FINAL EXAM | MATH 208 | DEC 2015 ①

① price $\$2.28 \begin{cases} S=7500 \\ D=7900 \end{cases}$ PRICE $\$2.37 \begin{cases} S=7900 \\ D=7800 \end{cases}$

A) SUPPLY EQUATION $p = mx + b$ (LINEAR)

SUPPLY $\overset{x}{(7500}, \overset{p}{2.28)}$
$(7900, 2.37)$

(NOTE: PRICE $p \Rightarrow$ "y-value")

$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$

SLOPE $= \frac{2.37 - 2.28}{7900 - 7500} = \frac{0.09}{400} \Rightarrow \boxed{m = 0.000225}$

$p = mx + b$

$2.28 = 0.000225(7500) + b$

$2.28 = 1.6875 + b$

$b = 2.28 - 1.6875$

$\boxed{b = 0.5925}$

SUPPLY EQUATION

$\boxed{p = 0.000225x + 0.5925}$

B) DEMAND EQUATION $p = mx + b$

DEMAND $\overset{x}{(7900}, \overset{p}{2.28)}$
$(7800, 2.37)$

SLOPE $= \frac{2.37 - 2.28}{7800 - 7900} = \frac{0.09}{-100} \Rightarrow \boxed{m = -0.0009}$

$p = mx + b$

$2.28 = -0.0009(7900) + b$

$2.28 = -7.11 + b$

$b = 2.28 + 7.11$

$\boxed{b = 9.39}$

DEMAND EQUATION

$\boxed{p = -0.0009x + 9.39}$

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① CONTINUED

C) EQUILIBRIUM POINT ⇒ SUPPLY = DEMAND

$$0.000225x + 0.5925 = -0.0009x + 9.39$$

$$0.000225x + 0.0009x = 9.39 - 0.5925$$

$$0.001125x = 8.7975$$

$$x = \frac{8.7975}{0.001125} \Longrightarrow \boxed{x = 7820} \text{ (million)}$$

AND

$$p = 0.000225(7820) + 0.5925$$

$$p = 1.7595 + 0.5925$$

$$p = 2.352 \Longrightarrow \boxed{p = \$2.35}$$

② A) $9^{2x-1} = 27^x$

$$(3^2)^{2x-1} = (3^3)^x$$

$$3^{4x-2} = 3^{3x} \longrightarrow 4x - 2 = 3x$$

$$4x - 3x = 2$$

$$\boxed{x = 2}$$

B) $(2)^{3x} = \frac{1}{32} = \frac{1}{2^5}$

$$2^{3x} = 2^{-5} \longrightarrow 3x = -5 \quad \boxed{x = \frac{-5}{3}}$$

C) $\log_2 \sqrt{2x^2} - 1 = \frac{3}{2}$

$$\log_2 (2x^2)^{1/2} = \frac{5}{2}$$

$$\frac{1}{2} \log_2 (2x^2) = \frac{5}{2}$$

$$\log_2 (2x^2) = 5$$

(APPLY DEF. $\log_b x = y \Leftrightarrow b^y = x$)

$$2^5 = 2x^2$$

$$32 = 2x^2 \longrightarrow 2x^2 - 32 = 0$$

$$2(x^2 - 16) = 0$$

$$2(x+4)(x-4) = 0$$

$$\boxed{\begin{array}{l} x = -4 \\ x = 4 \end{array}}$$

Checked OK ✓

BOTH VALID SOLUTIONS

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2 (CONTINUED)

D) $\log_5(x+2) + \log_5 x = \log_5(x+12)$

$\log_5 (x+2)x = \log_5(x+12)$

$\cancel{\log_5}(x^2+2x) = \cancel{\log_5}(x+12)$

$x^2+2x = x+12$

$x^2+x-12=0 \Rightarrow (x+4)(x-3)=0$ check

$\cancel{x=-4}$ Reject

$x=3$

$\boxed{x=3}$ ✓

E) $\log_{\frac{1}{3}}(27) = x+2$

$-3 = x+2$

$\boxed{x=-5}$

$\Rightarrow$ $\log_{\frac{1}{3}}(27) = y$

$\left(\frac{1}{3}\right)^y = 27 = 3^3 = \left(\frac{1}{3}\right)^{-3}$

$\left(\frac{1}{3}\right)^y = \left(\frac{1}{3}\right)^{-3}$

$\Rightarrow y=-3$

3 $f(x) = 360-60x$ $\quad g(x) = 10^{x-10}$

A) $\sum_{k=0}^{49} f(k)$ $\quad f(0) = 360-60(0) = 360$

$\vdots$

$f(49) = 360-60(49) = -2580$

$a_1 = 360$ FIRST TERM

$n=50$ $(d=-60)$

$a_{50} = -2580$ LAST TERM

$S_n = \frac{n}{2}(a_1+a_n) \Rightarrow S_{50} = \frac{50}{2}(360+(-2580)) = 25(-2220)$

$\Rightarrow \boxed{-55\,500}$

B) $\sum_{h=1}^{35} g(h)$ $\quad g(1) = 10^{1-10} = 10^{-9}$

$a_1 = 10^{-9}$ FIRST TERM

$n=35$ $\quad r=10$

$S_n = \frac{a_1(r^n-1)}{r-1}$

$S_{35} = \frac{10^{-9}(10^{35}-1)}{10-1}$ $\Rightarrow \boxed{1.111111\ldots \times 10^{25}}$

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4 A) 1^{ST} DEPOSIT $\Rightarrow$ DAY SHE WAS BORN $\Rightarrow t=0$

$0, 1, 2, 3, \dots 21 \Rightarrow$ SO $t = 22$ years

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$$

$$FV = 1000\left(\frac{(1+0.0525)^{22} - 1}{0.0525}\right)$$

$$\begin{cases} FV = ? \quad PMT = \$1000 \\ r = 0.0525 \quad m = 1 \\ i = \frac{r}{m} = \frac{0.0525}{1} = 0.0525 \\ n = mt = (1)(22) = 22 \end{cases}$$

$$\boxed{FV = \$39\,664.40}$$

B) TOTAL INTEREST = FV − TOTAL DEPOSITS

= 39 664.40 − 22(1000)

$$\boxed{\text{INT.} = \$17\,664.40}$$

5 A) 20% DOWN SO $0.8(83\,000) = \$66\,400$

$$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$$66\,400 = PMT\left(\frac{1-(1+0.007)^{-360}}{0.007}\right)$$

$$66\,400 = PMT(131.2615606\ldots)$$

$$\begin{cases} PV = 66\,400 \quad PMT = ? \\ r = 0.084 \quad m = 12 \\ \qquad t = 30 \\ i = \frac{r}{m} = \frac{0.084}{12} = 0.007 \\ n = mt = (12)(30) = 360 \end{cases}$$

$$\boxed{PMT = \$505.86}$$

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(5) (CONTINUED)

B) UNPAID BALANCE AFTER 96^{th} PAYMENT = 8 years
(8X12)

$\Rightarrow$ 30 − 8 = 22 YEARS LEFT OR 22X12 = 264 PAYMENTS REMAINING
(OR 360−96 = 264)

$$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$$PV = 505.86\left(\frac{1-(1+0.007)^{-264}}{0.007}\right) = \boxed{\$60\,806.57}$$

UNPAID LOAN BALANCE AFTER 96 PAYMENTS

C) EQUITY = NET CURRENT MARKET VALUE − UNPAID LOAN BALANCE

given = $95 000

So EQUITY = 95 000 − 60 806.57 = $\boxed{\$34\,193.43}$

EQUITY

So LOAN 60% of EQUITY

60% × 34193.43 $\Rightarrow$ $\boxed{\$20\,516.06}$

(6)

$$\left(\begin{array}{ccc|c} 3 & 2 & 4 & -1 \\ -2 & 1 & -2 & 6 \\ 3 & 3 & 6 & 3 \end{array}\right) \quad \tfrac{1}{3}R_3 \to R_3$$

$$\sim \left(\begin{array}{ccc|c} 3 & 2 & 4 & -1 \\ -2 & 1 & -2 & 6 \\ 1 & 1 & 2 & 1 \end{array}\right) \quad R_1 \leftrightarrow R_2 \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ -2 & 1 & -2 & 6 \\ 3 & 2 & 4 & -1 \end{array}\right)$$

$2R_1 + R_2 \to R_2$

$-3R_1 + R_3 \to R_3$

6 (CONTINUED) MATH 208 DEC 2015

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 3 & 2 & 8 \\ 0 & -1 & -2 & -4 \end{array}\right) -1R_3 \to R_3 \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 3 & 2 & 8 \\ 0 & 1 & 2 & 4 \end{array}\right) R_2 \leftrightarrow R_3$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 2 & 4 \\ 0 & 3 & 2 & 8 \end{array}\right) \begin{array}{l} -1R_2+R_1 \to R_1 \\ -3R_2+R_3 \to R_3 \end{array} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -4 & -4 \end{array}\right) -\frac{1}{4}R_3 \to R_3$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right) -2R_3+R_2 \to R_2 \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

SOLUTION: $\boxed{x_1 = -3, \quad x_2 = 2, \quad x_3 = 1}$

7 TECHNOLOGY MATRIX M

A) $$M = \begin{array}{c} \\ T \\ A \\ F \end{array} \begin{array}{c} \begin{array}{ccc} T & A & F \end{array} \\ \begin{pmatrix} 0.3 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} \end{array}$$

T: TOURISM
A: AGRICULTURE
F: FISHING

B) $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \begin{array}{c} T \\ A \\ F \end{array}$ FINAL DEMAND (IN MILLIONS) $D = \begin{pmatrix} 40 \\ 10 \\ 20 \end{pmatrix} \begin{array}{c} T \\ A \\ F \end{array}$

$$X = MX + D \Rightarrow \quad X = M \quad X + D$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \begin{array}{c} T \\ A \\ F \end{array} = \begin{array}{c} \begin{array}{ccc} T & A & F \end{array} \\ \begin{pmatrix} 0.3 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} \end{array} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 40 \\ 10 \\ 20 \end{pmatrix} \begin{array}{c} T \\ A \\ F \end{array}$$

$X = MX + D$

$X - MX = D$

$(I-M)X = D \Rightarrow \boxed{X = (I-M)^{-1}D}$

So... $$I - M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{array}{c} \begin{array}{ccc} T & A & F \end{array} \\ \begin{pmatrix} 0.3 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} \end{array} = \begin{array}{c} I-M \\ \begin{pmatrix} 0.7 & -0.1 & -0.3 \\ -0.2 & 0.9 & -0.2 \\ -0.1 & -0.1 & 0.9 \end{pmatrix} \end{array}$$

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7 (CONTINUED)

c) TO FIND $(I-M)^{-1}$ $\qquad X=(I-M)^{-1}D$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 0.7 & -0.1 & -0.3 & 1 & 0 & 0 \\ -0.2 & 0.9 & -0.2 & 0 & 1 & 0 \\ -0.1 & -0.1 & 0.9 & 0 & 0 & 1 \end{array}\right) \text{ ....etc..}$$

$I-M \qquad I$

$$(I-M)^{-1} = \begin{pmatrix} 1.58 & 0.24 & 0.58 \\ 0.4 & 1.2 & 0.4 \\ 0.22 & 0.16 & 1.22 \end{pmatrix} \Rightarrow X=(I-M)^{-1}D$$

$$D = \begin{pmatrix} 40 \\ 10 \\ 20 \end{pmatrix} \begin{array}{c} T \\ A \\ F \end{array}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \begin{array}{c} T \\ A \\ F \end{array} = \underbrace{\begin{pmatrix} 1.58 & 0.24 & 0.58 \\ 0.4 & 1.2 & 0.4 \\ 0.22 & 0.16 & 1.22 \end{pmatrix}}_{(I-M)^{-1}} \underbrace{\begin{pmatrix} 40 \\ 10 \\ 20 \end{pmatrix}}_{D} = \begin{pmatrix} 77.2 \\ 36 \\ 34.8 \end{pmatrix} \begin{array}{c} T \\ A \\ F \end{array}$$

8 EXTREMIZE $P(x,y) = 20x - 15y$

① $x+2y \geq 14$

x	y
0	7
14	0

SOLID
TP(0,0)
$0 \geq 14$
FALSE

② $x+3y \leq 42$

x	y
0	14
42	0
21	7

SOLID
TP(0,0)
$0 \leq 42$
TRUE

③ $2x+y \leq 42$

x	y
0	42
21	0
14	14
7	28

SOLID
TP(0,0)
$0 \leq 42$
TRUE

$x \geq 0$
$y \geq 0$
$\Downarrow$
QUAD I ONLY

POINT C: ② ∩ ③ $\begin{cases} ② & x+3y=42 \quad (\times -2) \\ ③ & 2x+y=42 \end{cases}$

$$\begin{array}{rl} +\ ②' \Rightarrow & -2x - 6y = -84 \\ ③ \Rightarrow & \cancel{2x} + y = 42 \\ \hline & -5y = -42 \\ & y = 42/5 = 8.4 \end{array}$$

$2x + 8.4 = 42$
$2x = 33.6$
$x = \frac{33.6}{2} = 16.8$

C(16.8, 8.4)

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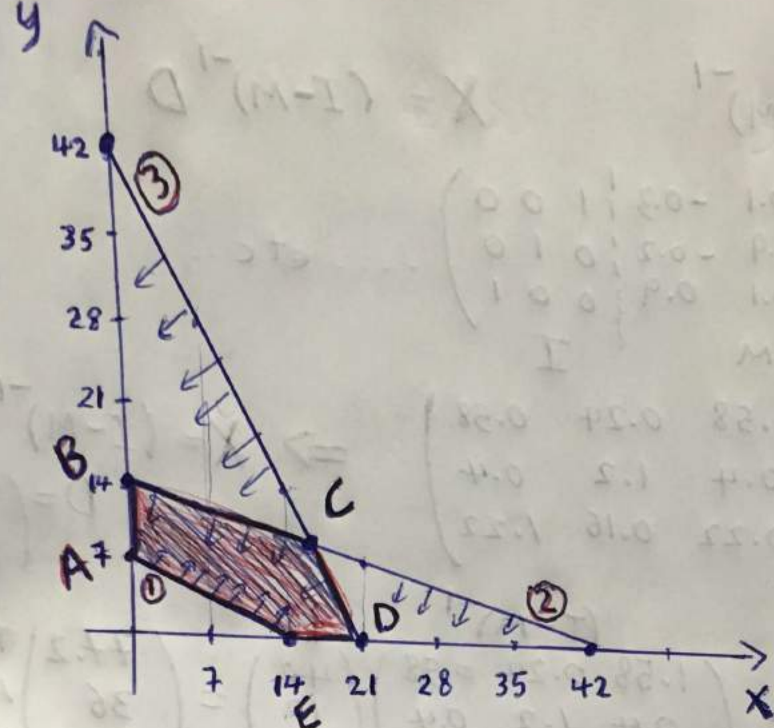
8 (CONTINUED)



CORNER POINTS	$P(x,y) = 20x - 15y$
$A(0,7)$	$20(0) - 15(7) = -105$
$B(0,14)$	$20(0) - 15(14) = -210 \leftarrow$ MIN
$C(16.8, 8.4)$	$20(16.8) - 15(8.4) = 210$
$D(21,0)$	$20(21) - 15(0) = 420 \leftarrow$ MAX
$E(14,0)$	$20(14) - 15(0) = 280$

$\Rightarrow$ MINIMUM VALUE of $P(x,y) = -210$ (at $B(0,14)$)

MAXIMUM VALUE of $P(x,y) = 420$ (at $D(21,0)$)

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6 WHEAT PLANTS
5 BARLEY PLANTS
3 RYE PLANTS

