ENGR 233: Lecture1
Introduction to vector calculus

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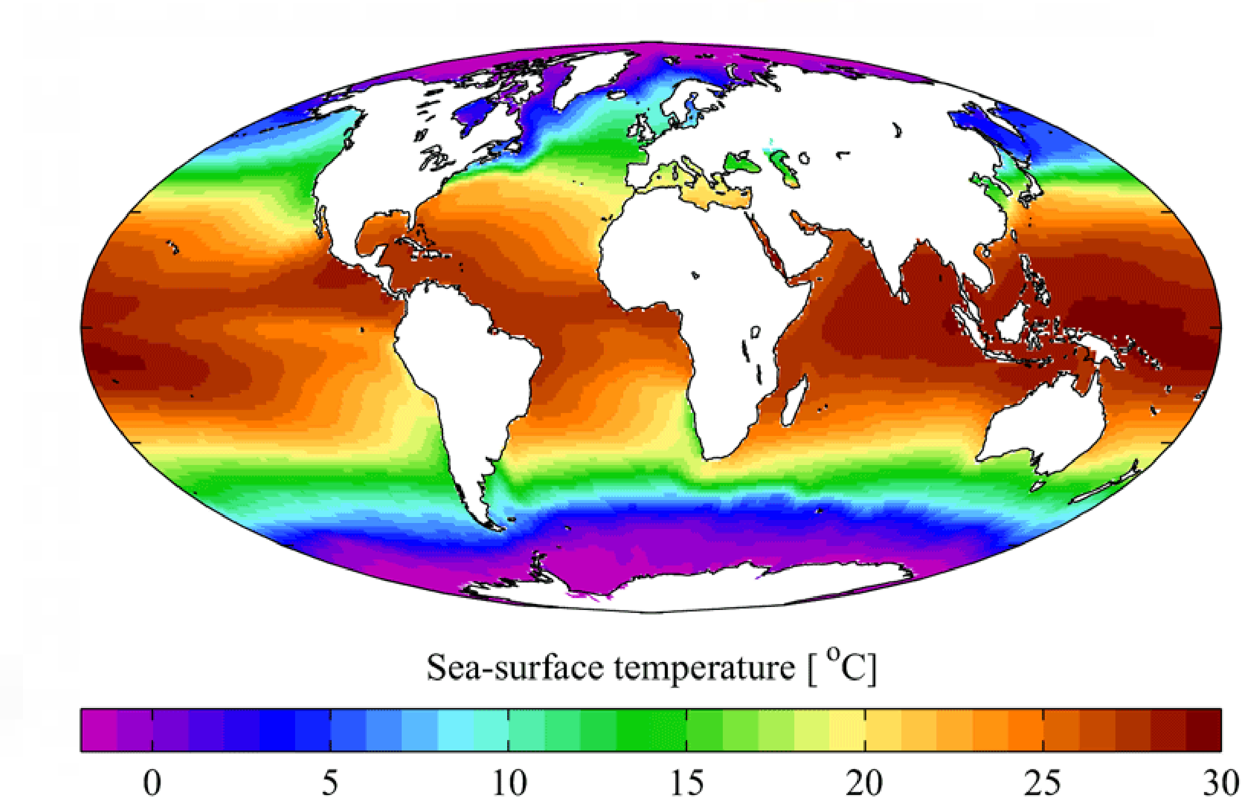
Vectors: Why are they important?

