Last Name Mao First Name Michael

Winter 2015 ENGG 317 - Mechanics of Solids

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* course coordinator

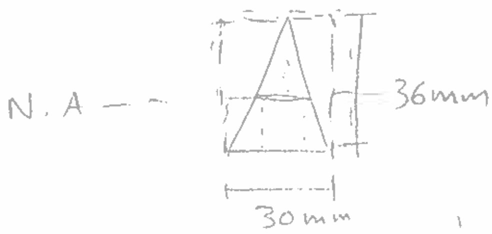
2015 ENGG317 Final Examination, Tues 28 Apr, 1900-2200, Red Gym

Instructions:

- **Most Important**: Indicate name, last name, student ID, and course section on the answer booklet
  - 09:00 – 09:50 MWF (L01) – Epstein
  - 09:00 – 09:50 MWF (L02) – Sudak
  - 15:00 – 15:50 MWF (L03) – Singh
  - 16:00 – 16:50 MWF (L04) – Sudak
- **Write clearly: bad writing may cause the markers not to understand what you mean**
- Clearly indicate your answer (e.g., underline or draw a box)
- Answer all questions; maximum marks for each question are indicated
- This is an open textbook and closed notes exam
- Calculators are permitted (Schulich calculators only)
- Exam duration is 180 minutes
- No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
- If during the course of the examination a student becomes ill or receives word of a domestic affliction, the student must report at once to the supervisor, hand in the unfinished paper and request that it be cancelled. If physical and/or emotional ill health is the cause, the student must report at once to a physician/counsellor so that subsequent application for a deferred examination is supported by a completed Physician/Counsellor statement form. **Once an examination has been handed in for marking a student cannot request that the examination be cancelled for whatever reason. Such a request will denied. Retroactive withdrawals will also not be considered.**

Question	Mark
1	8
2	8
3	10
4	4
5	8

38/50


N. A
36mm
30mm

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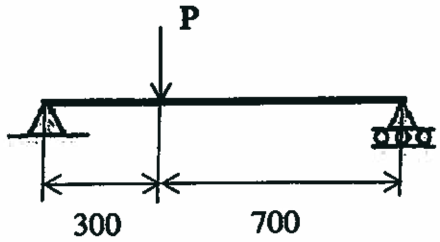
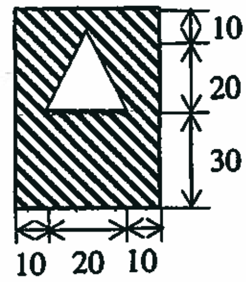
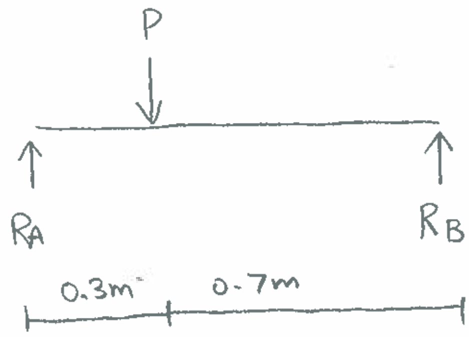
Question 1 [10 marks]

The beam shown is made of an elastoplastic material with σ_y = 200 MPa. Find the load P that would cause the beam to collapse. All dimensions are in millimetres. (Hint: collapse occurs when the bending moment under the load equals the fully plastic moment).





Statics:



$\circlearrowleft + \sum M_B = 0$

$P(0.7m) - R_A(1m) = 0$

$R_A = 0.7P$

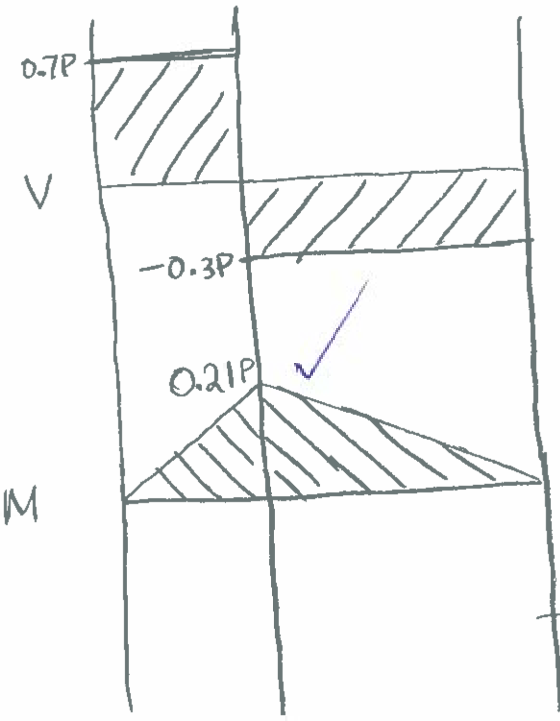
$+\uparrow \sum F = 0$

$R_A + R_B = P$

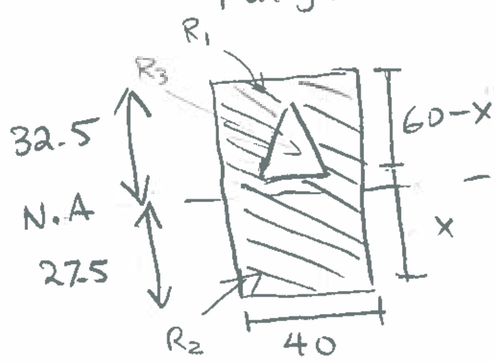
$R_B = P - 0.7P$

$R_B = 0.3P$

Fully Plastic: NA where areas equal



$M_{max} = 0.21P$



Area of triangle $= \frac{1}{2}(20)(20) = 200$

$(40)(x) = (60-x)(40) - 200$

$40x = 2400 - 40x - 200$

$80x = 2200$

$x = 27.5mm$ ✓

Check: Area above N.A must equal area below N.A ✓

$(32.5)(40) - 200 = (27.5)(40)$

$1100 = 1100$

$M_{max} = M$

$M_{max} = 0.21P$

R_1 is full rectangle including area

$R_1 = \sigma_y A_1 = \sigma_y (32.5)(40)$ $d_1 = 16.25$

$R_2 = \sigma_y A_2 = \sigma_y (40)(27.5)$ $d_2 = 13.75$ ✗

$R_3 = \sigma_y A_3 = \sigma_y (\frac{1}{2})(20)(20)$ $d_3 = 6.667$

$M = R_1 d_1 + R_2 d_2 - R_3 d_3$

$M = \sigma_y (2.1125\times10^{-5} + 1.5125\times10^{-5} - 1.334\times10^{-6})$

$M = 200\times10^6 (3.4916\times10^{-5}) = 6.98 \text{ kN·m}$

$0.21P = 6.98 \text{ kN·m}$

$P = 33.23 \text{ kN}$

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$$M_p = \frac{A_T}{2}(\bar{y}_1 + \bar{y}_2)$$

$$M_p = R_1 d_1$$

$$= \sigma_y \frac{(30)(36)}{2} ($$

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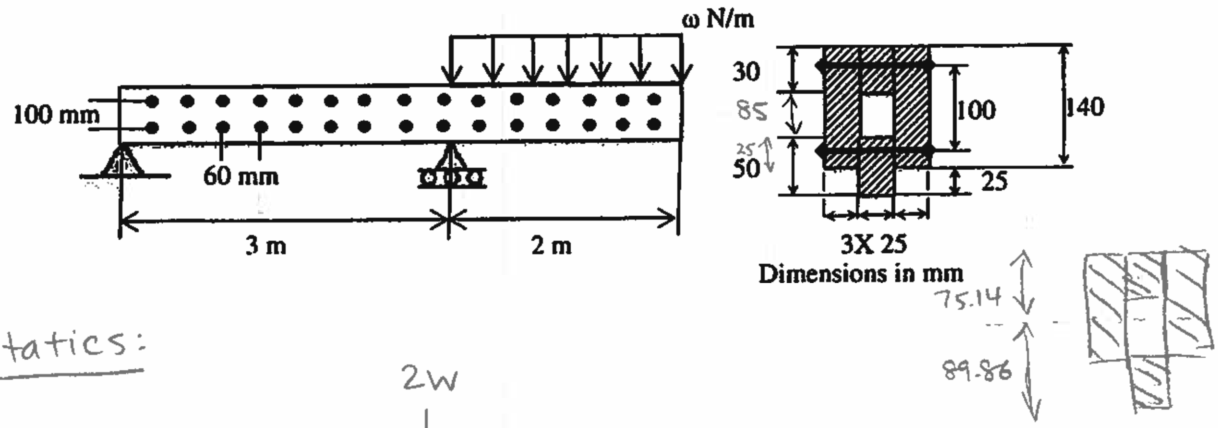
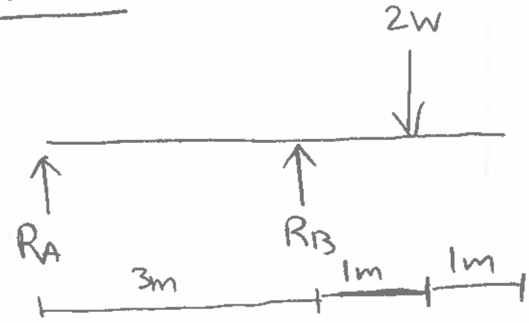
Question 2 [10 marks]
A beam is constructed from four planks connected by two 12-mm diameter bolts that are spaced at equal intervals along the longitudinal axis of the beam as shown.

a) Draw the shear force diagram for the beam and determine the maximum absolute value of the shear force in terms of ω.
b) If the allowable shearing force in each nail is 730 N determine the maximum load ω.



a) Statics:



$\circlearrowleft + \sum M_A = 0$

$R_B(3m) - 2w(4m) = 0$

$3R_B = 8w$

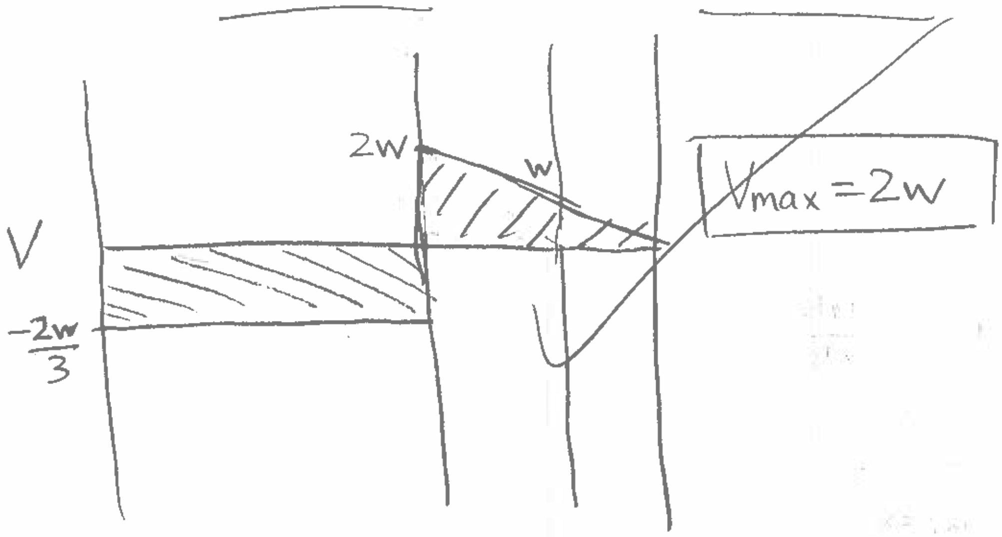
$R_B = \frac{8w}{3}$

$+\uparrow \sum F = 0$

$R_A + R_B = 2w$

$R_A = 2w - \frac{8w}{3}$

$R_A = \frac{-2w}{3}$



$V_{max} = 2w$

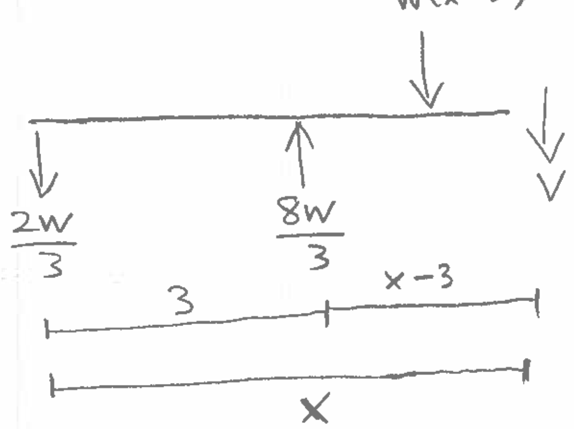
Section 1 $0 \le x < 3m$

$+\uparrow \sum F = 0$

$-V - \frac{2w}{3} = 0$

$V = \frac{-2w}{3}$

Section 2 $3 \le x < 5m$



At $x=3, V=2w$
$x=5, V=0$

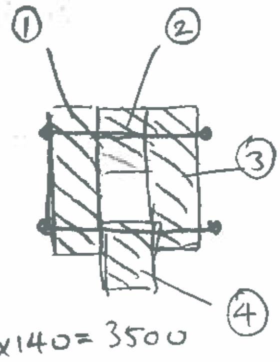
$V = 5w - wx$

$+\uparrow \sum F = 0$

$\frac{-2w}{3} + \frac{8w}{3} - w(x-3) = V$

$2w - wx + 3w = V$

b)