A) CHOOSE 4 PLANTS AT RANDOM OUT OF $6+5+3 = 14$ PLANTS
ORDER NOT IMPORTANT ⇒ COMBINATION

$$C_{14,4} = \frac{14!}{10!\,4!} = \boxed{1001}$$

B) CHOOSE 4 PLANTS, EXACTLY 2 MUST BE WHEAT PLANTS

⇒ CHOOSE 2 OUT OF 6 WHEAT PLANTS
AND ⇒ CHOOSE 2 OUT of REMAINING 8 plants

$$C_{6,2} \cdot C_{8,2} = (15)(28)$$
$$= \boxed{420}$$

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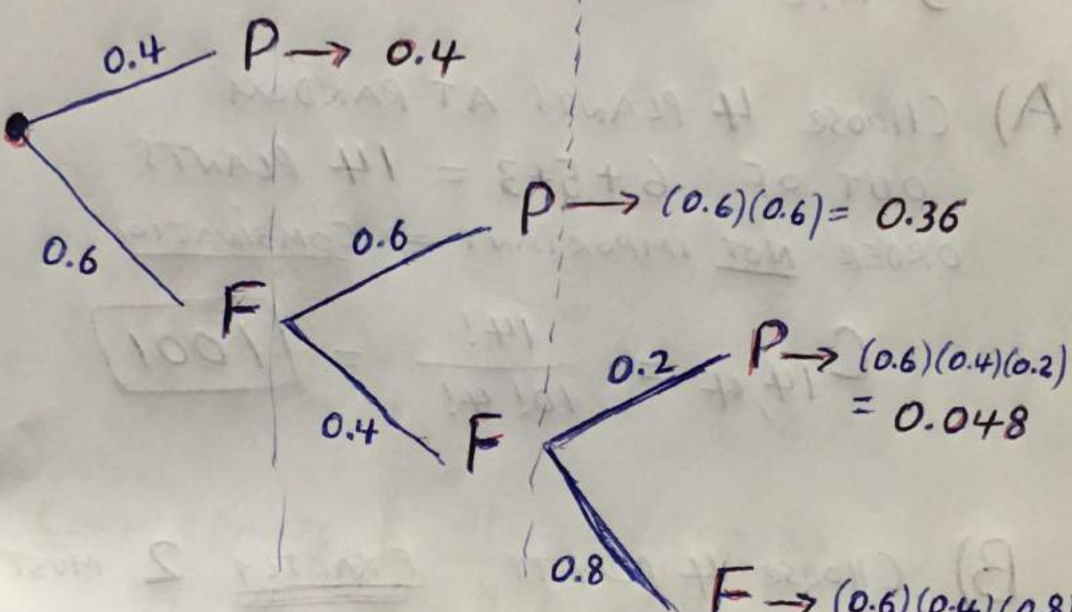
P = PASS F = FAIL

1^{ST} TRIAL | 2^{ND} TRIAL | 3^{RD} TRIAL



A) PROB. of P ON 1^{ST} OR 2^{ND} TRIAL

$$0.4 + 0.36 = \boxed{0.76}$$

B) PROB of F ON ALL 3 ATTEMPTS

$$P(F,F,F) = \boxed{0.192}$$

C) PROB of F ON 1^{ST} TWO TRIALS and P ON THIRD

$$P(F,F,P) = \boxed{0.048}$$

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(1) price 2.28 $\begin{cases} S = 7500 \\ D = 7900 \end{cases}$ price 2.37 $\begin{cases} S = 7900 \\ D = 7800 \end{cases}$

A) SUPPLY EQUATION $p = mx + b$ (linear)

(NOTE: PRICE $p \Rightarrow$ "y-value")

SUPPLY $\;(\overset{x}{7500}, \overset{p}{2.28})$

$(7900, 2.37)$

$$\text{SLOPE} = \frac{2.37 - 2.28}{7900 - 7500} = \frac{0.09}{400} \Rightarrow \boxed{m = 0.000225}$$

$p = mx + b$

$2.28 = 0.000225(7500) + b$

$2.28 = 1.6875 + b$

$b = 2.28 - 1.6875$

$\boxed{b = 0.5925}$

SUPPLY EQUATION

$$\boxed{p = 0.000225x + 0.5925}$$

B) DEMAND EQUATION $p = mx + b$

DEMAND $\;(\overset{x}{7900}, \overset{p}{2.28})$

$(7800, 2.37)$

$$\text{SLOPE} = \frac{2.37 - 2.28}{7800 - 7900} = \frac{0.09}{-100} \Rightarrow \boxed{m = -0.0009}$$

$p = mx + b$

$2.28 = -0.0009(7900) + b$

$2.28 = -7.11 + b$

$\boxed{b = 9.39}$

DEMAND EQUATION

$$\boxed{p = -0.0009x + 9.39}$$

1 ① CONTINUED

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C) EQUILIBRIUM POINT ⇒ SUPPLY = DEMAND

$0.000225x + 0.5925 = -0.0009x + 9.39$

$0.000225x + 0.0009x = 9.39 - 0.5925$

$0.001125x = 8.7975$

$x = \frac{8.7975}{0.001125}$ $\boxed{x = 7820}$ (million)

and $P = 0.000225(7820) + 0.5925$

$P = 2.352$ $\boxed{P = \$2.35}$

② A) $4^{x^2}(2^{5x}) = 8$ $(8 = 2^3 \quad 4 = 2^2)$

$(2^2)^{x^2}(2^{5x}) = 2^3$

$2^{2x^2} 2^{5x} = 2^3$

$2^{2x^2+5x} = 2^3$

$\Rightarrow 2x^2+5x = 3$

$2x^2+5x-3=0$ (OR USE QUADRATIC FORMULA)

$2x^2+6x-x-3$

$2x(x+3)-1(x+3)$

$(x+3)(2x-1)=0$

$\boxed{x=-3 \quad x = 1/2}$

B) $3^{x^2+x} = \sqrt{3} = 3^{1/2}$

$x^2+x = \frac{1}{2}$

$x^2+x-0.5=0$

$\Delta = b^2-4ac = (1)^2-4(1)(-0.5)$

$\Delta = 3$

$\frac{-1 \pm \sqrt{3}}{2(1)}$

$\frac{-1+\sqrt{3}}{2} \simeq 0.366$

$\frac{-1-\sqrt{3}}{2} \simeq -1.366$

$\boxed{x = 0.366 \quad x = -1.366}$

C) $\log_2 \sqrt{2y^2} - 1 = 3/2$

$\log_2 (2y^2)^{1/2} = 5/2$

$\frac{1}{2}\log_2(2y^2) = 5/2$

$\log_2(2y^2) = 5$

$\Rightarrow 2^5 = 2y^2$

$2y^2 - 32 = 0$ $(2^5 = 32)$

$2(y^2-16)=0$

$2(y+4)(y-4)=0$

2 SOLUTIONS (check ✓)

$\boxed{y=-4 \quad y = 4}$

2 CONTINUED

D) $\log_{11}(x+7) - \log_{11}(x+10) = \log_{11}(0.5)$

$\log_{11}\left(\frac{x+7}{x+10}\right) = \log_{11}\left(\frac{1}{2}\right)$

$\frac{x+7}{x+10} = \frac{1}{2} \Rightarrow 2x+14 = x+10$

$x = -4$ check ✓

$\boxed{X = -4}$

E) $\log_2(\log_2 x) = 1$ (APPLY DEF. TWICE $\log_c x = y \Leftrightarrow c^y = x$)

$2^1 = \log_2 x$

$2 = \log_2 x$

$2^2 = x \rightarrow$ check ✓

$\boxed{x = 4}$

3 $f(x) = -24x+32 \quad g(x) = 6(0.4)^x$

A) $\sum_{K=0}^{29} f(K) \quad f(0) = -24(0)+32 \quad a_1 = 32$ First Term

$\vdots \quad n = 30, \ d = -24$

$f(29) = -24(29)+32 = -664$

SUM $= S_n = \frac{n}{2}(a_1 + a_n) \quad a_{30} = -664$ LAST Term

So $S_{30} = \frac{30}{2}(32 + (-664)) = 15(-632) = \boxed{-9480}$

B) $\sum_{h=1}^{19} g(h) \quad g(1) = 6(0.4)^1 = 2.4 \quad a_1 = 2.4$ FIRST TERM

SUM $= S_n = a_1\left(\frac{r^n - 1}{r-1}\right) \Rightarrow S_{19} = 2.4\left(\frac{(0.4)^{19} - 1}{0.4-1}\right) \quad n = 19, \ r = 0.4$

$\boxed{S_{19} \approx 3.99999989\ldots}$

C) $\sum_{h=0}^{\infty} g(h) \quad g(0) = 6(0.4)^0 = 6 \quad a_1 = 6$ FIRST TERM

$r = 0.4 < 1$ SO SUM EXISTS

$\Rightarrow$ SUM $= S_\infty = \frac{a_1}{1-r} \Rightarrow S_\infty = \frac{6}{1-0.4} = \frac{6}{0.6} \Rightarrow \boxed{S_\infty = 10}$

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4) A) 6% SIMPLE INTEREST SEMI-ANNUALLY

$I = Prt$ $\quad P = \$4000 \quad t = \frac{1}{2}$ year $\quad r = 0.06$

$I = 4000(0.06)(\frac{1}{2}) = \$120$

SEMI-ANNUALY INTEREST PAYMENTS = $\boxed{\$120}$

B) SINKING FUND 8% Compounded ANNUALLY $\to m=1$

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$$

$$\begin{cases} FV = \$4000 \\ \left.\begin{matrix} r = 0.08 \\ m = 1 \end{matrix}\right\} i = \frac{r}{m} = \frac{0.08}{1} = 0.08 \\ t = 5 \rightarrow n = mt = (1)(5) = 5 \end{cases}$$

$$4000 = PMT\left(\frac{(1+0.08)^5 - 1}{0.08}\right) \longrightarrow \boxed{PMT = \$681.83}$$

5) \$110 000, 30 year mortgage at 6.6% compounded MONTHLY $\to m=12$

A) $$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

$$\begin{cases} PV = 110\,000 \\ i = \frac{r}{m} = \frac{0.066}{12} = 0.0055 \\ n = mt = (12)(30) = 360 \end{cases}$$

$$110\,000 = PMT\left(\frac{1-(1+0.0055)^{-360}}{0.0055}\right)$$

$\boxed{PMT = \$702.52}$

B) TOTAL INTEREST = TOTAL PAYMENTS − INITIAL LOAN

$= (360 \times 702.52) - 110\,000$

$\boxed{\text{TOTAL INTEREST} = \$142\,907.20}$

C) INTEREST ON 1ST PAYMENT $\Rightarrow 0.0055 \times 110\,000 = \$605$

1ST PAYMENT = $702.52 = 605 + 97.52$

605: INTEREST PART

97.52: PART TO REDUCING DEBT

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$$\left(\begin{array}{ccc|c} 2 & -1 & 1 & -9 \\ 4 & 4 & -4 & 24 \\ 1 & -2 & 3 & 1 \end{array}\right) \underset{\sim}{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 4 & 4 & -4 & 24 \\ 2 & -1 & 1 & -9 \end{array}\right) \begin{array}{l} -4R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \end{array} \sim$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 12 & -16 & 20 \\ 0 & 3 & -5 & -11 \end{array}\right) \underset{\sim}{\tfrac{1}{12}R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 1 & -4/3 & 5/3 \\ 0 & 3 & -5 & -11 \end{array}\right) \begin{array}{l} 2R_2 + R_1 \to R_1 \\ -3R_2 + R_3 \to R_3 \end{array} \sim$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1/3 & 13/3 \\ 0 & 1 & -4/3 & 5/3 \\ 0 & 0 & -1 & -16 \end{array}\right) \underset{\sim}{-1R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 0 & 1/3 & 13/3 \\ 0 & 1 & -4/3 & 5/3 \\ 0 & 0 & 1 & 16 \end{array}\right)$$

$$\sim \begin{array}{l} \tfrac{4}{3}R_3 + R_2 \to R_2 \\ -\tfrac{1}{3}R_3 + R_1 \to R_1 \end{array} \sim \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 23 \\ 0 & 0 & 1 & 16 \end{array}\right) \end{array} \quad \boxed{\begin{array}{l} x_1 = -1 \\ x_2 = 23 \\ x_3 = 16 \end{array}}$$

7

TECHNOLOGICAL MATRIX M

T: TOURISM
A= AGRICULTURE
F= FISHING

A)

$$M = \begin{array}{c} \begin{array}{ccc} T & A & F \end{array} \\ \begin{pmatrix} 0.2 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{pmatrix} \end{array} \begin{array}{c} T \\ A \\ F \end{array}$$

7 (CONTINUED) MATH 208 APRIL 2015

$$X = \begin{array}{c} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \end{array}\begin{array}{c} T \\ A \\ F \end{array} \qquad \text{FINAL DEMAND (IN MILLION)} \quad D = \begin{pmatrix} 10 \\ 15 \\ 20 \end{pmatrix}\begin{array}{c} T \\ A \\ F \end{array}$$

B) $X = MX + D \Rightarrow$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\begin{array}{c} T \\ A \\ F \end{array} = \underbrace{\begin{bmatrix} 0.2 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 10 \\ 15 \\ 20 \end{bmatrix}}_{D}\begin{array}{c} T \\ A \\ F \end{array}$$

$X - MX = D$

$(I - M)X = D \Rightarrow X = (I-M)^{-1}D$

$$I - M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{array}{c} \begin{array}{ccc} T & A & F \end{array} \\ \begin{pmatrix} 0.2 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{pmatrix} \end{array} = \underbrace{\begin{pmatrix} 0.8 & -0.4 & -0.3 \\ -0.2 & 0.9 & -0.1 \\ -0.2 & -0.1 & 0.9 \end{pmatrix}}_{I-M}$$

TO FIND $(I-M)^{-1} \Rightarrow$

$$\left(\begin{array}{ccc|ccc} 0.8 & -0.4 & -0.3 & 1 & 0 & 0 \\ -0.2 & 0.9 & -0.1 & 0 & 1 & 0 \\ -0.2 & -0.1 & 0.9 & 0 & 0 & 1 \end{array}\right) \text{ - - - - - - - etc..}$$

$I - M \qquad I$

C) $(I-M)^{-1} = \begin{pmatrix} 1.6 & 0.78 & 0.62 \\ 0.4 & 1.32 & 0.28 \\ 0.4 & 0.32 & 1.28 \end{pmatrix} \Rightarrow X = (I-M)^{-1}D$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\begin{array}{c} T \\ A \\ F \end{array} = \underbrace{\begin{pmatrix} 1.6 & 0.78 & 0.62 \\ 0.4 & 1.32 & 0.28 \\ 0.4 & 0.32 & 1.28 \end{pmatrix}}_{(I-M)^{-1}} \underbrace{\begin{pmatrix} 10 \\ 15 \\ 20 \end{pmatrix}}_{D} = \begin{pmatrix} 40.1 \\ 29.4 \\ 34.4 \end{pmatrix}\begin{array}{c} T \\ A \\ F \end{array}$$

FINAL EXAM | MATH 208 | APRIL 2015 | 4

8 EXTREMIZE $50x+50y = P(x,y)$

① $5x+8y \geq 200$

x	y
0	25
40	0
16	15

Solid
TP(0,0)
$0 \geq 200$
FALSE

② $25x-10y \geq 250$

x	y
10	0
16	15
40	75
50	100

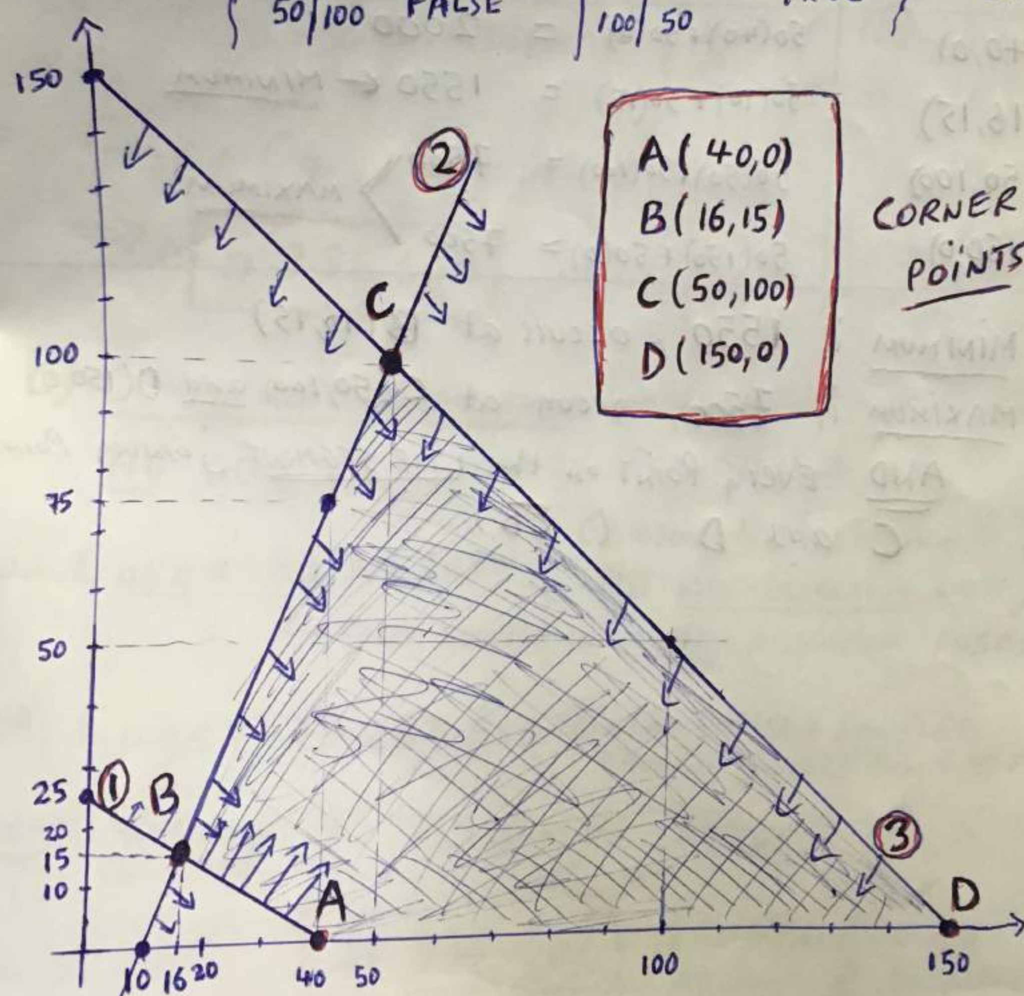
solid
TP(0,0)
$0 \geq 250$
FALSE

③ $4x+4y \leq 600$

x	y
0	150
150	0
50	100
100	50

solid
TP(0,0)
$0 \leq 600$
TRUE

$x \geq 0$, $y \geq 0$ ⇓ QUAD I. ONLY



A(40,0)
B(16,15)
C(50,100)
D(150,0)

