Concordia University

ENGR 233 - First Midterm Exam - February 10, 2012 Instructor: Galia Dafni

Total time: 75 minutes Total marks: 30

Allowable materials: writing utensils. You may **NOT** use notes, books, calculators or any other materials.

Write your answers in the examination booklet. Write clearly and neatly and show all your work in order to receive full marks.

Self-serve formula sheet (not marked).

$\hat{B} =$	$a_T =$	$\hat{T} =$	$\kappa =$	$s(t) =$	$a_N =$	$\hat{N} =$

$\frac{\vec{v}\cdot\vec{a}}{\lVert\vec{v}\rVert}$	$\hat{T}\times\hat{N}$	$\frac{\lVert\vec{v}\times\vec{a}\rVert}{\lVert\vec{v}\rVert^3}$	$\int_a^t \lVert\vec{r}\,'(\tau)\rVert d\tau$	$\frac{\lVert\vec{v}\times\vec{a}\rVert}{\lVert\vec{v}\rVert}$	$\frac{d\hat{T}/dt}{\lVert d\hat{T}/dt\rVert}$	$\frac{\vec{r}\,'(t)}{\lVert\vec{r}\,'(t)\rVert}$
(a)	**(b)**	**(c)**	**(d)**	**(e)**	**(f)**	**(g)**

Problem 1 (10 marks). Let

$$x(t) = \sin \pi t,\ y(t) = \cos \pi t,\ z(t) = t.$$

(i) Describe the shape of the curve given by the vector-valued function $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$.

(ii) Find the vector equation of the tangent line to this curve at the point $(0, -1, 1)$.

(iii) If $w = e^{xyz}$, write down the Chain Rule to find $\frac{dw}{dt}$, and evaluate it at $t = 1$. (Note: the answer is a number, don't express it in terms of t.)

Problem 2 (10 marks). Let

$$\vec{r}(t) = t\hat{\imath} + (t - 16t^2)\hat{\jmath}$$

be the equation of the trajectory of a moving particle in the xy plane.

(i) Find the velocity and acceleration of the particle at time t.

(ii) Find the tangential and normal components of the acceleration at time $t = \frac{1}{32}$.

(iii) What kind of motion is described by this equation? What happens at time $t = \frac{1}{32}$?

Problem 3 (10 marks). The following questions refer to the function

$$f(x, y) = x^2 + 4y^2.$$

(i) Sketch some level curves of f. How is the gradient of f positioned in relation to these curves?

(ii) Give a unit vector in the direction in which the function increases most rapidly at the point $(2, 1)$, and compute the maximum rate.

(iii) Find the point on the graph of the function at which the tangent plane is parallel to the plane $4x + 4y - z = 0$. (Hint: write the surface as $x^2 + 4y^2 - z = 0$).