Area

① 25×140 = 3500

② 25×30 = 750

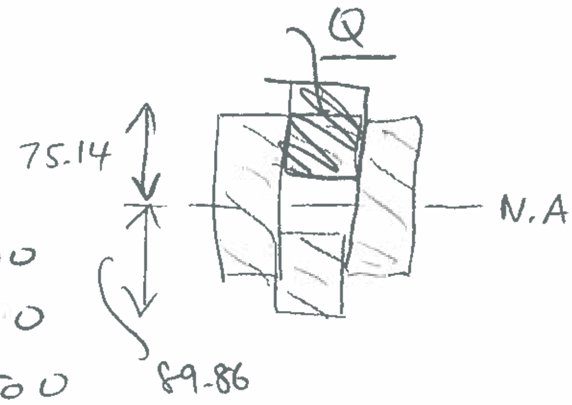
③ 25×140 = 3500

④ 25×50 = 1250

Locate N.A

	A	$\bar{y}$	$\bar{y}A$
①	3500	95	332500
②	750	150	112500
③	3500	95	332500
④	1250	25	31250
Σ	9000		808750

?



$$\bar{Y} = \frac{808750}{9000} = 89.86\,mm$$ ✓

$$I_1 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(25)(140)^3 + (3500)(5.14)^2$$

$$I_1 = 5716666.67 + 92468.6$$

$$I_1 = 5.809 \times 10^{-6}\,m^4$$

$$I_2 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(25)(30)^3 + (750)(60.14)^2$$

$$I_2 = 56250 + 2712614.7$$

$$I_2 = 2.77 \times 10^{-6}\,m^4$$

$$I_3 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(25)(140)^3 + (3500)(5.14)^2$$

$$I_3 = 5.809 \times 10^{-6}\,m^4$$

$$I_4 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(25)(50)^3 + (1250)(64.86)^2$$

$$I_4 = 260416.6667 + 5258524.5$$

$$I_4 = 5.519 \times 10^{-6}\,m^4$$

$$I = I_1 + I_2 + I_3 + I_4 = 1.991 \times 10^{-5}\,m^4$$

what about bottom block

2

$$Q = \bar{y}A = (25mm)(30mm)(60.14mm) = 4.5105 \times 10^{-5}\,m^3$$

b/c I chose the Q at ② it is subjected to double shear

$$F_{nail} = qs \Rightarrow F_{nail} = \frac{1}{2}qs$$

$$q = \frac{VQ}{I} \qquad F_{nail} = \frac{1}{2}s\left(\frac{VQ}{I}\right)$$

s = 60mm

V = 2w

$I = 1.991 \times 10^{-5}\,m^4$

$Q = 4.5105 \times 10^{-5}\,m^3$

F_{nail} = 730N

w = ?

$$730 = \frac{\frac{1}{2}(60 \times 10^{-3})(2w)(4.5105 \times 10^{-5})}{1.991 \times 10^{-5}}$$

W = 5371.6 N/m

2

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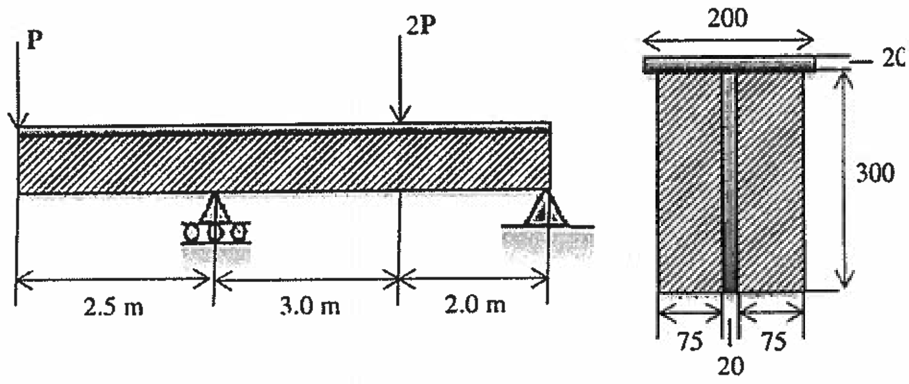
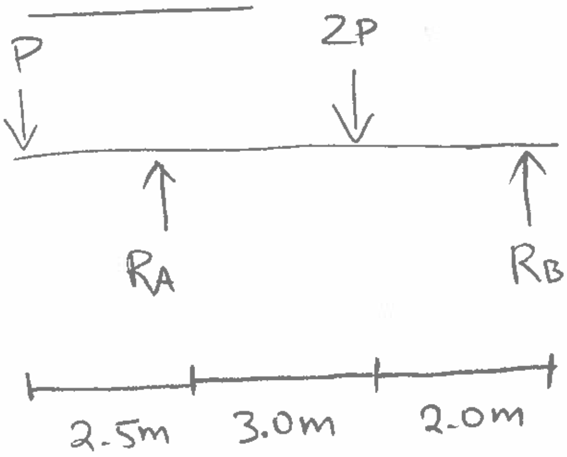
Question 3 [10 marks]

A simply supported composite beam is loaded by two concentrated forces as shown. The beam is constructed from two steel plates welded together to form a beam in the shape of a 'T'. The 'T' section has been strengthened by securely fastening two wood members to it as shown in the figure. The modulus of elasticity is 12.5 GPa for the wood and 200 GPa for the steel. Determine the largest force **P** that can be applied to the beam if the allowable stress for wood and steel is 5 MPa and 90 MPa, respectively.



dimensions in mm

Statics:



$\circlearrowleft + \sum M_B = 0$

$2P(2.0m) - R_A(5m) + P(7.5m) = 0$

$11.5P = 5R_A$

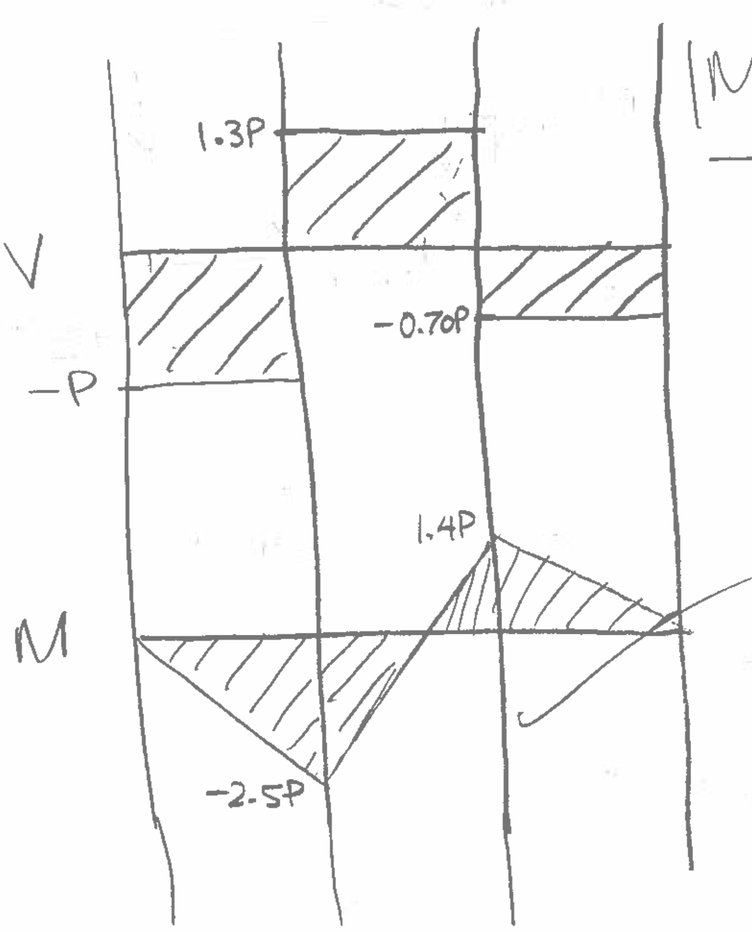
$R_A = 2.3P$

$+\uparrow \sum F = 0$

$R_A + R_B - P - 2P = 0$

$R_B = 3P - 2.3P$

$R_B = 0.7P$



$|M_{max}| = 2.5P$

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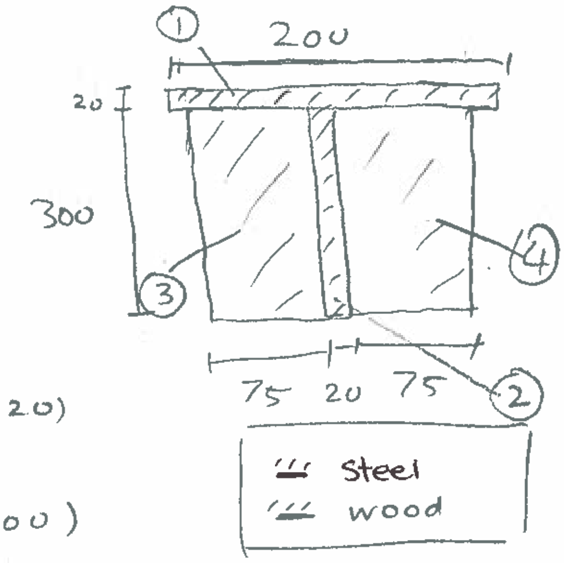
$$n = \frac{E_s}{E_w} = \frac{200}{12.5} = 16$$



Locate N.A

	nA	$\bar{y}$	$nA\bar{y}$
①	64000	310	19840000
②	96000	150	14400000
③	22500	150	3375000
④	22500	150	3375000
Σ	205000		40990000

nA:

① $16 \times (200 \times 20) = 64000$

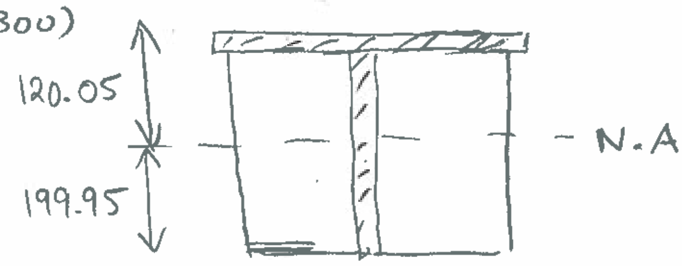
② $16 \times (20 \times 300) = 96000$

③ $1 \times (75 \times 300) = 22500$

④ $1 \times (75 \times 300) = 22500$

$$\bar{y} = \frac{40990000}{205000}$$

$$\bar{y} = 199.95\,mm$$



$$I_1 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(200)(20)^3 + (4000)(110.05)^2$$

$$I_1 = 133333.33 + 48444010$$

$$I_1 = 4.8577 \times 10^{-5}\,m^4$$

$$I_2 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(20)(300)^3 + (6000)(49.95)^2$$

$$I_2 = 45000000 + 14970015$$

$$I_2 = 5.997 \times 10^{-5}\,m^4$$

$$I_3 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(75)(300)^3 + (22500)(49.95)^2$$

$$I_3 = 168750000 + 56137556.25$$

$$I_3 = 2.249 \times 10^{-4}\,m^4$$

$$I_4 = I_3 = 2.249 \times 10^{-4}\,m^4$$

$$I = n(I_1 + I_2) + I_3 + I_4$$

$$I = 16(4.8577 \times 10^{-5} + 5.997 \times 10^{-5}) + 2.249 \times 10^{-4} + 2.249 \times 10^{-4}$$

$$I = 1.736752 \times 10^{-3} + 4.498 \times 10^{-4}$$

$$I = 2.186552 \times 10^{-3}\,m^4$$

P = 2181N is the largest force that can be applied

For wood: $n = 1$, $M = 2.5P$, $y = 199.95 \times 10^{-3}\,m$, $I = 2.186552 \times 10^{-3}\,m^4$, $\sigma = 5 \times 10^{6}\,Pa$

$$\sigma = \frac{nMy}{I}$$

$$nMy = \sigma I$$

$$M = \frac{\sigma I}{ny} = \frac{(5 \times 10^6)(2.186552 \times 10^{-3})}{(1)(199.95 \times 10^{-3})} = 2.5P$$

$$\frac{54677.46937}{2.5} = P$$

P = 21871N

For steel: $n = 16$, $M = 2.5P$, $I =$ same as wood, $y =$ same as wood, $\sigma = 90 \times 10^{6}\,Pa$

$$M = \frac{\sigma I}{ny} = \frac{(90 \times 10^6)(2.186552 \times 10^{-3})}{(16)\cdot(199.95 \times 10^{-3})}$$