CORNER POINTS

(8) (CONTINUED) **MATH 208** APRIL 2015

POINT B $\Rightarrow$ ① $\cap$ ②

① $5x + 8y = 200$ $(\times -5)$

② $25x - 10y = 250$

$$\Rightarrow + \begin{array}{r} -25x - 40y = -1000 \\ 25x - 10y = 250 \\ \hline -50y = -750 \\ \boxed{y = 15} \end{array}$$

$\rightarrow 5x + 8(15) = 200$

$5x = 80$

$\boxed{x = 16}$ B(16,15)

CORNER POINT	OBJECTIVE FUNCTION $P(x,y) = 50x + 50y$
A(40,0)	$50(40) + 50(0) = 2000$
B(16,15)	$50(16) + 50(15) = 1550$ ← <u>MINIMUM</u>
C(50,100)	$50(50) + 50(100) = 7500$ <u>MAXIMUM</u>
D(150,0)	$50(150) + 50(0) = 7500$ <u>MAXIMUM</u>

So • <u>MINIMUM</u> is 1550, occurs at B(16,15)

• <u>MAXIMUM</u> is 7500, occurs at C(50,100) <u>and</u> D(150,0)

<u>AND</u> Every Point on the <u>LINE SEGMENT</u> joining Points C and D. ($\overline{CD}$)

9. 100 FUSES, 10 DEFECTIVE, 5 FUSES SELECTED

A)

5 FUSES CHOSEN:
- 2 DEFECTIVE (OUT OF 10 DEFECTIVE ONES)
- 3 NON-DEFECTIVE (OUT OF 100−10 = 90 NON-DEFECTIVE ONES)

SO $C_{10,2} \cdot C_{90,3} = (45)\cdot(117480)$

$\Rightarrow$ $\boxed{5\,286\,600}$ SAMPLES CONTAIN 2 DEFECTIVE FUSES

B) COMPLEMENT of "AT LEAST ONE DEFECTIVE FUSE" is NO DEFECTIVE FUSES IN OUR SAMPLE

So $C_{90,5} = 43\,949\,268$ (Choosing 5 FUSES from 90 NON-DEFECTIVE ONES)

$\hookrightarrow$ TOTAL SAMPLES WITH NO DEFECTIVE FUSES

AND $C_{100,5} = 75\,287\,520$ (TOTAL POSSIBLE SAMPLES Choosing 5 FUSES from 100 FUSES)

COMPLEMENT $\Rightarrow$ $75\,287\,520 - 43\,949\,268$

$\Rightarrow$ $\boxed{31\,338\,252}$ Samples CONTAIN AT LEAST 1 DEFECTIVE FUSE.

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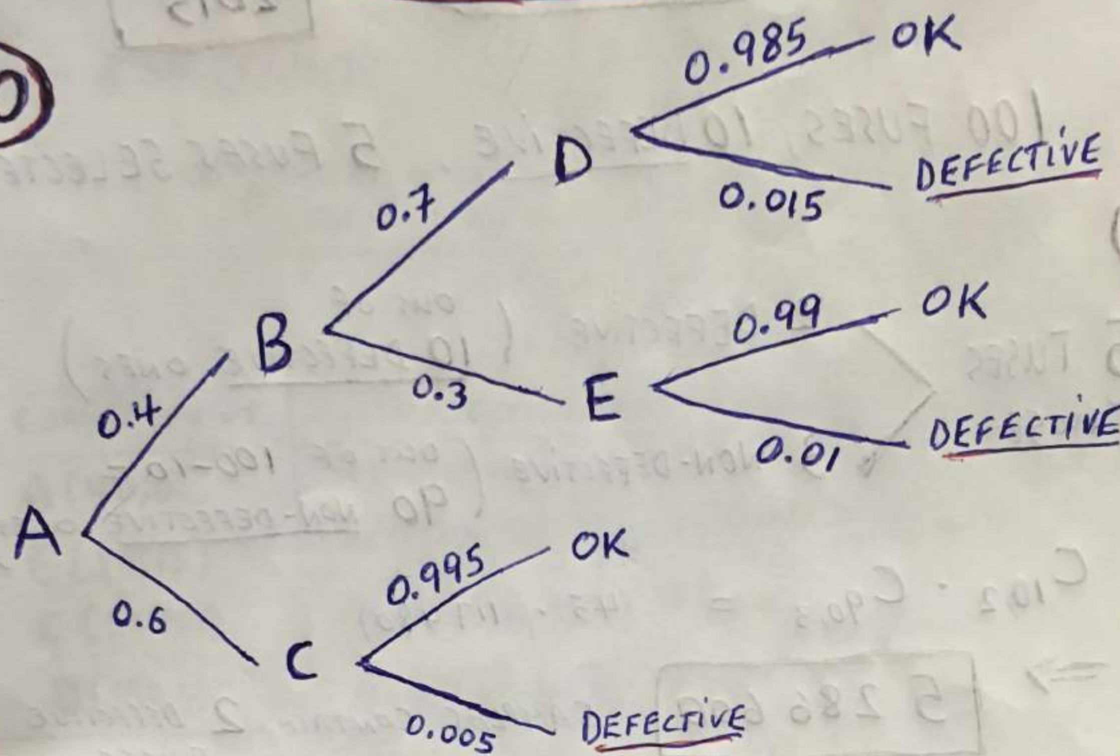
10



A) 3 paths leading to <u>DEFECTIVE</u>

$P(\text{DEFECTIVE}) = (0.4)(0.7)(0.015) + (0.4)(0.3)(0.01) + (0.6)(0.005) = \boxed{0.0084}$

B) 2 paths leading to <u>E OR C</u>

$P(E \text{ OR } C) = (0.4)(0.3) + 0.6 = \boxed{0.72}$

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1 a) $p(x) = 360 - 20x \quad C(x) = 300 + 95x$

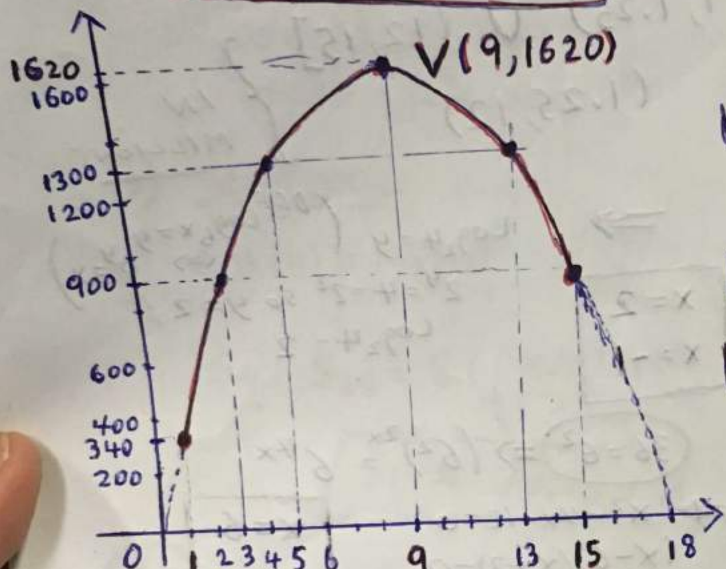
$R(x) = px = (360-20x)x \quad (1 \leq x \leq 15) \quad (a \ominus n)$

$\boxed{R(x) = -20x^2 + 360x} \Rightarrow$ y-INT : $y = 0$

X-INT : $-20x(x-18) = 0$

$\begin{cases} x = 0 \\ x = 18 \end{cases}$

$V\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right) \begin{cases} \Delta = b^2 - 4ac \\ \Delta = (360)^2 - 4(-20)(0) \\ \Delta = (360)^2 = 129600 \end{cases}$



$V\left(\frac{-360}{2(-20)}, \frac{-129600}{4(-20)}\right)$

$V(9, 1620)$

b) MAX REVENUE OCCURS AT THE VERTEX

OUTPUT $x = 9$ million tablet computers

MAXIMUM Revenue $R(9) = \$1620$ million $\quad p(x) = 360 - 20x$

use price-demand for PRICE $\Rightarrow p(9) = 360 - 20(9) = 180$

PRICE $p = \$180$

c) FIND BREAK-EVEN POINTS $R(x) = C(x)$

$-20x^2 + 360x = 300 + 95x$

SO $20x^2 - 265x + 300 = 0$ $\Rightarrow$

$\Delta = b^2 - 4ac = (-265)^2 - 4(20)(300) \Rightarrow \Delta = 46225$ .....

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① c) CONTINUED

$$x = \frac{265 \pm \sqrt{46225}}{2(20)} = \frac{265 \pm 215}{40} \begin{cases} \rightarrow \frac{265+215}{40} = \frac{480}{40} = 12 \\ \rightarrow \frac{265-215}{-40} = \frac{50}{40} = 1.25 \end{cases}$$

SO

LOSS OCCURS $\Rightarrow$ $[1, 1.25) \cup (12, 15]$

PROFIT OCCURS $\Rightarrow$ $(1.25, 12)$

IN MILLIONS

② a) $x^2 - x = \log_2 4 \Rightarrow \log_2 4 = y$ (DEF $\log_b x = y \Leftrightarrow b^y = x$)

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2 \quad x = -1}$

$2^y = 4 = 2^2$ so $y = 2$!

$\log_2 4 = 2$

b) $36^{2x} - 6^{x^2-12} = 0$ $\quad$ $36 = 6^2 \Rightarrow (6^2)^{2x} = 6^{4x}$

$6^{4x} = 6^{x^2-12}$

$\Rightarrow 4x = x^2 - 12 \rightarrow x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$\boxed{x = 6 \quad x = -2}$

c) $2\log_3(x+1) - 2\log_3 9 = 2 \Rightarrow \log_3 9 = y$

$3^y = 9 = 3^2$

so $y = 2$

$\log_3 (x+1)^2 - 2(2) = 2$

$\log_3 (x+1)^2 = 6$

$3^6 = (x+1)^2 \rightarrow x^2 + 2x + 1 = 729$

$x^2 + 2x - 728 = 0$

$(x-26)(x+28) = 0$

$\boxed{x = 26 \text{ ONLY}}$

$x = \cancel{-28}$ Reject

Can't have ⊖ log

d) $\log_8(x-1) + \log_8(x+1) = 1$

$\log_8 (x-1)(x+1) = 1$

$8^1 = (x-1)(x+1) \rightarrow x^2 - 1 = 8$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$\boxed{x = 3 \text{ ONLY}}$

($x = \cancel{-3}$ Reject)

e) $\log_{10}(x+7) - \log_{10}(x-2) = 1$

$\log_{10}\left(\frac{x+7}{x-2}\right) = 1 \rightarrow \frac{x+7}{x-2} = 10^1$

$10x - 20 = x + 7 \rightarrow 9x = 27$

$\boxed{x = 3}$ ✓ check ✓

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3 a) SUM = $\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$

$n = 100$
$a_{100} = 1000$
$S_{100} = 50000$
FIND a_1

$50\,000 = \frac{100}{2}(a_1 + 1000)$

$50000 = 50(a_1 + 1000)$

$1000 = a_1 + 1000$ so $\boxed{a_1 = 0}$

b) geometric

$h(n) = 10(0.9)^n$ First term $\Rightarrow h(2) = 10(0.9)^2 = 8.1 = a_1$

$r = 0.9$ and $n = 19$ terms

So $S_{19} = 8.1\left(\frac{(0.9)^{19} - 1}{0.9 - 1}\right)$ $\left(S_n = a_1\left(\frac{r^n - 1}{r - 1}\right)\right)$

$\boxed{S_{19} \approx 70.0581\ldots}$

4 a)
$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left(\begin{array}{ccc|c} 3 & 6 & 9 & 3 \\ 2 & 3 & 4 & 3 \\ 3 & 6 & 3 & 9 \end{array}\right) \quad \frac{1}{3}R_1 \to R_1$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & 3 \\ 3 & 6 & 3 & 9 \end{array}\right) \quad \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & -6 & 6 \end{array}\right) \quad -1R_2 \to R_2 \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -6 & 6 \end{array}\right) \quad \begin{array}{l} -2R_2 + R_1 \\ \to R_1 \end{array}$$

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(4) a) CONTINUED

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -6 & 6 \end{array}\right) \quad -\frac{1}{6}R_3 \to R_3$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right) \quad \begin{array}{l} -2R_3 + R_2 \to R_2 \\ 1R_3 + R_1 \to R_1 \end{array}$$

$$\sim \begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right) \quad \text{so} \quad \boxed{\begin{array}{l} x = 2 \\ y = 1 \\ z = -1 \end{array}}$$

b) $3(2) + 6(1) + 9(-1) = 3$ ✓

$2(2) + 3(1) + 4(-1) = 3$ ✓

$3(2) + 6(1) + 3(-1) = 9$ ✓

(5) a) $A = P(1+i)^n$

$A = 150\,000\left(1 + \frac{1}{300}\right)^{24}$

$\boxed{A = \$162\,471.44}$

$P = 150\,000 \quad A = ?$

$r = 0.04, \ m = 12, \ t = 2$

$i = \frac{r}{m} = \frac{0.04}{12} = \frac{1}{300}$

$n = mt = 12(2) = 24$

b) $PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$

$8000 = PMT\left(\frac{1-(1+0.0025)^{-60}}{0.0025}\right)$

$\boxed{PMT = \$143.75}$

$PV = 8000 \quad PMT = ?$

$r = 0.03, \ m = 12, \ t = 5$

$i = \frac{r}{m} = \frac{0.03}{12} = 0.0025$

$n = mt = 12(5) = 60$

and INTEREST = TOTAL PAYMENTS − INITIAL LOAN

$(60 \times 143.75) - 8000 \Rightarrow$ INTEREST $= \boxed{\$625}$

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6

a) $FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$

PMT = 5000 FV = ?

$r = 0.08$, $m = 1$, $t = 30$

$i = \frac{r}{m} = \frac{0.08}{1} = 0.08$

$n = mt = (1)(30) = 30$

$FV = 5000\left(\frac{(1+0.08)^{30} - 1}{0.08}\right)$

$\boxed{FV = \$\,566\,416.06}$

b) PMT = ?

$r = 0.06$, $m = 12$, $t = 10$

$i = \frac{r}{m} = \frac{0.06}{12} = 0.005$

$n = mt = 12(10) = 120$

$PV = FV = 566\,416.06$

$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$

$566\,416.06 = PMT\left(\frac{1-(1+0.005)^{-120}}{0.005}\right)$

$\boxed{PMT = \$\,6288.38}$

7

Extremize $P(x,y) = 10x + 12y$

① $2x + 3y \leq 12$

x	y
0	4
6	0
1	10/3
3	2

≤ SOLID

TP(0,0)

$0 \leq 12$

TRUE

② $4x - 5y \leq 2$

x	y
0	-0.4
1	0.4
$\frac{1}{2}$	0
$\frac{7}{4}$	1
3	2

≤ SOLID

TP(0,0)

$0 \leq 2$

TRUE

INTERSECTION POINT

① $2x + 3y = 12\ (\times -2)$

② $4x - 5y = 2$

①′ $-\cancel{4x} - 6y = -24$

\+ ② $\cancel{4x} - 5y = 2$

$-11y = -22$

$y = 2$

and

$2x + 3(2) = 12$

$2x = 6$

$x = 3$

So $(3,2)$

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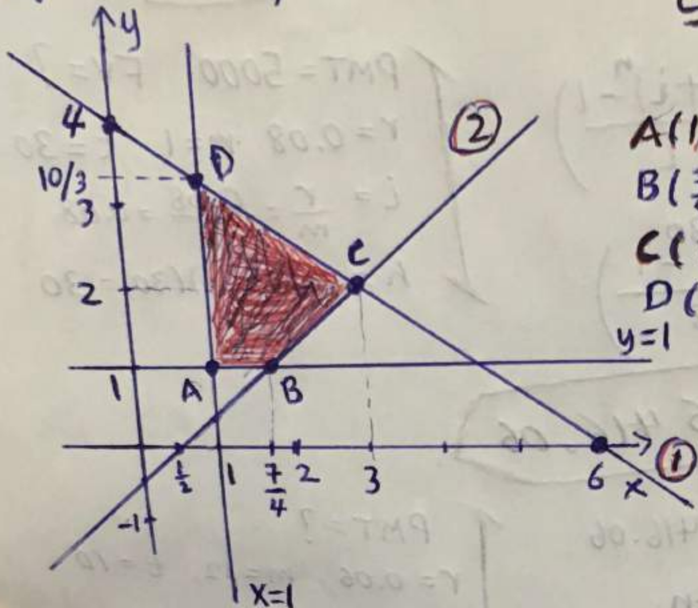
7 (CONTINUED)



CORNER POINT TABLE

$P(x,y) = 10x + 12y$

$A(1,1) \Rightarrow 10(1) + 12(1) = \boxed{22}$ MIN

$B(\frac{7}{4},1) \Rightarrow 10(7/4) + 12(1) = 29.5$

$C(3,2) \Rightarrow 10(3) + 12(2) = \boxed{54}$ MAX

$D(1,\frac{10}{3}) \Rightarrow 10(1) + 12(\frac{10}{3}) = 50$

8 a) $PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$

$130000 = PMT\left(\frac{1-(1+0.0035)^{-300}}{0.0035}\right)$

PMT = ? PV = 130000

$r = 0.042$, $m = 12$, $t = 25$

$i = \frac{r}{m} = \frac{0.042}{12} = 0.0035$

$n = mt = 12(25) = 300$

$\boxed{PMT = \$700.63}$

b) UNPAID LOAN BALANCE AFTER 15 years

$\Rightarrow$ 10 years LEFT so $n = 10(12) = 120$

$PV = 700.63\left(\frac{1-(1+0.0035)^{-120}}{0.0035}\right) = \boxed{68\,555.88}$

UNPAID LOAN BALANCE

c) INTEREST = TOTAL PAYMENTS − INITIAL LOAN

$= (300 \times 700.63) - 130000$

$\boxed{\text{INTEREST} = 80189}$

9

a) $C_{9,3} = 84$

b) $\dfrac{C_{5,3}}{C_{9,3}} = \dfrac{10}{84} = \dfrac{5}{42} \approx 0.11905\ldots$

c) $\dfrac{C_{5,1} \cdot C_{4,2}}{C_{9,3}} = \dfrac{30}{84} = \dfrac{5}{14} \approx 0.35714\ldots$

10

a) PERMUTATION $\Longrightarrow$ ORDER MATTERS

$$\frac{P_{6,2}}{P_{10,2}} = \frac{30}{90} = \frac{1}{3} \approx 0.3333\ldots$$

b) COMBINATION – MAJORITY MEN

2 MEN + 1 WOMAN & 3 MEN

$$\frac{C_{4,2}\, C_{6,1}}{C_{10,3}} + \frac{C_{4,3}}{C_{10,3}}$$

$$\frac{36}{120} + \frac{4}{120} = \frac{40}{120}$$

$$= \frac{1}{3} \approx 0.3333\ldots$$

FINAL EXAM

DEC 2012

MATH 208

1

1

A) REVENUE ⇒ Quadratic so MAXIMUM is given by VERTEX

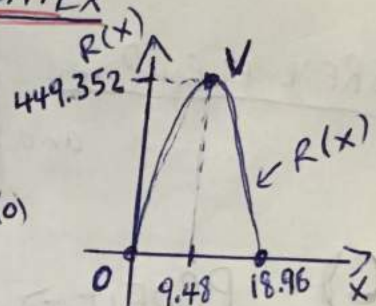
$R(x) = x(94.8 - 5x)$

$R(x) = -5x^2 + 94.8x$

$\Delta = b^2 - 4ac$

$\Delta = (94.8)^2 - 4(-5)(0)$

$\Delta = 8987.04$



$\text{Vertex} = V\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right)$

$= V\left(\frac{-94.8}{2(-5)}, \frac{-8987.04}{4(-5)}\right)$

$= V(9.48, 449.352)$

You must produce 9.48 million cameras to achieve (or 9 480 000 cameras)

The MAXIMUM Revenue which is \$449.352 millions (or \$449 352 000)

(Note: $R(9.48) = -5(9.48)^2 + 94.8(9.48) = 449.352$)

B) BREAK-EVEN POINTS ⇒ $R(x) = C(x)$

given $C(x) = 156 + 19.7x$ ⇒ $-5x^2 + 94.8x = 156 + 19.7x$

⇒ $5x^2 - 75.1x + 156 = 0$ USE QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \qquad \Delta = b^2 - 4ac$$

$$\Delta = (-75.1)^2 - 4(5)(156)$$

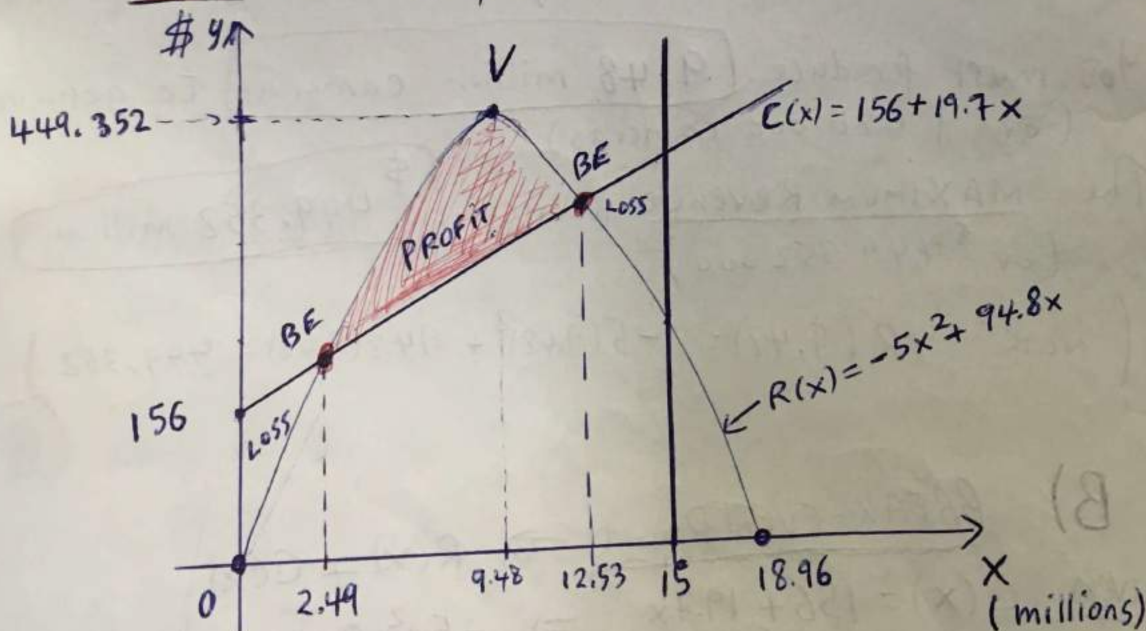
$$\Delta = 2520.01$$

$$x = \frac{75.1 \pm \sqrt{2520.01}}{2(5)} = \frac{75.1 \pm 50.2}{10} \begin{cases} \frac{75.1+50.2}{10} \approx 12.53 \\ \frac{75.1-50.2}{10} \approx 2.49 \end{cases}$$

- BREAK-EVEN POINTS $\boxed{\begin{array}{ll} x \approx 2.49 \text{ million} & (2\,490\,000) \\ \text{and } x \approx 12.53 \text{ million} & (12\,530\,000) \end{array}}$

C) PROFIT ⇒ (in millions) $(2.49, 12.53)$

LOSS ⇒ $[0, 2.49) \cup (12.53, 15]$



FINAL EXAM $\boxed{\text{DEC 2012}}$

$\boxed{2}$

MATH 208

2

A) $\left(\frac{3}{4}\right)^x = \frac{64}{27} = \frac{4^3}{3^3} = \left(\frac{4}{3}\right)^3 = \left(\frac{3}{4}\right)^{-3} \Rightarrow \boxed{x = -3}$

B) $e^{-3x^2+15x-72} = e^{-x^2+35x-22}$

$-3x^2+15x-72 = -x^2+35x-22$

$2x^2+20x+50=0 \rightarrow 2(x^2+10x+25)=0$

$2(x+5)^2=0$

$\boxed{x=-5}$

C) $\log_b x = \frac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3$

$\log_b x = \log_b 27^{2/3} + \log_b 2^2 - \log_b 3$

$\log_b x = \log_b \left(\frac{27^{2/3} \times 2^2}{3}\right)$ $\quad \begin{cases} 27^{2/3} = 9 \\ 2^2 = 4 \end{cases}$

$\log_b x = \log_b \left(\frac{9 \times 4}{3}\right) \Rightarrow x = \frac{9\times 4}{3} = 12$ $\boxed{x=12}$

check ok ✓

D) $\log_a x + \log_a (x-4) = \log_a (x+6)$

$\log_a [x(x-4)] = \log_a (x+6)$

$x^2 - 4x = x+6$

$x^2-5x-6=0 \rightarrow (x-6)(x+1)=0$

$\boxed{x=6}$ ✓ check

$\cancel{x=-1} \rightarrow$ Reject

E) $\log_{\frac{1}{3}}(x^2+x) - \log_{\frac{1}{3}}(x^2-x) = -1$

$\log_{\frac{1}{3}}\left(\frac{x^2+x}{x^2-x}\right) = -1$

$\Rightarrow 3 \Rightarrow \left(\frac{1}{3}\right)^{-1} = \frac{x^2+x}{x^2-x}$

$3x^2-3x = x^2+x$

$2x^2-4x=0$

$2x(x-2)=0$

check

$\cancel{x=0}$ Reject

$\boxed{x=2}$ ✓

$\boxed{\text{FINAL EXAM}}$ (DEC 2012)

3

A) $f(x) = 72 - 6x$

$$\sum_{K=0}^{50} f(k) = f(0) + f(1) + f(2) + \cdots f(49) + f(50)$$

$f(0) = 72 - 6(0) = 72 \rightarrow 1^{ST}$ Term

$f(1) = 72 - 6(1) = 66$

$f(2) = 72 - 6(2) = 60$

$\vdots$

$f(50) = 72 - 6(50) = -228$ Last Term

ARITHMETIC $\boxed{d = -6}$

$a_1 = 72$ $n = 51$

$a_{51} = -228$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{51} = \frac{51}{2}\left(72 + (-228)\right) = \frac{51}{2}(-156) = -3978$ $\boxed{S_{51} = -3978}$

(OR $S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right) \Rightarrow S_{51} = \frac{51}{2}\left[2(72) + (51-1)(-6)\right] = \frac{51}{2}(-156) = -3978$)

B) $g(x) = 5(0.5)^x$

$$\sum_{h=0}^{30} g(h) = g(0) + g(1) + \cdots g(29) + g(30)$$

$g(0) = 5(0.5)^0 = 5 \rightarrow 1^{ST}$ Term

$g(1) = 5(0.5)^1 = 2.5$

$g(2) = 5(0.5)^2 = 1.25$

$\vdots$

GEOMETRIC $\boxed{r = 0.5}$

$a_1 = 5$ $n = 31$

$$S_n = \frac{a_1(r^n - 1)}{r - 1}$$

$$S_{31} = \frac{5\left((0.5)^{31} - 1\right)}{0.5 - 1} = 9.999999995 \simeq \boxed{10}$$

$S_{31} = 10$

FINAL EXAM $\boxed{\text{DEC 2012}}$

$\boxed{3}$

4

A) EARL

$PMT = 5000 \quad FV = ?$

$r = 0.06 \quad m = 1 \quad t = 12$

$i = \frac{r}{m} = \frac{0.06}{1} = 0.06 \quad n = mt = (1)(12) = 12$

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$$

$$= 5000\left(\frac{(1+0.06)^{12} - 1}{0.06}\right) = 5000(16.8699412)$$

$$\boxed{FV = 84\,349.71}$$

Then use COMPOUND INTEREST FORMULA $A = P(1+i)^n$

with $P = 84\,349.71 \quad i = 0.06 \quad m = 1 \quad t = 36$

$A = ? \quad n = mt = (1)(36) = 36$

$$A = P(1+i)^n = 84\,349.71(1+0.06)^{36}$$

$$= 84\,349.71(8.147252) = \boxed{687\,218.34}$$

EARL'S AMOUNT WHEN HE RETIRES

B) LARRY

$PMT = 5000 \quad r = 0.06 \quad m = 1 \quad t = 36$

$FV = ? \quad i = 0.06 \quad n = mt = (1)(36) = 36$

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$$

$$\doteq 5000\left(\frac{(1+0.06)^{36} - 1}{0.06}\right)$$

$$= 5000(119.1208667)$$

$$\boxed{595\,604.33}$$

LARRY'S AMOUNT WHEN HE RETIRES

FINAL EXAM $\boxed{\text{DEC 2012}}$

C) Who paid the most Money INTO his IRA

EARL → $5000 \times 12 = \$60\,000$
(12 years)

LARRY → $5000 \times 36 = \$180\,000$
(36 years)

- LARRY PAID the MOST, However he got LESS IN his IRA (595 604.33) than EARL (687 218.34) UPON Retirement.

(5) A) PV = ? $\quad r = 0.0648 \quad m = 12 \quad t = 20$

PMT = 1500 $\quad i = \frac{r}{m} = \frac{0.0648}{12} = 0.0054 \quad n = mt = (12)(20) = 240$

(1) $PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$

$$= 1500\left(\frac{1-(1+0.0054)^{-240}}{0.0054}\right) = 1500(134.3370739)$$

$$= \boxed{\$201\,505.61}$$

(2) $FV = 201\,505.61 \quad i = 0.0054 \quad m = 12 \quad t = 15$

PMT = ? $\quad n = mt = (12)(15) = 180$

$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$

$$201505.61 = PMT\left(\underbrace{\frac{(1+0.0054)^{180}-1}{0.0054}}_{303.0227126}\right)$$

$\boxed{PMT = \$664.99}$

5 CONTINUED

B) INTEREST OVER 35 years

- TOTAL DEPOSITS : $\$664.99 \times 180 = \$119\,698.20$
  (15 years)

- TOTAL WITHDRAWALS : $\$1500 \times 240 = \$360\,000$
  (20 years)

So TOTAL INTEREST earned over 35 years

$$360\,000 - 119\,698.20 = \boxed{240\,301.80}$$

C)

$FV = ?$ $\quad i = 0.0054 \quad m = 12 \quad t = 15$

$PMT = 1000 \quad n = mt = (12)(15) = 180$

① $FV = PMT\left(\dfrac{(1+i)^n - 1}{i}\right) = 1000\left(\dfrac{(1+0.0054)^{180} - 1}{0.0054}\right)$

$$\boxed{FV = \$303\,022.71}$$

② $PV = 303\,022.71 \quad i = 0.0054 \quad m = 12 \quad t = 20$

$PMT = ? \quad n = mt = (12)(20) = 240$

$$PV = PMT\left(\frac{1 - (1+i)^{-n}}{i}\right)$$

$$303\,022.71 = PMT\left(\frac{1 - (1+0.0054)^{-240}}{0.0054}\right)$$

$$\boxed{PMT = \$2255.69}$$

FINAL EXAM (DEC 2012)

6 A) $x+y+z = 16$

$300x + 900y + 1500z = 19200$

B) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 300 & 900 & 1500 & 19200 \end{array}\right) \quad \frac{1}{300}R_2 \to R_2$

$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 1 & 3 & 5 & 64 \end{array}\right) \xrightarrow{-1R_1+R_2 \to R_2} \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 0 & 2 & 4 & 48 \end{array}\right) \quad \frac{1}{2}R_2 \to R_2$

$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 0 & 1 & 2 & 24 \end{array}\right) -1R_2+R_1 \to R_1 \sim \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 1 & 0 & -1 & -8 \\ 0 & 1 & 2 & 24 \end{array}\right) \end{array}$

- LET $z=t$ (VARIABLE WITHOUT Leading 1)

SO $x = -8+t$ • SINCE x, y, and z is # of TRUCKS
$y = 24-2t$ $\quad x, y, z \geq 0$

$x \geq 0 \Rightarrow -8+t \geq 0$ AND $y \geq 0 \Rightarrow 24-2t \geq 0$

$t \geq 8$ $\quad -2t \geq -24$

$t \leq 12$

THEREFORE $t = 8, 9, 10, 11, 12$

SOLUTIONS

	x	y	z
t=8	0	8	8
t=9	1	6	9
t=10	2	4	10
t=11	3	2	11
t=12	4	0	12

C) DAILY INCOME

$19.95(0) + 29.95(8) + 39.95(8) = 559.20$

$19.95(1) + 29.95(6) + 39.95(9) = 559.20$

$19.95(2) + 29.95(4) + 39.95(10) = 559.20$

$19.95(3) + 29.95(2) + 39.95(11) = 559.20$

$19.95(4) + 29.95(0) + 39.95(12) = 559.20$

DAILY INCOME IS THE SAME (559.20) FOR ANY COMBINATION

FINAL EXAM $\boxed{\text{DEC. 2012}}$

7) A) TECHNOLOGICAL MATRIX M

$$M = \begin{array}{c} \\ A \\ M \\ E \end{array} \begin{array}{c} \begin{array}{ccc} A & M & E \end{array} \\ \begin{bmatrix} 0.2 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{bmatrix} \end{array}$$

A = AGRICULTURE

M = MANUFACTURING

E = ENERGY

B) $X = MX + D$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{array}{c} A \\ M \\ E \end{array} \quad D = \begin{bmatrix} 20 \\ 10 \\ 30 \end{bmatrix} \begin{array}{c} A \\ M \\ E \end{array}$$

BILLION

$$\begin{array}{c} A \\ M \\ E \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{array}{c} \\ A \\ M \\ E \end{array} \begin{array}{c} \begin{array}{ccc} A & M & E \end{array} \\ \begin{bmatrix} 0.2 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 20 \\ 10 \\ 30 \end{bmatrix} \begin{array}{c} A \\ M \\ E \end{array}$$

$$X \quad = \quad M \quad X \quad + \quad D$$

AND $X - MX = D$

$(I-M)X = D \Rightarrow X = (I-M)^{-1}D$

C) $$I - M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.2 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.8 & -0.4 & -0.3 \\ -0.2 & 0.9 & -0.1 \\ -0.2 & -0.1 & 0.9 \end{bmatrix}$$

- TO FIND $(I-M)^{-1}$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 0.8 & -0.4 & -0.3 & 1 & 0 & 0 \\ -0.2 & 0.9 & -0.1 & 0 & 1 & 0 \\ -0.2 & -0.1 & 0.9 & 0 & 0 & 1 \end{array}\right] \Longrightarrow (I-M)^{-1} = \begin{bmatrix} 1.6 & 0.78 & 0.62 \\ 0.4 & 1.32 & 0.28 \\ 0.4 & 0.32 & 1.28 \end{bmatrix}$$

$$\quad M \qquad I$$

$\Rightarrow$ SOLUTION $X = (I-M)^{-1}D$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{array}{c} A \\ M \\ E \end{array} = \underbrace{\begin{bmatrix} 1.6 & 0.78 & 0.62 \\ 0.4 & 1.32 & 0.28 \\ 0.4 & 0.32 & 1.28 \end{bmatrix}}_{(I-M)^{-1}} \underbrace{\begin{bmatrix} 20 \\ 10 \\ 30 \end{bmatrix}}_{D} = \begin{bmatrix} 58.4 \\ 29.6 \\ 49.6 \end{bmatrix} \begin{array}{c} A - x_1 \\ M - x_2 \\ E - x_3 \end{array}$$

BILLION

⑧ $P(x,y) = 20x + 10y$

① $3x + y \geq 24$

x	y
0	24
8	0
4	12
6	6

SOLID
TP(0,0)
$0 \geq 24$
FALSE

② $6x + 6y \leq 96$

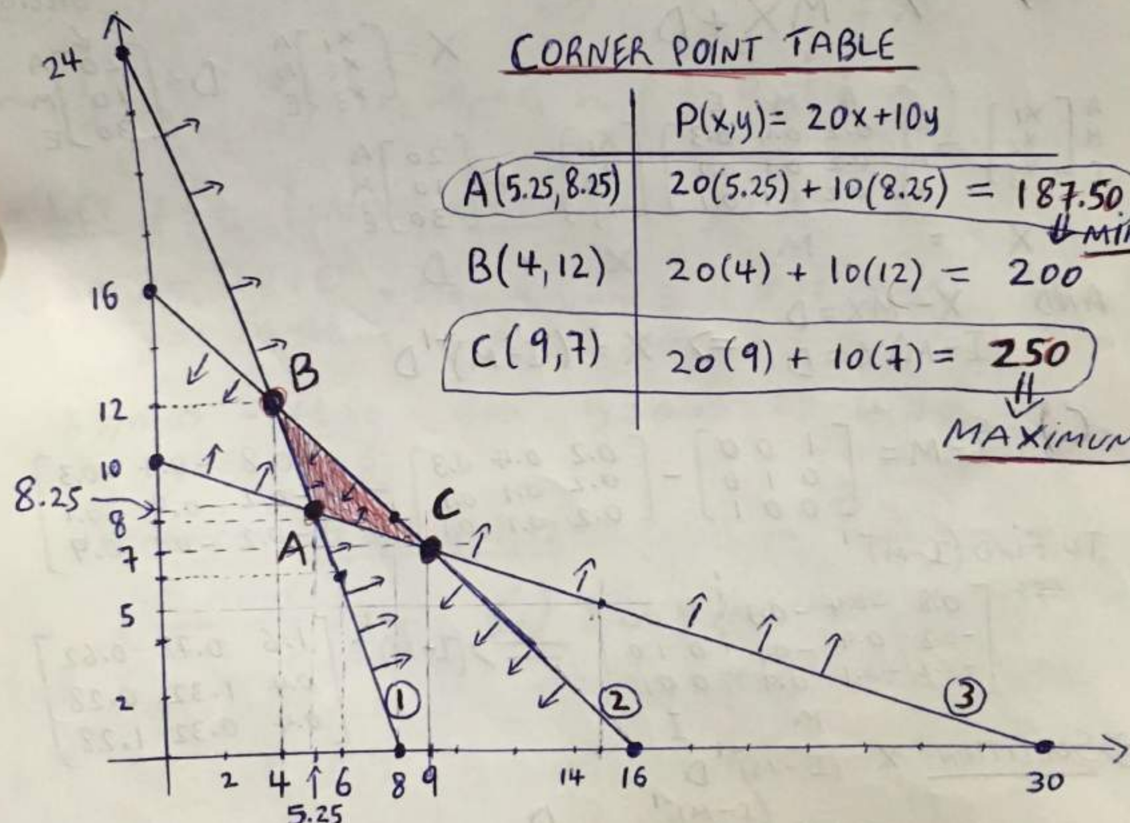
x	y
0	16
16	0
8	8
4	12
9	7

Solid
TP(0,0)
$0 \leq 96$
TRUE

③ $3x + 9y \geq 90$

x	y
0	10
30	0
9	7
15	5

Solid
TP(0,0)
$0 \geq 90$
FALSE



CORNER POINT TABLE

	$P(x,y) = 20x + 10y$
A(5.25, 8.25)	$20(5.25) + 10(8.25) = 187.50$ → MINIMUM
B(4, 12)	$20(4) + 10(12) = 200$
C(9, 7)	$20(9) + 10(7) = 250$ → MAXIMUM

POINT A ① ∩ ③

$$\begin{aligned} 3x + y &= 24 \\ - \quad 3x + 9y &= 90 \\ \hline -8y &= -66 \\ y = \frac{33}{4} &= 8.25 \end{aligned}$$

and $3x + 8.25 = 24$
$3x = 15.75$
$x = 5.25$

A(5.25, 8.25)

FINAL EXAM $\boxed{\text{DEC 2012}}$

6

$\boxed{\text{MATH 208}}$

9

A) $\begin{cases} \text{PLANT A} \Rightarrow 6 \\ \text{PLANT B} \Rightarrow 8 \end{cases}$

O_1: Selecting 1 officer from Plant A to Fill V_1 $\Rightarrow$ N_1: 6 WAYS

O_2: Selecting 1 officer from PLANT B to Fill V_2 $\Rightarrow$ N_2: 8 WAYS

$\Rightarrow$ MULTIPLICATION PRINCIPLE: $N_1 \cdot N_2 = 6 \times 8 = \boxed{48 \text{ ways}}$

(OR $P_{6,1} \times P_{8,1} = 6 \times 8 = 48$)

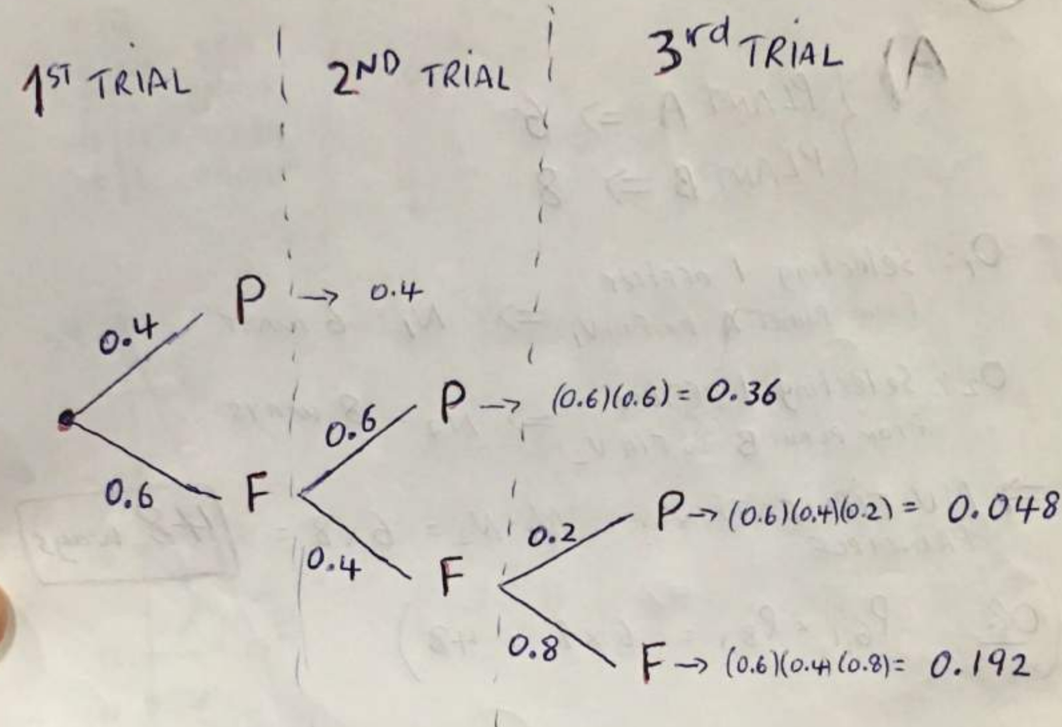
B) WITHOUT REGARD TO PLANT: So $6+8 = 14$ OFFICERS TOTAL

Choose 2 out of 14 (WITH ORDER and NO REPLACEMENTS)

$P_{14,2} = 14 \times 13 = \boxed{182 \text{ ways}}$

FINAL EXAM DEC 2012

10 P = PASS F = FAIL



A) PROB. OF P 1ST <u>OR</u> 2ND TRIAL

$$0.4 + 0.36 = \boxed{0.76}$$

B) PROB OF F all 3 ATTEMPTS

$$P(F,F,F) = \boxed{0.192}$$

C) PROB. OF F ON 1ST <u>TWO</u> TRIALS and P on Third.

$$P(F,F,P) = \boxed{0.048}$$

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/2		All except EC
Examination	**Date**	**Time**	**Pages**
Final	December 2015	3 Hours	3

Instructors — **Course Examiner**

E. Duma, F. Romanelli, H. Greenspan, J. Ortmann, L. Aggoun, L. Dube — D. Sen
M. Padamadan, P. Gauthier, U. Tiwari, V. Kalvin

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\,\frac{(1+i)^n - 1}{i}, \quad PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- Answer all questions.
- **Only approved calculators are allowed.**

MARKS

[10] **1.** At a price of \$2.28 per bushel, the supply of barley is 7,500 million bushels and the demand is 7,900 million bushels. At a price of \$2.37 per bushel, the supply of barley is 7,900 million bushels and the demand is 7,800 million bushels.

(A) Find a price-supply equation of the form $p = mx + b$.

(B) Find a price-demand equation of the form $p = mx + b$.

(C) Find the equilibrium point.

[10] **2.** Solve for x in the following equations:

(A) $9^{2x-1} = 27^x$

(B) $(2)^{3x} = \frac{1}{32}$

(C) $\log_2 \sqrt{2x^2} - 1 = \frac{3}{2}$

(D) $\log_5(x+2) + \log_5 x = \log_5(x+12)$

(E) $\log_{\frac{1}{3}}(27) = x + 2$

[10] **3.** For $f(x) = 360 - 60x$ and $g(x) = 10^{x-10}$ find the following by only using a proper formula:

(A) $\sum_{k=0}^{49} f(k) = f(0) + f(1) + f(2) + \cdots + f(49)$.

(B) $\sum_{h=1}^{35} g(h) = g(1) + g(2) + g(3) + \cdots + g(35)$.

[10] **4.** A father opened a savings account for his daughter on the day she was born, depositing \$1000. Each year on her birthday he deposits another \$1000, making the last deposit on her 21st birthday.

(A) If the account pays 5.25% interest compounded annually, how much is in the account at the end of the day on his daughter's 21st birthday?

(B) How much interest has been earned?

[10] **5.** A family purchased their home 8 years ago for \$83,000. The home was financed by paying 20% down and signing a 30−year mortgage at 8.4% compounded monthly for the balance. Equal monthly payments were made to amortize the loan over the 30−year period.

(A) What is the monthly payment?

(B) Find the unpaid balance just after the 96th payment is made.

(C) The market value of the house is now \$95,000. After making the 96th payment, the family applied to the loan company for the maximum loan. The loan company will loan up to 60% of the equity in a home. How much will the family receive?

[10] **6.** Solve by using Gauss-Jordan Elimination:

$$3x_1 + 2x_2 + 4x_3 = -1$$

$$-2x_1 + x_2 - 2x_3 = 6$$

$$3x_1 + 3x_2 + 6x_3 = 3$$

No other method of solving this system of equations will be accepted!

[10] **7.** An island economy consists of the sectors of tourism, agriculture and fishing. To produce a dollar's worth of tourism requires an input of \$0.3, \$0.2 and \$0.1 from tourism, agriculture and fishing respectively. A dollar's worth of agriculture requires inputs of \$0.1 from each sector. On the other hand, a dollar's worth of fishing requires inputs of \$0.3, \$0.2 and \$0.1 from the sectors of tourism, agriculture and fishing.

(A) Write the technology matrix M for this island economy.

(B) If a final demand of \$40 million, \$10 million and \$20 million from tourism, agriculture and fishing is to be met, then set up the equation to be satisfied by the inputs from the respective sectors.

(C) Solve the respective inputs satisfying these demands.

[10] **8.** Extremize $P(x, y) = 20x - 15y$ subject to

$$x + 2y \geq 14,\ x + 3y \leq 42,\ 2x + y \leq 42,\ x \geq 0,\ y \geq 0.$$

[10] **9.** In an experiment on plant hardiness, a researcher gathers 6 wheat plants, 5 barley plants, and 3 rye plants. She wishes to select 4 plants at random.

(A) In how many ways can this be done?

(B) In how many ways can this be done if exactly 2 wheat plants must be included?

[10] **10.** To transfer into a particular technical department, a company requires an employee to pass a screening test. A maximum of 3 attempts is allowed at 6-month intervals between trials. From past record it is found that 40% pass on the first trial; of those that fail the first trial and take the test a second time; 60% pass; and of those that fail on the second trial and take the test a third time, 20% pass. For an employee wishing to transfer:

(A) What is the probability of passing the test on the first or second try?

(B) What is the probability of failing on all 3 attempts?

(C) What is the probability of failing on the first 2 trials and passing on the third?

FINAL EXAM | MATH 208 | DEC 2014

(1) $C(x) = 20x + 100$

A) PRICE = \$24 so $R(x) = 24x$ -- REVENUE

and BREAK-EVEN $\Rightarrow R(x) = C(x)$

$24x = 20x + 100$

$4x = 100$

$\boxed{x = 25}$

B) $\begin{cases} \text{LOSS } R < C \Rightarrow [0, 25) \\ \text{PROFIT } R > C \Rightarrow (25, +\infty) \end{cases}$

C) PROFIT $= P(x) = R(x) - C(x)$

$P(x) = 24x - (20x + 100) = 24x - 20x - 100$

$\boxed{P(x) = 4x - 100} \Rightarrow$ PROFIT

AT $x = 100$ $P(100) = 4(100) - 100 = \boxed{\$300}$

D) FIND x when PROFIT = \$900

$P(x) = 900 \Rightarrow 900 = 4x - 100$

$1000 = 4x \Rightarrow \boxed{x = 250}$

(2) A) $9^{-x+15} = 27^{x}$

$(3^2)^{-x+15} = (3^3)^x$

$3^{-2x+30} = 3^{3x}$

$\Rightarrow -2x + 30 = 3x$

$5x = 30$

$\boxed{x = 6}$

MATH 208 DEC 2014

2 (CONTINUED)

B) $e^{-3x^2+15x-72} = e^{-x^2+35x-22}$

$\Rightarrow -3x^2+15x-72 = -x^2+35x-22$

$2x^2+20x+50=0$

$2(x^2+10x+25)=0$

$2(x+5)^2=0$

(OR USE QUADRATIC FORMULA)

$\boxed{x=-5}$

C) $\log_7\left(\frac{x}{5}\right) + \log_7 625 + 2\log_7\sqrt{5} = 3\log_7\sqrt[3]{7} + 4\log_7 5$

$\log_7\left(\frac{x}{5}\right) + \cancel{\log_7 625} + \log_7(5^{1/2})^2 = \log_7(7^{1/3})^3 + \cancel{\log_7(5^4)}$

$5^4 = 625$

$\log_7\left(\frac{x}{5}\right) = \log_7 7 - \log_7 5$

$\log_7\left(\frac{x}{5}\right) = \log_7\left(\frac{7}{5}\right) \Rightarrow \frac{x}{5} = \frac{7}{5} \Rightarrow \boxed{x=7}$

Check; ok ✓

D) $\log_5(x+6) + \log_5(x+2) = 1$

$\log_5 (x+6)(x+2) = 1$

$5^1 = (x+6)(x+2)$

$5^1 = x^2+8x+12$

$x^2+8x+7=0$

$(x+1)(x+7)=0$

Check $\cancel{x=-7} \Rightarrow$ Reject

ok $\boxed{x=-1}$ ✓ 1 SOLUTION ONLY

E) $\log_3\sqrt{x-2} = 2$

$3^2 = \sqrt{x-2}$

$9 = \sqrt{x-2}$ (SQUARE BOTH SIDES)

$9^2 = x-2$

$x-2=81$

$\boxed{x=83}$

Check, ok ✓

FINAL EXAM | MATH 208 | DEC 2014 | 2

3

A) $f(x) = 180 - 30x$ $\quad \sum_{K=1}^{50} f(K) = f(1) + f(2) + \cdots + f(50)$

$f(1) = 180 - 30(1) = 150 \Rightarrow$ 1ST TERM (a_1) $\quad \boxed{d = -30}$

$\vdots$

$f(50) = 180 - 30(50) = 180 - 1500 = -1320 \Rightarrow$ LAST TERM (a_n)

$\boxed{n = 50}$ TERMS $\quad$ SUM $= S_n = \frac{n}{2}(a_1 + a_n)$

$\Rightarrow S_{50} = \frac{50}{2}(150 + (-1320)) = 25(-1170) \Rightarrow \boxed{-29\,250}$

B) $g(x) = 5^{x-5}$ $\quad \sum_{h=0}^{29} g(h) = g(0) + g(1) + \cdots + g(29)$

$g(0) = 5^{0-5} = 5^{-5} = a_1 \Rightarrow$ 1ST Term $\quad \boxed{r = 5}$

$\boxed{n = 30}$ *

SUM $S_n = \dfrac{a_1(r^n - 1)}{r - 1}$

$S_{30} = \dfrac{5^{-5}(5^{30} - 1)}{5 - 1} \approx \boxed{7.45 \times 10^{16}}$

4

A) $FV = PMT\left(\dfrac{(1+i)^n - 1}{i}\right)$ $\quad \Rightarrow \begin{cases} PMT = \$2000, \; r = 0.08, \; m = 1 \\ i = \frac{r}{m} = \frac{0.08}{1} = 0.08 \quad t = 9 \\ n = mt = (1)(9) = 9 \end{cases}$

$FV = 2000\left(\dfrac{(1+0.08)^9 - 1}{0.08}\right) = \boxed{FV = 24975.12}$ AND $\Rightarrow$

MATH 208 DEC 2014

4 A) (CONTINUED)

$A = P(1+i)^n$ $\left\{ P = \$24\,975.12 \quad i = 0.08 \quad n = 36 \right.$

$A = 24\,975.12(1+0.08)^{36} = \boxed{398\,807.01}$

AMOUNT ON 65th birthday

B) $FV = PMT\left(\frac{(1+i)^n - 1}{i}\right) \Rightarrow \left\{ PMT = \$2000 \quad i = \frac{r}{m} = \frac{0.08}{1} = 0.08 \quad n = mt = (1)(36) = 36 \quad t = 36 \right.$

$FV = 2000\left(\frac{(1+0.08)^{36} - 1}{0.08}\right) = \boxed{374\,204.30}$

5 A) UNPAID LOAN BALANCE AFTER 5 years

2 years LEFT TO PAY ON 7 year LOAN (8% QUARTERLY)

→ So $n = 2 \times 4 = 8$ PAYMENTS LEFT.

$PV_5 = PMT\left(\frac{1-(1+i)^{-n}}{i}\right) \Rightarrow \left\{ PMT = 1000 \quad r = 0.08,\ m = 4 \quad i = \frac{r}{m} = \frac{0.08}{4} = 0.02,\ n = 8 \right.$

$PV_5 = 1000\left(\frac{1-(1+0.02)^{-8}}{0.02}\right) = \boxed{PV_5 = \$7325.28}$

unpaid balance after 5 years

B) INTEREST IN 5th YEAR ONLY

⇒ FIND UNPaid Loan balance AFTER 4 years

→ 3 years LEFT

→ so $n = 3 \times 4 = 12$

$PV_4 = 1000\left(\frac{1-(1+0.02)^{-12}}{0.02}\right) = \$10\,575.34$

AMOUNT PAID-OFF DURING the 5th year ⇒ $PV_4 - PV_5$

→ $10\,575.34 - 7325.28 = 3249.86$

TOTAL PAYMENTS IN the 5th YEAR = $4 \times \$1000 = \$4000$

INTEREST PAID DURING The 5th year ⇒ $4000 - 3249.86 = \boxed{\$750.14}$

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5 (CONTINUED)

C) INTEREST on the 1^{ST} PAYMENT (PAYMENT = \$1000)

$\Rightarrow \$21\,281.27 \times 0.02 = \$425.63$

So $1000 - 425.63 = \boxed{574.37} \Rightarrow$ PRINCIPAL REPAID IN the 1^{ST} Payment.

D) TOTAL INTEREST = TOTAL PAYMENTS − INITIAL LOAN

$= \underbrace{7 \times 4 \times \$1000}_{(28000)} - 21\,281.27$

$= \boxed{6718.73} \Rightarrow$ TOTAL INTEREST

6 A) $x + y + z = 16$

$300x + 900y + 1500z = 19200$

B) $\begin{array}{c} \begin{matrix} x & y & z & \end{matrix} \\ \left(\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 300 & 900 & 1500 & 19200 \end{array}\right) \end{array} \quad \frac{1}{300}R_2 \to R_2 \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 1 & 3 & 5 & 64 \end{array}\right) \; -R_1 + R_2 \to R_2$

$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 0 & 2 & 4 & 48 \end{array}\right) \; \frac{1}{2}R_2 \to R_2 \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 0 & 1 & 2 & 24 \end{array}\right) \; -R_2 + R_1 \to R_1$

$\sim \begin{array}{c} \begin{matrix} x & y & z & \end{matrix} \\ \left(\begin{array}{ccc|c} 1 & 0 & -1 & -8 \\ 0 & 1 & 2 & 24 \end{array}\right) \end{array} \Rightarrow$ Let $z = t \Rightarrow \begin{cases} x = -8 + t \\ y = 24 - 2t \\ z = t \end{cases}$

Since x, y, z are # of TRUCKS $\Rightarrow$ must be (+) or 0.

So $x, y, z \geq 0 \Rightarrow x \geq 0$ OR $-8 + t \geq 0$

$\boxed{t \geq 8}$

$z \geq 0 \Rightarrow t \geq 0$

$\Rightarrow y \geq 0$ OR $24 - 2t \geq 0$

$-2t \geq -24$

$\boxed{t \leq 12}$

So $8 \leq t \leq 12$

$\Rightarrow t = 8, 9, 10, 11, 12$

Rr. P(x) EXAM

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6 (CONTINUED)

B) SOLUTIONS (FIVE)

	x	y	z
t=8	0	8	8
t=9	1	6	9
t=10	2	4	10
t=11	3	2	11
t=12	4	0	12

C) LOWEST DAILY INCOME

$24.95x + 39.95y + 49.95z$

$24.95(0) + 39.95(8) + 49.95(8) = \$719.20$

$24.95(1) + 39.95(6) + 49.95(9) = 714.20$

$24.95(2) + 39.95(4) + 49.95(10) = 709.20$

$24.95(3) + 39.95(2) + 49.95(11) = 704.20$

$24.95(4) + 39.95(0) + 49.95(12) = \boxed{699.20}$

$\Rightarrow$ So $(\overset{x}{4}, \overset{y}{0}, \overset{z}{12})$ PRODUCES LOWEST INCOME.

7 A)

$$M = \begin{array}{c c} & \begin{array}{ccc} C & O & T \end{array} \\ \begin{array}{c} C \\ O \\ T \end{array} & \begin{bmatrix} 0.2 & 0 & 0.4 \\ 0 & 0.1 & 0.2 \\ 0.4 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

C = COAL
O = OIL
T = TRANSPORTATION

B) EQUATION $\Rightarrow X = MX + D$

X = OUTPUT MATRIX $X = \begin{array}{cc} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} & \begin{array}{c} C \\ O \\ T \end{array} \end{array}$

D = FINAL DEMAND $D = \begin{array}{cc} \begin{bmatrix} 20 \\ 5 \\ 10 \end{bmatrix} & \begin{array}{c} C \\ O \\ T \end{array} \end{array}$

M = TECHNOLOGICAL MATRIX

$$\begin{array}{c} C \\ O \\ T \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{array}{c c} & \begin{array}{ccc} C & O & T \end{array} \\ \begin{array}{c} C \\ O \\ T \end{array} & \begin{bmatrix} 0.2 & 0 & 0.4 \\ 0 & 0.1 & 0.2 \\ 0.4 & 0.2 & 0.2 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 20 \\ 5 \\ 10 \end{bmatrix} \begin{array}{c} C \\ O \\ T \end{array}$$

$X = M \quad X + D$

SOLUTION $\Rightarrow X = MX + D$

$X - MX = D$

$(I - M)X = D \Rightarrow$ OR $\boxed{X = (I-M)^{-1}D}$

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7 (CONTINUED)

C) SOLVE...

$$I - M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{array}{c} \begin{matrix} C & O & T \end{matrix} \\ \begin{pmatrix} 0.2 & 0 & 0.4 \\ 0 & 0.1 & 0.2 \\ 0.4 & 0.2 & 0.2 \end{pmatrix} \end{array} = \begin{array}{c} \begin{matrix} C & O & T \end{matrix} \\ \begin{pmatrix} 0.8 & 0 & -0.4 \\ 0 & 0.9 & -0.2 \\ -0.4 & -0.2 & 0.8 \end{pmatrix} \\ I-M \end{array}$$

$$\boxed{X = (I-M)^{-1} D}$$

AND

$$\Rightarrow (I-M)^{-1} = \begin{pmatrix} 1.7 & 0.2 & 0.9 \\ 0.2 & 1.2 & 0.4 \\ 0.9 & 0.4 & 1.8 \end{pmatrix} \qquad D = \begin{pmatrix} 20 \\ 5 \\ 10 \end{pmatrix}\begin{matrix} C \\ O \\ T \end{matrix}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\begin{matrix} C \\ O \\ T \end{matrix} = \begin{pmatrix} 1.7 & 0.2 & 0.9 \\ 0.2 & 1.2 & 0.4 \\ 0.9 & 0.4 & 1.8 \end{pmatrix}\begin{pmatrix} 20 \\ 5 \\ 10 \end{pmatrix} = \begin{pmatrix} 44 \\ 14 \\ 38 \end{pmatrix}\begin{matrix} C \\ O \\ T \end{matrix}$$

$$X = (I-M)^{-1} D$$

SOLUTION INPUTS

- $x_1 = 44$ BILLION COAL
- $x_2 = 14$ BILLION OIL
- $x_3 = 38$ BILLION TRANSPORTATION

8 $P(x,y) = 50x + 150y$

① $6x + 3y \geq 48$

$\geq$ solid

TP(0,0) $0 \geq 48$ FALSE

x	y
0	16
8	0
4	8
6	4

② $5x + 5y \geq 60$

$\geq$ Solid

TP(0,0) $0 \geq 60$ FALSE

x	y
0	12
12	0
4	8
10	2

③ $3x + 6y \geq 42$

$\geq$ SOLID

TP(0,0) $0 \geq 42$ FALSE

x	y
0	7
14	0
10	2
6	4

AND $x \geq 0, y \geq 0 \Rightarrow$ QUADRANT I ONLY

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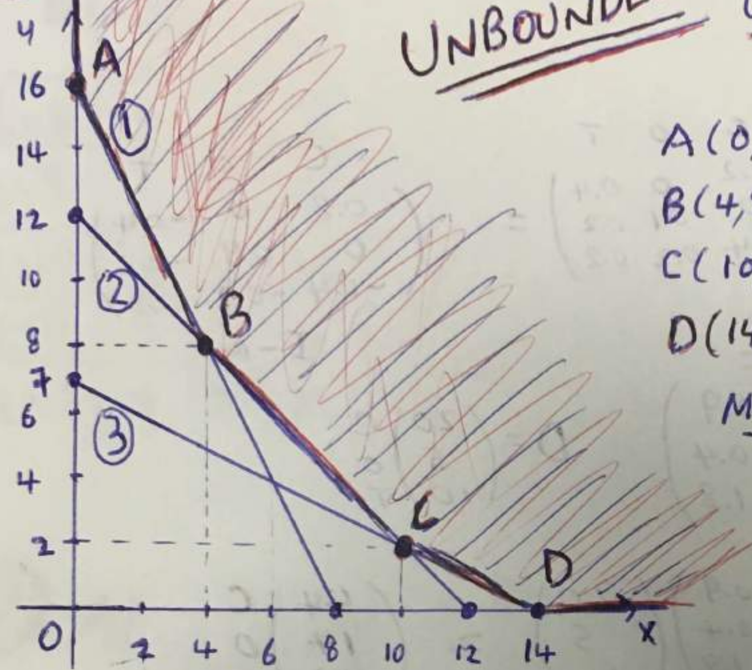
8 (CONTINUED)



UNBOUNDED

CORNER POINT TABLE

$P(x,y) = 50x + 150y$

$A(0,16) \Rightarrow 50(0) + 150(16) = 2400$

$B(4,8) \Rightarrow 50(4) + 150(8) = 1400$

$C(10,2) \Rightarrow 50(10) + 150(2) = 800$

$D(14,0) \Rightarrow 50(14) + 150(0) = \boxed{700}$

MINIMUM is 700 (AT D(14,0))

NO MAXIMUM SINCE UNBOUNDED

9 A) $\begin{cases} \text{PLANT A} \Rightarrow 6 \\ \text{PLANT B} \Rightarrow 8 \end{cases}$

O_1: Selecting 1 officer From Plant A to Fill V_1 $\Rightarrow N_1$: 6 ways

O_2: Selecting 1 officer from PLANT B to fill V_2 $\Rightarrow N_2$: 8 ways

* Using multiplication PRINCIPLE $\Rightarrow$ $N_1 \cdot N_2 = 6 \times 8 = \boxed{48 \text{ ways}}$

(OR USE $P_{6,1} \times P_{8,1} = 6 \times 8 = 48$)

B) WITHOUT Regard to PLANT $\Rightarrow$ $6 + 8 = 14$ OFFICERS TOTAL

Choose 2 out of 14 (with ORDER & NO Replacement)

$\Rightarrow P_{14,2} = 14 \times 13 = \boxed{182 \text{ WAYS}}$

↓ Permutation

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(10) • 40 Mickey Mouse Watches with 6 Defective ones
• choose 7 for Testing
• REJECT entire shipment IF 1 or MORE defective

A) EVENT E : AT LEAST 1 watch is DEFECTIVE (out of the chosen 7)
(shipment REJECTED)

$\Rightarrow$ So Complement
EVENT E' : NONE are DEFECTIVE
(shipment ACCEPTED)

• Since there are 40 watches TOTAL and 6 are DEFECTIVE then $40-6 = 34$ NON-DEFECTIVE WATCHES

So –
$n(E') = C_{34,7}$ (# of ways of choosing 7 watches out of 34 non-def. watches)

$n(S) = C_{40,7}$ (# of ways of choosing 7 watches out of 40 TOTAL watches)

Sample space

$$P(E') = \frac{C_{34,7}}{C_{40,7}} \approx \boxed{0.29}$$

(Shipment ACCEPTED)
since NONE are defective

B) $P(E) = 1 - P(E') = 1 - 0.29 \approx \boxed{0.71}$

Shipment Rejected

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①

(1) a) $p(x) = 360 - 20x \quad C(x) = 300 + 95x$

$R(x) = px = (360-20x)x \quad (1 \le x \le 15) \quad (a \ominus n)$

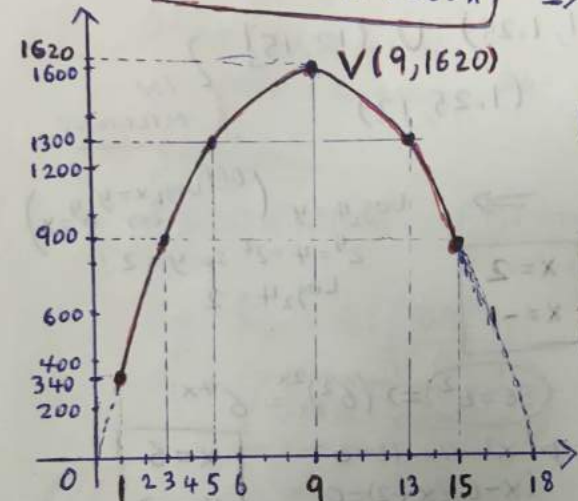
$\boxed{R(x) = -20x^2 + 360x} \Rightarrow$ y-INT : $y = 0$

X-INT : $-20x(x-18)=0$ $\begin{cases} x=0 \\ x=18 \end{cases}$

$V\left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right)$ $\begin{cases} \Delta = b^2 - 4ac \\ \Delta = (360)^2 - 4(-20)(0) \\ \Delta = (360)^2 = 129600 \end{cases}$

$V\left(\frac{-360}{2(-20)}, \frac{-129\,600}{4(-20)}\right)$

$V(9, 1620)$



b) MAX REVENUE OCCURS AT THE VERTEX

OUTPUT $x = 9$ million tablet computers

MAXIMUM REVENUE $R(9) = \$1620$ million $\quad p(x) = 360 - 20x$

use price-demand for PRICE $\Rightarrow p(9) = 360 - 20(9) = 180$

PRICE $p = \$180$

c) FIND BREAK-EVEN POINTS $R(x) = C(x)$

$-20x^2 + 360x = 300 + 95x$

SO $20x^2 - 265x + 300 = 0$

$\Delta = b^2 - 4ac = (-265)^2 - 4(20)(300) \Rightarrow \Delta = 46225$ ..... $\Rightarrow$

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(1) c) CONTINUED

$$x = \frac{265 \pm \sqrt{46225}}{2(20)} = \frac{265 \pm 215}{40} \begin{cases} \frac{265+215}{40} = \frac{480}{40} = 12 \\ \frac{265-215}{40} = \frac{50}{40} = 1.25 \end{cases}$$

SO

LOSS OCCURS $\Rightarrow$ $[1, 1.25) \cup (12, 15]$

PROFIT OCCURS $\Rightarrow$ $(1.25, 12)$

IN MILLIONS

(2)

a) $x^2 - x = \log_2 4 \Rightarrow \log_2 4 = y$ (DEF $\log_b x = y \Leftrightarrow b^y = x$)

$x^2 - x = 2$ $\qquad 2^y = 4 = 2^2$ so $y = 2$!

$x^2 - x - 2 = 0$ $\qquad \log_2 4 = 2$

$(x-2)(x+1) = 0$

$\boxed{x = 2 \\ x = -1}$

b) $36^{2x} - 6^{x^2-12} = 0$ $\qquad$ $36 = 6^2 \Rightarrow (6^2)^{2x} = 6^{4x}$

$6^{4x} = 6^{x^2-12}$

$\Rightarrow 4x = x^2 - 12 \to x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$\boxed{x = 6 \\ x = -2}$

c) $2\log_3(x+1) - 2\log_3 9 = 2 \Rightarrow \log_3 9 = y$

$\log_3 (x+1)^2 - 2(2) = 2$ $\qquad 3^y = 9 = 3^2$

$\log_3 (x+1)^2 = 6$ $\qquad$ so $y = 2$

$3^6 = (x+1)^2 \to x^2 + 2x + 1 = 729$

$x^2 + 2x - 728 = 0$

$(x-26)(x+28) = 0$

$\boxed{x = 26 \text{ ONLY}}$

$x = \cancel{-28}$ Reject

Can't have $\ominus$ log

d) $\log_8(x-1) + \log_8(x+1) = 1$

$\log_8 (x-1)(x+1) = 1$

$8^1 = (x-1)(x+1) \to x^2 - 1 = 8$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$\boxed{x = 3 \text{ ONLY}}$

($x = \cancel{-3}$ Reject)

e) $\log_{10}(x+7) - \log_{10}(x-2) = 1$

$\log_{10}\left(\frac{x+7}{x-2}\right) = 1 \to \frac{x+7}{x-2} = 10^1$

$10x - 20 = x + 7 \to 9x = 27$

$\boxed{x = 3}$ check ✓

FINAL EXAM MATH 208 APRIL 2014 (2)

(3) a) SUM = $\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$

$n = 100$
$a_{100} = 1000$
$S_{100} = 50000$
FIND a_1

$50\,000 = \frac{100}{2}(a_1 + 1000)$

$50000 = 50(a_1 + 1000)$

$1000 = a_1 + 1000$ so $\boxed{a_1 = 0}$

b) geometric

$h(n) = 10(0.9)^n$ First term $\Rightarrow h(2) = 10(0.9)^2 = 8.1 = a_1$

$r = 0.9$ and $n = 19$ Terms

So $S_{19} = 8.1\left(\frac{(0.9)^{19} - 1}{0.9 - 1}\right)$ $\left[S_n = a_1\left(\frac{r^n - 1}{r - 1}\right)\right]$

$\boxed{S_{19} \approx 70.0581\ldots}$

(4) a)

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left(\begin{array}{ccc|c} 3 & 6 & 9 & 3 \\ 2 & 3 & 4 & 3 \\ 3 & 6 & 3 & 9 \end{array}\right) \quad \frac{1}{3}R_1 \to R_1$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & 3 \\ 3 & 6 & 3 & 9 \end{array}\right) \quad \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \end{array}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & -6 & 6 \end{array}\right) \quad -1R_2 \to R_2 \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -6 & 6 \end{array}\right) \quad \begin{array}{l} -2R_2 + R_1 \\ \to R_1 \end{array}$$

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(4) a) CONTINUED

$$\sim\left(\begin{array}{ccc|c}1&0&-1&3\\0&1&2&-1\\0&0&-6&6\end{array}\right) \quad -\tfrac{1}{6}R_3\to R_3$$

$$\sim\left(\begin{array}{ccc|c}1&0&-1&3\\0&1&2&-1\\0&0&1&-1\end{array}\right) \quad \begin{array}{l}-2R_3+R_2\to R_2\\ 1R_3+R_1\to R_1\end{array}$$

$$\sim\begin{array}{c}\begin{array}{ccc} x & y & z\end{array}\\ \left(\begin{array}{ccc|c}1&0&0&2\\0&1&0&1\\0&0&1&-1\end{array}\right)\end{array} \quad \text{so} \quad \boxed{\begin{array}{l}x=2\\y=1\\z=-1\end{array}}$$

b) $3(2)+6(1)+9(-1)=3$ ✓

$2(2)+3(1)+4(-1)=3$ ✓

$3(2)+6(1)+3(-1)=9$ ✓

(5) a) $A=P(1+i)^n$

$A=150000\left(1+\frac{1}{300}\right)^{24}$

$\boxed{A=\$162\,471.44}$

$P=150000 \quad A=?$

$r=0.04, \ m=12, \ t=2$

$i=\frac{r}{m}=\frac{0.04}{12}=\frac{1}{300}$

$n=mt=12(2)=24$

b) $PV=PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$

$8000=PMT\left(\frac{1-(1+0.0025)^{-60}}{0.0025}\right)$

$\boxed{PMT=\$143.75}$

$PV=8000 \quad PMT=?$

$r=0.03, \ m=12, \ t=5$

$i=\frac{r}{m}=\frac{0.03}{12}=0.0025$

$n=mt=12(5)=60$

and INTEREST = TOTAL PAYMENTS − INITIAL LOAN

$(60\times 143.75)-8000 \Rightarrow$ INTEREST $=\boxed{\$625}$

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6 a) $FV = PMT\left(\frac{(1+i)^n-1}{i}\right)$

$PMT = 5000 \quad FV = ?$
$r = 0.08,\ m = 1,\ t = 30$
$i = \frac{r}{m} = \frac{0.08}{1} = 0.08$
$n = mt = (1)(30) = 30$

$FV = 5000\left(\frac{(1+0.08)^{30}-1}{0.08}\right)$

$\boxed{FV = \$\,566\,416.06}$

b) $PV = FV = 566\,416.06$

$PMT = ?$
$r = 0.06,\ m = 12,\ t = 10$
$i = \frac{r}{m} = \frac{0.06}{12} = 0.005$
$n = mt = 12(10) = 120$

$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$

$566\,416.06 = PMT\left(\frac{1-(1+0.005)^{-120}}{0.005}\right)$

$\boxed{PMT = \$\,6288.38}$

7 Extremize $P(x,y) = 10x + 12y$

① $2x + 3y \le 12$

x	y
0	4
6	0
1	10/3
3	2

$\le$ SOLID
TP(0,0)
$0 \le 12$
TRUE

② $4x - 5y \le 2$

x	y
0	-0.4
1	0.4
$\frac{1}{2}$	0
$\frac{7}{4}$	1
3	2

$\le$ SOLID
TP(0,0)
$0 \le 2$
TRUE

INTERSECTION POINT

① $2x + 3y = 12 \quad (\times -2)$
② $4x - 5y = 2$

$+$ ①' $-\cancel{4x} - 6y = -24$
② $\cancel{4x} - 5y = 2$

$-11y = -22$
$y = 2$

and
$2x + 3(2) = 12$
$2x = 6$
$x = 3$

So $(3,2)$

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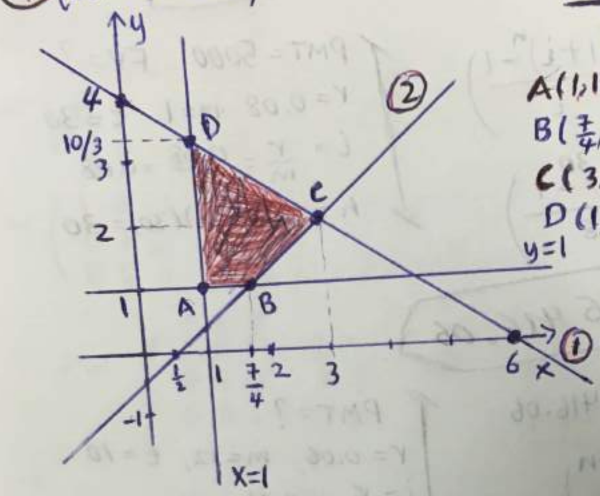
(7) (CONTINUED)



CORNER POINT TABLE

$P(x,y) = 10x + 12y$

$A(1,1) \Rightarrow 10(1) + 12(1) = \boxed{22}$ MIN

$B(\frac{7}{4},1) \Rightarrow 10(7/4) + 12(1) = 29.5$

$C(3,2) \Rightarrow 10(3) + 12(2) = \boxed{54}$ MAX

$D(1,\frac{10}{3}) \Rightarrow 10(1) + 12(\frac{10}{3}) = 50$

(8) a) $PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$

$PMT = ?$ $PV = 130000$

$r = 0.042$, $m = 12$, $t = 25$

$i = \frac{r}{m} = \frac{0.042}{12} = 0.0035$

$n = mt = 12(25) = 300$

$130000 = PMT\left(\frac{1-(1+0.0035)^{-300}}{0.0035}\right)$

$\boxed{PMT = \$700.63}$

b) UNPAID LOAN BALANCE AFTER 15 years

$\Rightarrow$ 10 years LEFT so $n = 10(12) = 120$

$PV = 700.63\left(\frac{1-(1+0.0035)^{-120}}{0.0035}\right) = \boxed{68\,555.88}$

UNPAID LOAN BALANCE

c) INTEREST = TOTAL PAYMENTS − INITIAL LOAN

$= (300 \times 700.63) - 130\,000$

$\boxed{\text{INTEREST} = 80189}$

9

a) $C_{9,3} = 84$

b) $\dfrac{C_{5,3}}{C_{9,3}} = \dfrac{10}{84} = \dfrac{5}{42} \approx 0.11905\ldots$

c) $\dfrac{C_{5,1} \cdot C_{4,2}}{C_{9,3}} = \dfrac{30}{84} = \dfrac{5}{14} \approx 0.35714\ldots$

10

a) PERMUTATION $\Rightarrow$ ORDER MATTERS

$$\frac{P_{6,2}}{P_{10,2}} = \frac{30}{90} = \frac{1}{3} \approx 0.3333\ldots$$

b) COMBINATION – MAJORITY MEN

2 MEN + 1 WOMAN & 3 MEN

$$\frac{C_{4,2}\, C_{6,1}}{C_{10,3}} + \frac{C_{4,3}}{C_{10,3}}$$

$$\frac{36}{120} + \frac{4}{120} = \frac{40}{120}$$

$$= \frac{1}{3} \approx 0.3333\ldots$$

APRIL 2017 (1)

MATH 208 Final EXAM Dec 2013

1

#1 Price 2.28 $\begin{cases} S: 7500 \\ D: 7900 \end{cases}$

Price 2.37 $\begin{cases} S: 7900 \\ D: 7800 \end{cases}$

$p = mx + b$!!

* PRICE = y-value !!

a) SUPPLY $\begin{pmatrix} x & p \\ 7500, & 2.28 \\ 7900, & 2.37 \end{pmatrix}$

Slope $\frac{2.37 - 2.28}{7900 - 7500} = \frac{0.09}{400}$

$m = 0.000225$

$p = mx + b$

$2.28 = 0.000225(7500) + b$

$2.28 = 1.6875 + b$

$b = 2.28 - 1.6875$

$b = 0.5925$

SUPPLY

$p = 0.000225x + 0.5925$

b) Demand (x, p)

$(7900 \quad 2.28)$

$(7800 \quad 2.37)$

Slope $\frac{2.37 - 2.28}{7800 - 7900} = \frac{0.09}{-100}$

$m = -0.0009$

$p = mx + b$

$2.28 = -0.0009(7900) + b$

$2.28 = -7.11 + b$

$b = 7.11 + 2.28$

$b = 9.39$

Demand

$p = -0.0009x + 9.39$

c) Equilibrium Supply = Demand

$0.000225x + 0.5925 = -0.0009x + 9.39$

$0.000225x + 0.0009x = 9.39 - 0.5925$

$0.001125x = 8.7975$

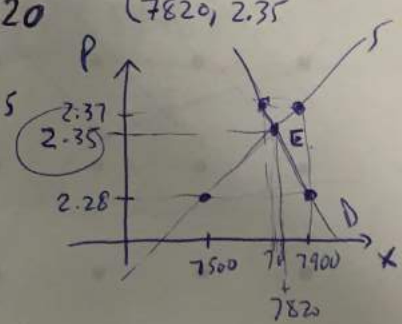
$X = \frac{8.7975}{0.001125} = 7820$

and $p = 0.000225(7820) + 0.5925$

$p = 2.352 \overset{\$}{=} 2.35$

d)

Equilibrium
(x, p)
$(7820, 2.35)$



FINA

208

#2 A) $2^{x+1}\, 8^{-x} = 4$

$2^{x+1} (2^3)^{-x} = 2^2$

$2^{x+1}\, 2^{-3x} = 2^2$

$2^{x+1+(-3x)} = 2^2$

$2^{-2x+1} = 2^2$

$-2x+1 = 2$

$-2x = +1$

$\boxed{x = -1/2}$

B) $(e^4)^x e^{x^2} = e^{12}$

$e^{4x} e^{x^2} = e^{12}$

$e^{4x+x^2} = e^{12}$

$x^2+4x = 12$

$x^2+4x-12=0$

$(x+6)(x-2)=0$

$\boxed{x=-6}$ ✓

$\boxed{x=2}$ ✓

C) $\log x + \log_{10}(x+15) = 2$

$\log_{10} x(x+15) = 2$

$10^2 = x(x+15)$

$x^2+15x-100=0$

$(x+20)(x-5)=0$

$x = \cancel{-20}$ Reject

$x=5$ ✓ Check

D) $\log_{10}(x-1) - \log_{10}(x+1) = 1$

$\log_{10}\left(\frac{x-1}{x+1}\right) = 1$

$10^1 = \frac{x-1}{x+1}$

$10x+10 = x-1$

$9x = -11$

$x = \cancel{-11/9}$ check

No solution

E) $\log_6(x+3) + \log_6(x+4) = 1$

$\log_6((x+3)(x+4)) = 1$

$6^1 = (x+3)(x+4)$

$6 = x^2+7x+12$

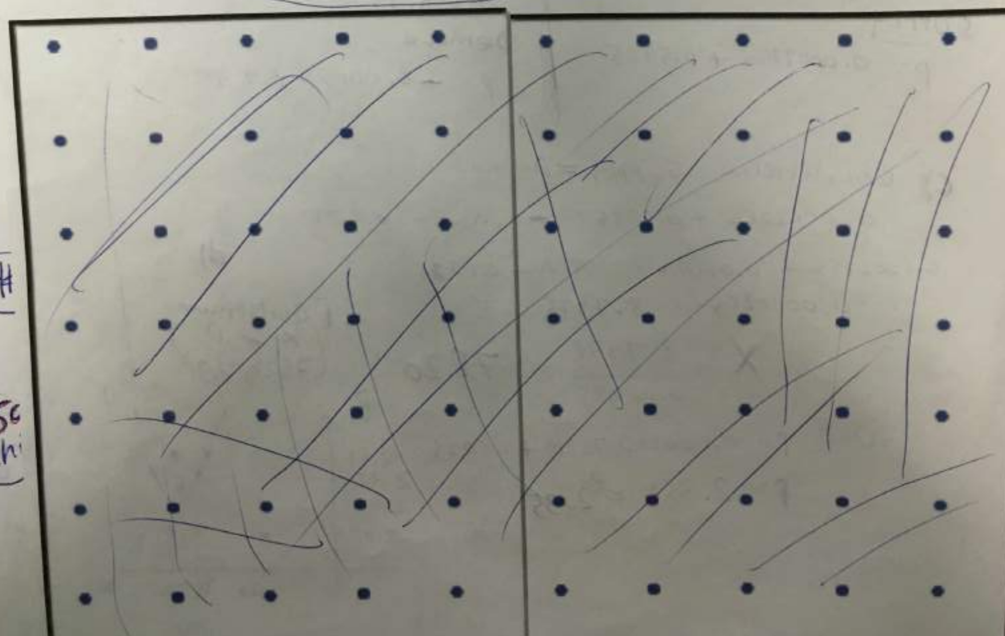
$\Rightarrow x^2+7x+6=0$

$(x+6)(x+1)=0$

$x=-1$ ✓ check

$x = \cancel{-6}$ reject

** $\boxed{\text{Def log} \Rightarrow \log_b x = y \Rightarrow b^y = x}$



#

SC

ch

$\text{PROB}_1 = 1/12^4 = \frac{1}{20736} \approx 0.000048$

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#3

A) $f(x) = 100 - 20x$

50 Terms

$\sum_{k=1}^{50} f(x) = f(1) + f(2) + \dots f(50)$

$f(1) = 100 - 20(1) = 80 \rightarrow a_1$ (First Term)

$f(2) = 100 - 20(2) = 60$

$f(3) = 100 - 20(3) = 40$

$(d = -20)$

$\vdots$

$f(50) = 100 - 20(50) = -900 \Rightarrow$ Last term

a_{50} $n = 50$ Terms

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{50} = \frac{50}{2}\left(80 + (-900)\right) = 25(-820) = \boxed{-20500}$

B) $g(x) = 8(0.8)^x$

$\sum_{h=0}^{39} g(h) = g(0) + g(1) + g(2) + \dots + g(39)$

$g(0) = 8(0.8)^0 = 8 \rightarrow a_1$ (First Term)

$g(1) = 8(0.8)^1 = 6.4$

$g(2) = 8(0.8)^2 = 5.12$

$(r = 0.8)$

$\vdots$

40 Terms

$S_n = \frac{a_1(r^n - 1)}{r - 1}$

$S_{40} = \frac{8\left((0.8)^{40} - 1\right)}{0.8 - 1} = \boxed{39.99468309}$

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#4

A) \$2000 IN IRA ON 21^{st} B-Day and up to and including 29 B-Day

8% compounded Annually

$21 \rightarrow 29 \Rightarrow$ 9 years

$\left.\begin{matrix}21\\22\\23\\24\\25\\26\\27\\28\\29\end{matrix}\right\}$ 9 years

ANNUITY

$FV = ?$ $PMT = \$2000$ $r = 0.08$

$m = 1$ $t = 9$

$i = \frac{r}{m} = \frac{0.08}{1} = 0.08$ $n = mt$

$n = (1)(9) = 9$

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right) = 2000\left(\frac{(1+0.08)^9 - 1}{0.08}\right)$$

$\boxed{FV = \$24\,975.12}$ $* 65 - 29 = 36$

Now $A = P(1+i)^n$

$A = 24\,975.12\,(1+0.08)^{36}$

$\boxed{A = 398\,807.01}$

B) $FV = ?$ $t = 36$ years

$PMT = 2000$ $i = \frac{r}{m} = \frac{0.08}{1}$ $n = mt = (1)(36) = 36$

$m = 1$ $r = 0.08$

$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right) = 2000\left(\frac{(1+0.08)^{36} - 1}{0.08}\right)$$

$$\boxed{FV = \$374\,204.30}$$

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5 A) $PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$

$PMT = \$5000$

$r = 0.0732$

$m = 4$ $\quad i = \frac{r}{m} = \frac{0.0732}{4} = 0.018$

$t = 10$ $\quad n = mt = 4 \cdot 10 = 40$

$PV = 5000\left(\frac{1-(1.0183)^{-40}}{0.0183}\right)$

$\boxed{PV = 140\,945.57}$

B) $FV = PV$ found in a) $\quad n = mt = 4 \times 20 = 80$

$i = \frac{r}{m} = 0.0183$

$FV = 140\,945.57$

Find PMT = ?

$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$

$140\,945.57 = PMT\left(\frac{(1.0183)^{80} - 1}{0.0183}\right)$ $\quad \boxed{PMT = 789.65}$

C) Total interest over 30 year period

Total withdrawals $= \underbrace{40}_{10 \text{ years}} \times 5000 = 200\,000$

$-$ Total deposits $= \underbrace{80}_{20 \text{ years}} \times 789.65 = -\ 63\,172$

$\boxed{136\,828}$

Total interest over 30 years

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(6) $\left(\begin{array}{ccc|c} 3 & -1 & -2 & -10 \\ 2 & 4 & -1 & -1 \\ 4 & -2 & 3 & 3 \end{array}\right)$ $\frac{1}{2} R_2 \to R_2$

$\sim \left(\begin{array}{ccc|c} 3 & -1 & -2 & -10 \\ 1 & 2 & -1/2 & -1/2 \\ 4 & -2 & 3 & 3 \end{array}\right)$ $R_1 \leftrightarrow R_2$ $\sim \left(\begin{array}{ccc|c} 1 & 2 & -1/2 & -1/2 \\ 3 & -1 & -2 & -10 \\ 4 & -2 & 3 & 3 \end{array}\right)$ $-3R_1 + R_2 \to R_2$, $-4R_1 + R_3 \to R_3$

[redacted] $\sim \left(\begin{array}{ccc|c} 1 & 2 & -1/2 & -1/2 \\ 0 & -7 & -1/2 & -17/2 \\ 0 & -10 & 5 & 5 \end{array}\right)$ $-\frac{1}{10} R_3 \to R_3$

$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1/2 & -1/2 \\ 0 & -7 & -1/2 & -17/2 \\ 0 & 1 & -1/2 & -1/2 \end{array}\right)$ $R_2 \leftrightarrow R_3$ $\sim \left(\begin{array}{ccc|c} 1 & 2 & -1/2 & -1/2 \\ 0 & 1 & -1/2 & -1/2 \\ 0 & -7 & -1/2 & -17/2 \end{array}\right)$

$-2R_2 + R_1 \to R_1$

$7R_2 + R_3 \to R_3$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & -1/2 & -1/2 \\ 0 & 0 & -4 & -12 \end{array}\right)$ $-\frac{1}{4} R_3 \to R_3$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & -1/2 & -1/2 \\ 0 & 0 & 1 & 3 \end{array}\right)$ $1/2\, R_3 + R_2 \to R_2$, $-1/2\, R_3 + R_1 \to R_1$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right)$ so $\boxed{\begin{array}{l} X_1 = -1 \\ X_2 = 1 \\ X_3 = 3 \end{array}}$

[redacted]

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A)

$$M = \begin{array}{c|ccc} & T & A & F \\ \hline T & 0.3 & 0.3 & 0.1 \\ A & 0.1 & 0.1 & 0.1 \\ F & 0.2 & 0.2 & 0.1 \end{array}$$

B) Final demand $D = \begin{bmatrix} 25 \\ 15 \\ 20 \end{bmatrix} \begin{matrix} T \\ A \\ F \end{matrix}$ → millions

Output matrix $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{matrix} T \\ A \\ F \end{matrix}$

M = Technological matrix

Equation: $X = MX + D$

$$\begin{matrix} T \\ A \\ F \end{matrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0.3 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.1 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 25 \\ 15 \\ 20 \end{bmatrix}}_{D} \begin{matrix} T \\ A \\ F \end{matrix}$$

$$X = \quad M \; X \; + D$$

Solution $X = MX + D$

$X - MX = D$ or $X = (I - M)^{-1} D$

$(I - M) X = D \Rightarrow$

C) Solution

$$I - M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{array}{c} \begin{matrix} T & A & F \end{matrix} \\ \begin{pmatrix} 0.3 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.1 \end{pmatrix} \end{array} = \underbrace{\begin{pmatrix} 0.7 & -0.3 & -0.1 \\ -0.1 & 0.9 & -0.1 \\ -0.2 & -0.2 & 0.9 \end{pmatrix}}_{I-M}$$

So
$$(I - M)^{-1} = \begin{array}{c|ccc} & T & A & F \\ \hline T & 1.58 & 0.58 & 0.24 \\ A & 0.22 & 1.22 & 0.16 \\ F & 0.4 & 0.4 & 1.2 \end{array} \qquad D = \begin{pmatrix} 25 \\ 15 \\ 20 \end{pmatrix}$$

So
$$X = (I - M)^{-1} D = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1.58 & 0.58 & 0.24 \\ 0.22 & 1.22 & 0.16 \\ 0.4 & 0.4 & 1.2 \end{pmatrix} \begin{pmatrix} 25 \\ 15 \\ 20 \end{pmatrix} \begin{matrix} T \\ A \\ F \end{matrix} = \begin{pmatrix} 53 \\ 27 \\ 40 \end{pmatrix} \begin{matrix} T \\ A \\ F \end{matrix}$$

→ millions

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8 $P(x,y) = 60x + 20y$

(1) $12x + 12y \geq 24$

x	y
0	2
2	0

solid TP(0,0) $0 \geq 24$ FALSE

(2) $6x + 4y \leq 36$

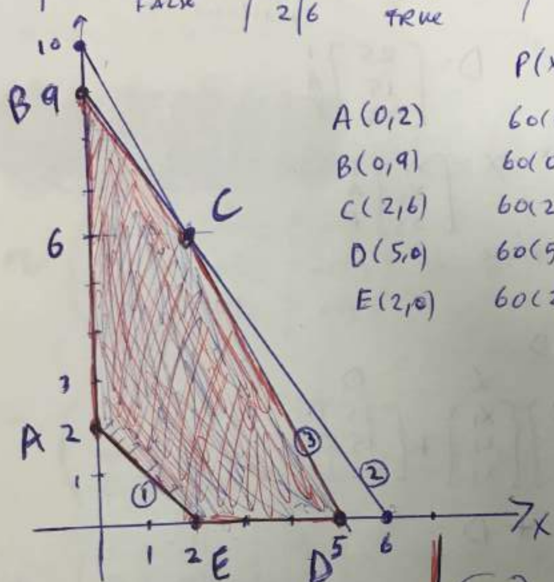
x	y
0	9
6	0
2	6

solid TP(0,0) $0 \leq 36$ TRUE

(3) $10x + 5y \leq 50$

x	y
0	10
5	0
2	6

$\leq$ Solid TP(0,0) $0 \leq 50$ TRUE



$P(x,y) = 60x + 20y$

A(0,2) $60(0) + 20(2) = \boxed{40}$ MIN

B(0,9) $60(0) + 20(9) = 180$

C(2,6) $60(2) + 20(6) = 240$

D(5,0) $60(5) + 20(0) = \boxed{300}$ MAX

E(2,0) $60(2) + 20(0) = 120$

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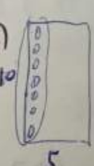
a)



$P_{10,10} \cdot P_{40,40}$

$10! \cdot 40!$

$= 2.96 \times 10^{54}$

50 chips

b) $P_{40,10} \cdot P_{40,40}$

$\frac{40!}{30!} \cdot 40! = 2.51 \times 10^{63}$

10 A)

ways you can guess Correctly = 1

Sample Space: # of ways you can choose 4 out of 12 WITH ORDER NO Repetition $\Rightarrow P_{12,4}$

PROB. ALL 4 BRANDS Could be identified By guessing $= \frac{1}{P_{12,4}} = \frac{1}{11880} = 0.000084$

B) # ways choose 4 out 12 WITH Repetition $\Rightarrow 12 \times 12 \times 12 \times 12 = 12^4$

PROB = $1/12^4 = \frac{1}{20736} = 0.000048$