

QUIZ 3

24th October 2016**Time Allowed: 30 minutes**

A mass of 10 g of water at an initial temperature of 400°C in a vertical piston-cylinder assembly (see figure below) is gradually cooled. The piston has a mass of 50 kg, an area of 0.01 m² and moves freely in the cylinder with negligible friction. The local atmospheric pressure and acceleration of gravity are 101 kPa and 9.80 m/s². As the water is cooled the height of the piston h falls until it reaches the stops, when it can no longer move down. Cooling continues until the temperature reaches 30°C.

- Assuming that the volume of the stops is negligible, determine the initial height of the piston and the initial pressure of the water.
- If the quality of the water is 80% when the piston just reaches the stops, determine the height of the stops h_s .
- Determine the final quality of the water and the total heat and work transfer.
- Sketch the process on a p - v diagram.

(a) Force balance on the piston:

$$P_1 A = P_{atm} A + mg$$

$$\therefore P_1 = P_{atm} + \frac{mg}{A}$$

$$P_1 = 101 \frac{\text{kN}}{\text{m}^2} + \frac{50 \times 9.80 \text{ N}}{0.01 \text{ m}^2} \cdot \frac{1 \text{ kN}}{1000 \text{ N}} = 150 \frac{\text{kN}}{\text{m}^2}$$

$$P_1 = 150 \text{ kPa or } 1.50 \text{ bar}$$

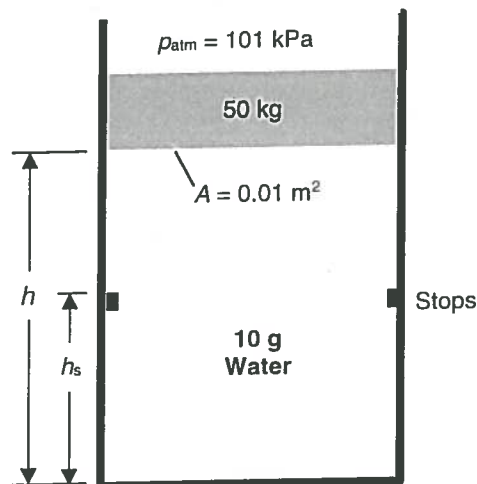


TABLE A-4 $T_1 = 400^\circ\text{C}$ $P_1 = 1.5 \text{ bar}$ $v_1 = 2.067 \text{ m}^3/\text{kg}$.

$$h_1 = \frac{V_1}{A} = \frac{m v_1}{A} = \frac{2.067 \times 0.01 \text{ m}^3}{0.01 \text{ m}^2} = 2.067 \text{ m}$$

$$h_1 = 2.067 \text{ m} \quad P_1 = 150 \text{ kPa}$$

(b) $P_2 = P_1$ (force balance on piston has not changed).
 $x_2 = 0.8$ TABLE A-3 $P_2 = 1.5 \text{ bar}$ $v_f = 1.0528 \times 10^{-3} \text{ m}^3/\text{kg}$
 $v_g = 1.159 \text{ m}^3/\text{kg}$

$$v_2 = x v_g + (1-x) v_f = (0.8 \cdot 1.159 + 0.2 \cdot 1.0528 \times 10^{-3}) = 0.9274 \text{ m}^3/\text{kg}$$

$$h_2 = h_s = \frac{V_2}{A} = \frac{m v_2}{A} = \frac{0.01 \times 0.9274}{0.01} = 0.9274 \text{ m}$$

$$h_2 = h_s = 0.9274 \text{ m}$$

(c) $h_3 = h_2 = h_s$ $T_3 = 30^\circ\text{C}$

$$\therefore v_3 = v_2 = 0.9274 \text{ m}^3/\text{kg}$$

TABLE A-2 $T = 30^\circ\text{C}$
 $v_f = 1.0043 \times 10^{-3} \text{ m}^3/\text{kg}$ $v_g = 32.894 \text{ m}^3/\text{kg}$

$$v_3 = x_3 v_g + (1-x_3) v_f$$

$$\therefore x_3 = \frac{v_3 - v_f}{v_g - v_f}$$

$$x_3 = \frac{(0.9274 - 1.0043 \times 10^{-3})}{(32.894 - 1.0043 \times 10^{-3})} = 0.02816 \text{ or } 2.816\%$$

$$x_3 = 0.02816$$

(Continue overleaf if you need more space)

CLOSED SYSTEM: 0.01 kg H₂O.

$$W_{12} = \int_1^2 p dV = m p (v_2 - v_1)$$

$W_{23} = 0$ (constant volume)

$$Q_{13} - W_{13} = m(u_3 - u_1)$$

1st LAW (assuming: $\Delta KE = \Delta PE = 0$)

$$\therefore Q_{13} = m(u_3 - u_1) + m p (v_2 - v_1)$$

$$W_{13} = W_{12} = 150 \frac{\text{kN}}{\text{m}^2} \cdot 0.01 \text{ kg} (0.9274 - 2.067) \frac{\text{m}^3}{\text{kg}} \cdot \frac{1 \text{ kJ}}{1 \text{ kNm}}$$

$$\therefore W_{13} = \underline{\underline{-1.7094 \text{ kJ}}}$$

ie Net work of 1.71 kJ done on the water.

TABLE A-2: $T_3 = 30^\circ\text{C}$ $u_g = 2416.6 \text{ kJ/kg}$
 $u_f = 125.78 \text{ kJ/kg}$

$$u_3 = [0.02816 \times 2416.6 + (1 - 0.02816) \times 125.78] \text{ kJ/kg}$$

$$u_3 = 190.3 \text{ kJ/kg}$$

TABLE A-4 $T_1 = 400^\circ\text{C}$ $P_1 = 1.5 \text{ bar}$: $u_1 = 2967.3 \text{ kJ/kg}$

$$Q_{13} = 0.01 \text{ kg} (190.3 - 2967.3) \frac{\text{kJ}}{\text{kg}} - 1.709 \text{ kJ}$$

$$Q_{13} = \underline{\underline{-29.48 \text{ kJ}}}$$

ie 29.5 kJ net heat transfer from the water to the surroundings.

(d) p-v diagram:

