ELEC 242 – Mid-Term Exam (REVIEW)

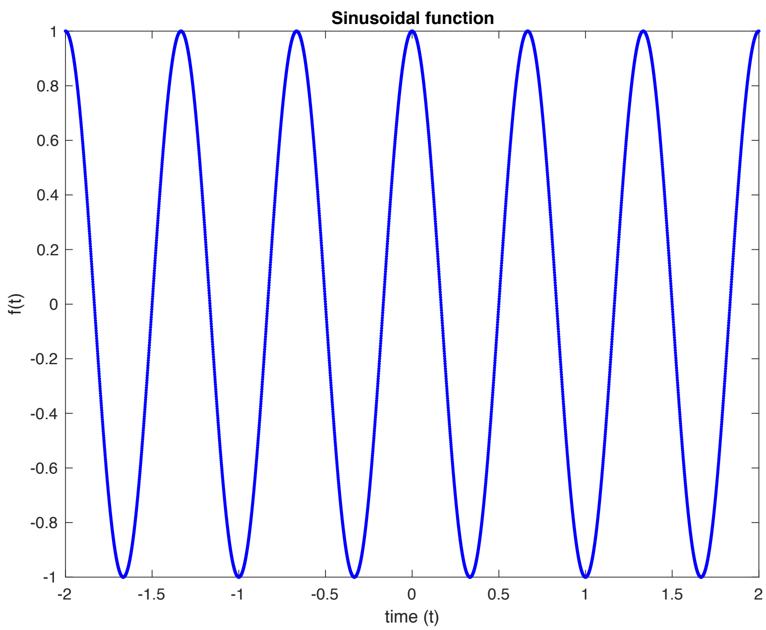
1. Sketch the following functions of time:

a) $x_1(t) = \cos(3\pi t)$

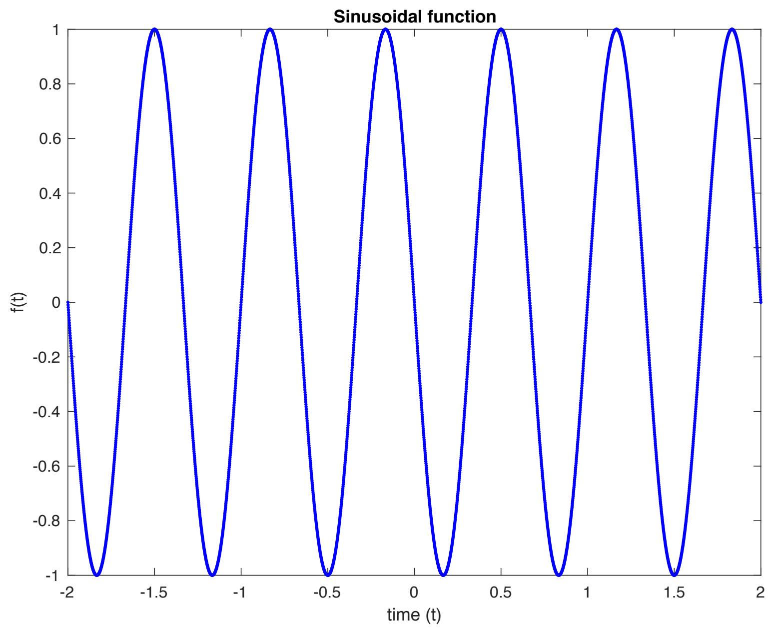
b) $x_2(t) = \cos(3\pi t + \pi / 2)$

Solution:

a)



b)



2. Determine if the following signals are periodic. If the signals are periodic, calculate the fundamental period of the overall signal.

a) $x_1(t) = \sin(4\pi t/9) + 2\cos(2t/5)$

b) $x_2(t) = 2 + t + \cos(4\pi t/9)$

<u>Solution:</u>

a)

$$\omega_1 = 4\pi/9 \tag{1}$$

$$T_1 = \frac{2\pi}{4\pi/9} = \frac{9}{2} \tag{2}$$

$$\omega_1 = 2/5 \tag{3}$$

$$T_2 = \frac{2\pi}{2/5} = 5\pi \tag{4}$$

The ratio of T_1 / T_2 is not a rational number, therefore the overall signal is not periodic.

b) The signal is not periodic because of a function *t* that is being added!

3. Determine if the following signals are energy or power signals or neither. Calculate the energy and power of the signals in each case:

a) $x_1(t) = -e^{-3t}$

b) $x_2(t) = -e^{-3t}u(t)$

<u>Solution:</u>

a)

$$E_x = \int_{-\infty}^{\infty} \left|-e^{-3t}\right|^2 dt = \int_{-\infty}^{\infty} e^{-6t}\,dt = -\frac{1}{6}e^{-6t}\Big|_{-\infty}^{\infty} = \frac{1}{6}\left(e^{\infty} - e^{-\infty}\right) = \frac{1}{6}\left(e^{\infty} - 0\right) = \infty \qquad (5)$$

Average power:

$$\begin{aligned} P_x &= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} \left|x_1(t)\right|^2 dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} e^{-6t}\,dt = \lim_{T\to\infty}\frac{1}{T}\left[-\frac{1}{6}e^{-6t}\Big|_{-T/2}^{T/2}\right] \\ &= \frac{1}{6T}\left(e^{3T} - e^{-3T}\right) = \infty \end{aligned} \qquad (6)$$

This is not an energy nor a power signal!

b)

$$E_x = \int_{-\infty}^{\infty} \left|-e^{-3t}\right|^2 u(t)\,dt = \int_{0}^{\infty} e^{-6t}\,dt = -\frac{1}{6}e^{-6t}\Big|_{0}^{\infty} = \frac{1}{6}\left(e^{0} - e^{-\infty}\right) = \frac{1}{6}(1-0) = \frac{1}{6} \qquad (7)$$

Average power:

$$P_x = \lim_{T\to\infty}\frac{1}{T}E_x = 0 \qquad (8)$$

This is an energy signal.

4. Determine if the following signals are even, odd, or neither.

a) $x_1(t) = 3 + \cos(4\pi t)$

b) $x_2(t) = \cos(4\pi t + 3\pi / 2)$

Solution:

a)

$$x_1(-t) = 3 + \cos(-4\pi t) = 3 + \cos(4\pi t) = x_1(t) \tag{9}$$

This is even signal.

b)

$$x_2(t) = \cos(4\pi t + 3\pi / 2) = \sin(4\pi t) \tag{10}$$

$$x_2(-t) = \sin(-4\pi t) = -\sin(4\pi t) \tag{11}$$

This is odd signal.

5. Calculate the following integrals:

a) $\int_{-\infty}^{\infty}(t-2)\delta(t-4)\,dt =$

b) $\int_{-5}^{0}t^2\delta(t-2)\,dt =$

<u>Solution</u>:

a)

$$\int_{-\infty}^{\infty}(t-2)\delta(t-4)\,dt = f(4) = 4-2 = 2 \qquad (12)$$

b)

$$\int_{-5}^{0}t^2\delta(t-2)\,dt = 0\ , \qquad (13)$$

because integration interval does not capture the delta function.

6. Consider the following signal

$$x(t) = \begin{cases} t+2, & -2 \le t \le -1 \\ 1, & -1 < t \le 1 \\ -t+2, & 1 < t \le 2 \\ 0, & \text{otherwise} \end{cases}$$

a) Find the function $x(t-4)$

b) Find the function $x(-2t-3)$.

<u>Solution</u>:

a)

$$x(t-4)=\begin{cases} t-4+2, & -2\le t-4\le -1 \\ 1, & -1<t-4\le 1 \\ -(t-4)+2, & 1<t-4\le 2 \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

$$x(t-4)=\begin{cases} t-2, & 2\le t\le 3 \\ 1, & 3<t\le 5 \\ -t+6, & 5<t\le 6 \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

b)

$$x(-2t-3)=\begin{cases} -2t-3+2, & -2\le -2t-3\le -1 \\ 1, & -1<-2t-3\le 1 \\ -(-2t-3)+2, & 1<-2t-3\le 2 \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

$$x(-2t-3)=\begin{cases} -2t-1, & 1\le -2t\le 2 \\ 1, & 2<-2t\le 4 \\ 2t+5, & 4<-2t\le 5 \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

$$x(-2t-3)=\begin{cases} -2t-1, & -0.5\ge t\ge -1 \\ 1, & -1>t\ge -2 \\ 2t+5, & -2>t\ge -2.5 \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

7. Determine if the system $x(t)\to y(t)=x(3t-2)$ is

a) Linear?

b) Stable?

Solution:

a) Linearity:

$$x_1(t) \to y_1(t) = x_1(3t-2) \tag{19}$$

$$x_2(t) \to y_2(t) = x_2(3t-2) \tag{20}$$

$$\alpha x_1(t) + \beta x_2(t) \to \alpha x_1(3t-2) + \beta x_2(3t-2) = \alpha y_1 + \beta y_2 \tag{21}$$

The system is linear.

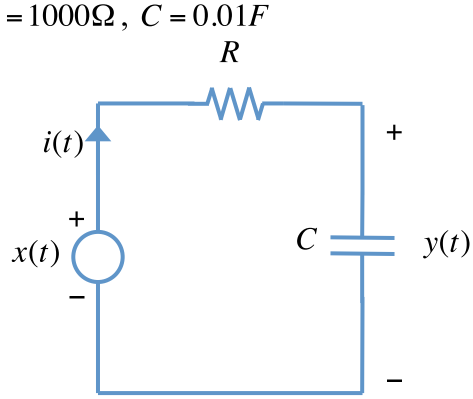
b) Stability: The system is stable; for a bounded input, the output is bounded.

8. Determine if the integrator system $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$ is a stable system?

Solution:

The system is unstable. For the step input, the output grows to infinity, and therefore the system is not a BIBO stable system.

9. Given the circuit below, $R = 1000\Omega$, $C = 0.01F$



a) Derive a differential equation between the input $x(t)$ and the output $y(t)$.

b) Find natural response if $y(0^-) = 5V$.

c) Find $y(3)$.

Solution:

a)

$$x(t) = Ri + y(t) \tag{22}$$

$$i = C\frac{dy(t)}{dt} \tag{23}$$

$$x(t) = RC\frac{dy(t)}{dt} + y(t) \tag{24}$$

$$10\frac{dy(t)}{dt} + y(t) = x(t) \tag{25}$$

b) Find natural response if $y(0^-) = 5V$.

$$\frac{dy(t)}{dt} + 0.1y(t) = 0 \tag{26}$$

$$y(t) = Ae^{-0.1t},\ t \geq 0\ . \tag{27}$$

$$y(0) = A = 5 \tag{28}$$

$$y(t) = 5e^{-0.1t},\ t \geq 0\ . \tag{29}$$

c)

$$y(3) = 5e^{-0.1\times 3}\ . \tag{30}$$