$$2.5P = \frac{196789.68}{3.1992}$$

$$P = 24605\,N$$

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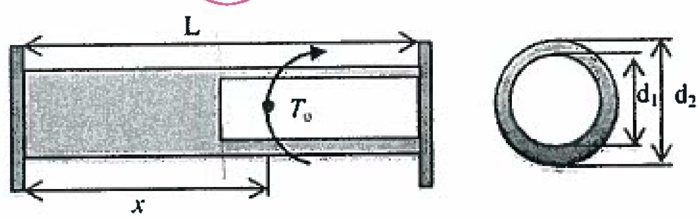
Question 4 [10 marks]

A circular bar *AB* with ends fixed against rotation contains a hole extending for half its length as shown. The outer diameter of the bar is d_2 and the inner diameter of the hole is d_1 and the total length of the bar is *L*. If and external torque T_o is applied at a distance '*x*' from the left hand side determine:



a) the location '*x*' (in terms of L, d_2, d_1) where the external torque is applied assuming the reactive torques at the supports are equal,

b) the location and the maximum angle of twist assuming the following parameters: $L = 1270$ mm, $d_2 = 76$ mm, $d_1 = 61$ mm, $T_o = 10$ kNm, G = 73 GPa.

Statically Interdeterminate bar

$C_2 = \frac{1}{2}d_2$

FBD of Bar AB $C_1 = \frac{1}{2}d_1$

a) $J_2 = \frac{\pi}{2}(C_2^4 - C_1^4)$

$J_2 = \frac{\pi}{2}((\frac{1}{2}d_2)^4 - (\frac{1}{2}d_1)^4)$

$J_2 = \frac{\pi}{2}(\frac{1}{16}d_2^4 - \frac{1}{16}d_1^4)$

$J_2 = \frac{\pi}{2}\cdot\frac{1}{16}(d_2^4 - d_1^4)$

$J_2 = \frac{\pi}{32}(d_2^4 - d_1^4)$

$J_2 = \frac{\pi(d_2^4 - d_1^4)}{32}$

$T_A = T_B = T$

$2T = T_o$

The total angle of twist must be zero b/c both of its end are restrained.

$\phi = \phi_1 + \phi_2 = 0$

$\phi = \frac{Tx}{J_1G} + \frac{T(L-x)}{J_2G} = 0$

-2 should have 3 terms here

$\frac{Tx}{J_1G} = \frac{-T(L-x)}{J_2G}$

$\frac{Tx}{J_1} = \frac{-TL+Tx}{J_2}$

$T(-L+x)J_1 = Tx(J_2) \Rightarrow J_2x = -J_1L + J_1x$

$J_1L = x(J_1 - J_2)$

$x = \frac{J_1L}{J_1 - J_2} = \frac{\frac{\pi}{32}(d_2^4 - d_1^4)L}{\frac{\pi}{32}d_2^4 - \frac{\pi}{32}d_1^4 - \frac{\pi}{32}d_1^4}$

$J_1 = \frac{\pi}{2}C_1^4 = \frac{\pi}{2}(\frac{1}{2}d_1)^4 = \frac{\pi}{32}d_1^4$

$\rightarrow x = \frac{\pi}{32}(d_2^4 - d_1^4)L \times \frac{32}{\pi d_2^4}$

$x = \frac{(d_2^4 - d_1^4)L}{d_2^4}$ ✗ -2

b) $J = \frac{\pi}{2}(d_2^4 - d_1^4) = \frac{\pi}{2}((76\times10^{-3})^4 - (61\times10^{-3})^4) = 3.0656\times10^{-5}\ m^4$

$x = \frac{(d_2^4 - d_1^4)L}{d_2^4}$

$x = \frac{((76\times10^{-3})^4 - (61\times10^{-3})^4)(1.27)}{(76\times10^{-3})}$

$x = 3.26\times10^{-4}\ m$ ✗ -1

$\phi = \frac{T_oL}{GJ}$

$\phi = \frac{(10\times10^3 Nm)(1.27m)}{(73\times10^9 Pa)(3.0656\times10^{-5} m^4)}$

