made of a perfectly elastoplastic material with shear modulus G and yielding stress τ_y , and is fixed at one end. An external torque is applied at the free end.

- a) Calculate the values T_y of the torque and ϕ_y of the twist angle at the onset of yielding.

$$T_y = \left(\frac{15}{32}\right) \pi \tau_y R^3, \phi_y = \frac{\tau_y L}{GR}$$

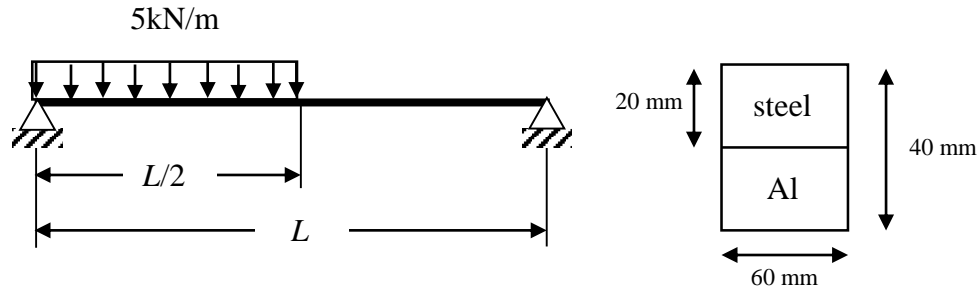
- b) Calculate the values T_p of the torque and ϕ_p of the twist angle when the cross-section is first fully plasticised.

$$T_p = \frac{7}{12} \pi \tau_y R^3, \phi_p = 2 \frac{\tau_y L}{GR}$$

- c) For both cases (a) and (b), draw the diagram of the internal torque T .

Question 2 [10 marks]

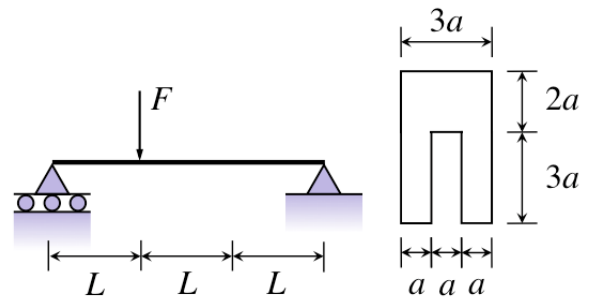
Steel (Young's modulus $E_s = 210 \text{ GPa}$, allowable stress $S_{all,s} = 150 \text{ MPa}$) and aluminum ($E_{Al} = 70 \text{ GPa}$, $S_{all,Al} = 100 \text{ MPa}$) are bonded together to form a composite beam. The beam is simply supported, and loaded by a uniformly distributed load of 5 kN/m as shown. Determine the largest allowable span L .



$L_{all} = 2220 \text{ mm}$

Question 3 [10 marks]

A beam, with supports, cross-section, dimensions, and loads as in the figure, is made of a perfectly elastoplastic material with yielding stress S_Y . Find the value of the concentrated force F for which the most loaded cross-section is completely plasticised.

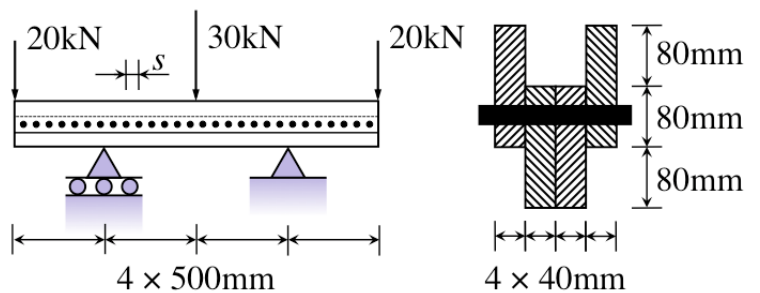


$$F_p = \frac{45}{2L} a^3 \sigma_y$$

Question 4 [10 marks]

A beam is made of four equal planks connected by 12-mm diameter bolts and spaced at equal intervals along the longitudinal axis of the beam, as shown.

- Draw the shear-force diagram of the beam and determine its maximum absolute value.
- If the allowable average shearing stress in the bolts is of 60 MPa , determine the maximum allowable spacing s of the bolts.



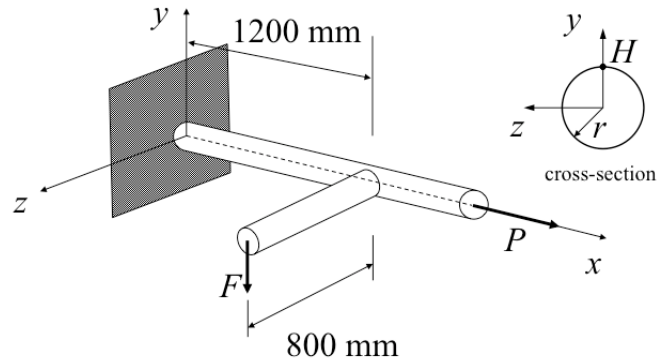
Most loaded section has a shear force of 20 kN ,

$S_{all} = 126 \text{ mm}$

Question 5 [10 marks]

A cantilevered bar of radius $r = 25$ mm supports an axial load $P = 450$ N and a vertical load $F = 900$ N acting at the end of an 800 mm long arm.

- Find the most loaded cross-section.
- At the most loaded cross-section, determine the state of stress at point H located on the y axis and outer surface (see figure).
- Draw the original stress state as well the principal stresses on respective properly oriented elements.



$$\sigma_x = \frac{12LF}{\pi r^3}, \tau_{xy} = 0, \tau_{xz} = \frac{4LF}{\pi r^3}$$

$$\sigma_1 = 2160 \frac{F}{\pi r^2}, \sigma_2 = -190 \frac{F}{\pi r^2}, \theta_p = 16.5 \text{ deg}, \theta_s = 49.5 \text{ deg}$$