$\phi = \frac{12700}{2237888} = 0.325°$ ✗ -1

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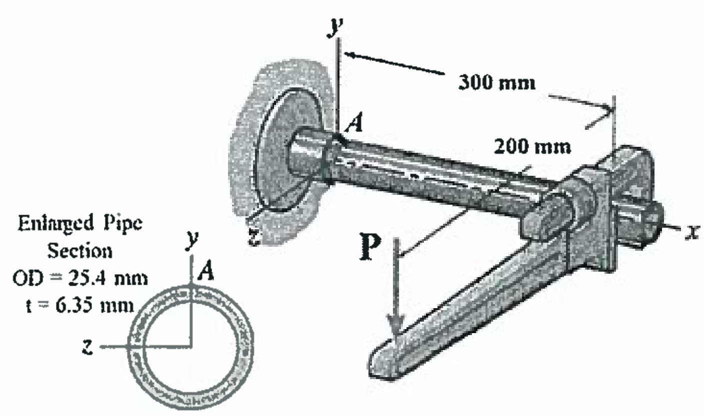
Question 5 [10 marks]

A vertical force P is applied to a pipe wrench whose handle is parallel to the z-axis. The pipe has an outside diameter of 25.4 mm and a wall thickness of t = 6.35 mm. The pipe material has a yield stress of σ_y = 240 MPa, an elastic modulus of E=200 GPa, and a shear modulus G= 75 GPa.

a) Determine the components of stress at point A on the cross-section where the pipe threads begin. Leave the stress components in terms of a multiple of the force P.

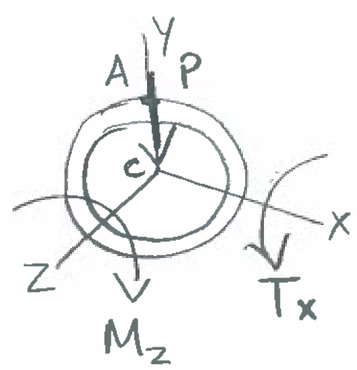
b) If the maximum principal stress at point A is not to exceed the yield stress, determine the maximum load P that can be applied to the wrench.



$\frac{25.4mm}{2} = 12.7mm$

Force Couple System

a)



$T = P(0.2m) = 0.2P(N\cdot m)$

$M_z = P(0.3m) = 0.3P(N\cdot m)$

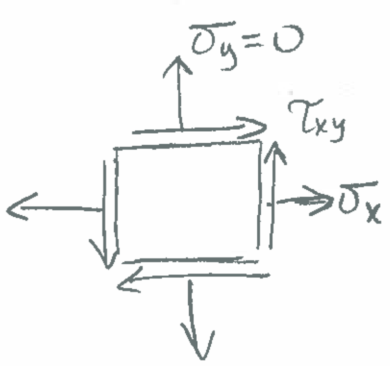
$\boxed{\sigma_y = 0}$ $\sigma_x = \frac{-Mc}{I} = \frac{-0.3P(12.7\times10^{-3})}{\frac{\pi}{4}(12.7\times10^{-3})^4}$ -0.5

$\sigma_x = \frac{3.81\times10^{-3}P}{2.043\times10^{-8}} = 0.186P(MPa)$

$\tau_{xz} = \frac{Tc}{J}$

$\tau_{xz} = \frac{(0.2P)(12.7\times10^{-3})}{\frac{\pi}{2}(12.7\times10^{-3})^4} = \frac{2.54\times10^{-3}P}{4.0863\times10^{-8}}$ -0.5

$\tau_{xz} = 0.0622P(MPa)$

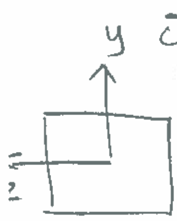


b)

$\sigma_{max,min} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$

$\sigma_{max,min} = \frac{0.186P}{2} \pm \sqrt{\left(\frac{0.186P}{2}\right)^2+(0.0622P)^2}$

$\sigma_{max,min} = 0.093P \pm 0.1119P$



$\sigma_x = 0.186P$
$\sigma_y = 0$
$\tau_{xy} = 0.0622P$

$\sigma_{max} = 0.093P + 0.1119P = 0.2049P$

$\sigma_{min} = 0.093P - 0.1119P = -0.0189P$

Only required max principal stress

yield stress : $\sigma_y = 240MPa$

$\sigma_{max} = 0.2049P$

$240 = 0.2049P$

$\boxed{P = 1171.3N}$

-1

~~$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$~~

$\tan 2\theta_p = \frac{2(0.0622P)}{0.186P}$

$2\theta_p = 33.78°$

$\theta_p = 16.89°$, 106.89°