- Two forms of quantities

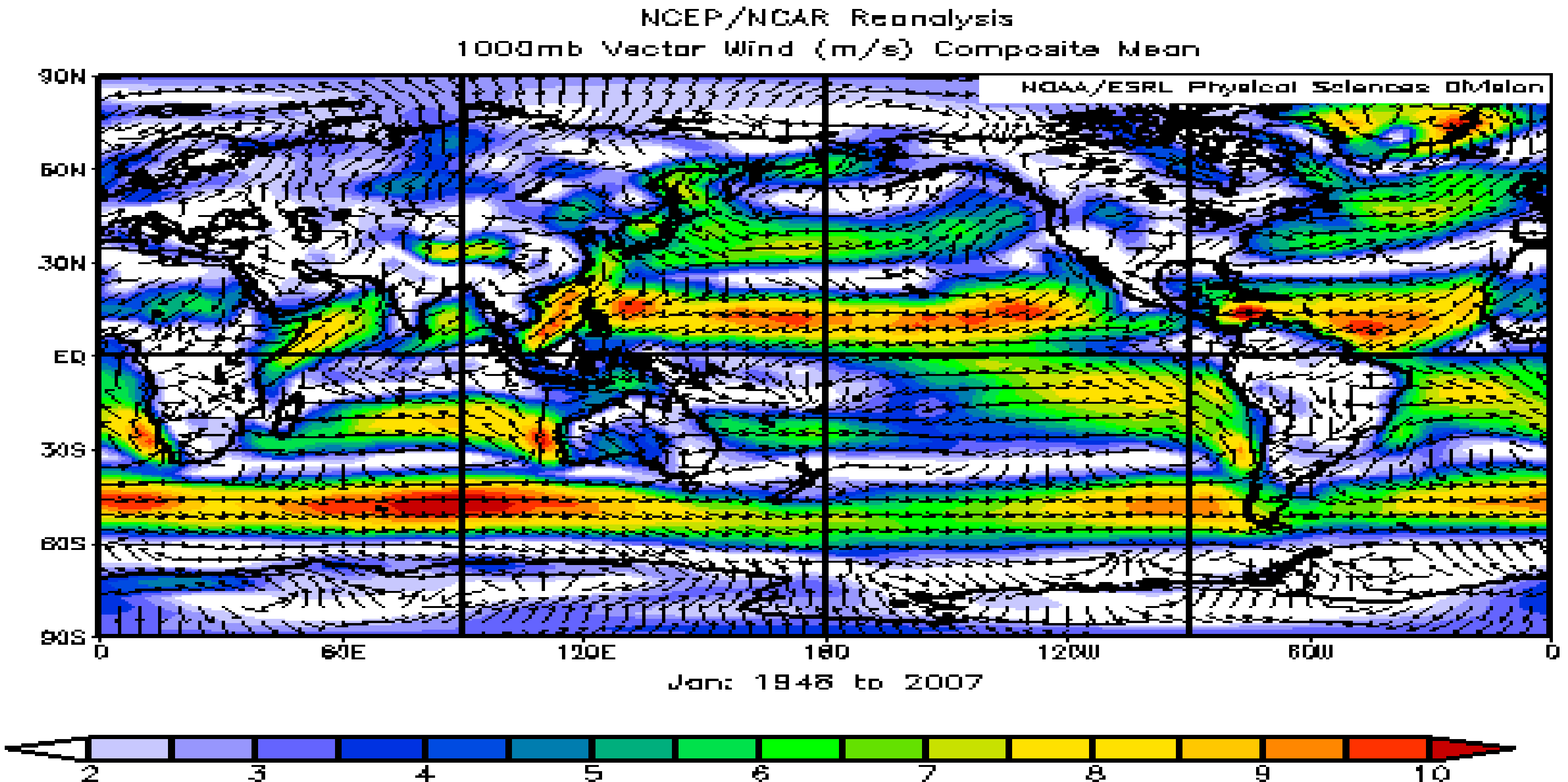
Scalars



Vectors: Why are they important?

- Two forms of quantities

Vectors



Vectors: What do they entail?

A wind of 80 km/h from the Southwest.

A car going 80 km/h northeast.

A plane traveling 1000 km/h on a 180 heading.

These issues are described by a **magnitude** and a **direction**.

Vectors: Notion and terminology

- A vector with starting point A and end point B is written as $\overrightarrow{AB}$
- Magnitude of this vector is written as: $\| \overrightarrow{AB} \|$
- The textbook uses **boldface** to represent vectors,

$$\boldsymbol{F} \qquad \boldsymbol{u}$$

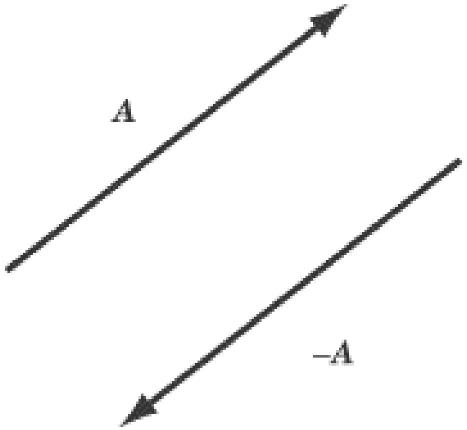
- You may also place an arrow above general vectors and a hat over unit vectors.

$$\vec{F} \qquad \hat{u}$$

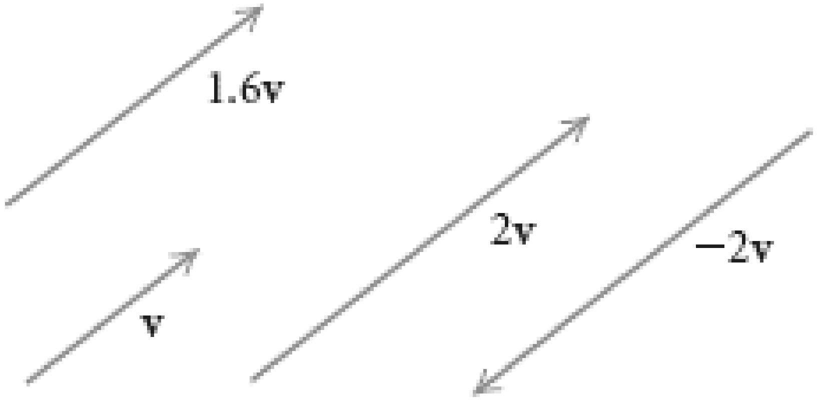
- From now on you should clearly identify vectors in your work and distinguish them from scalars.

Vectors: Notion and terminology

- Negative of a vector

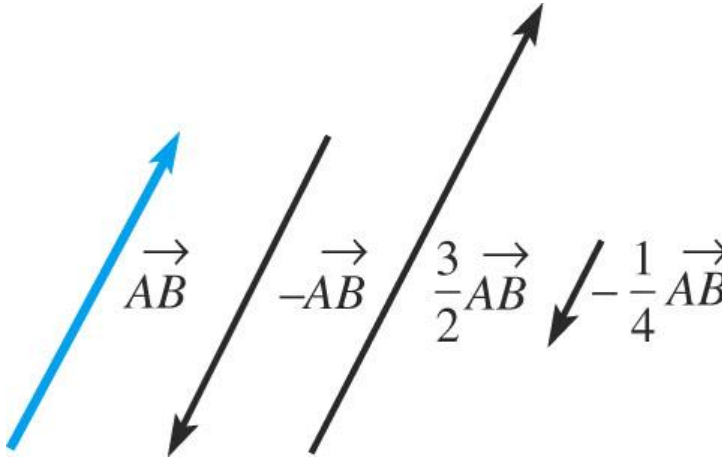


- Scalar multiplication of vector

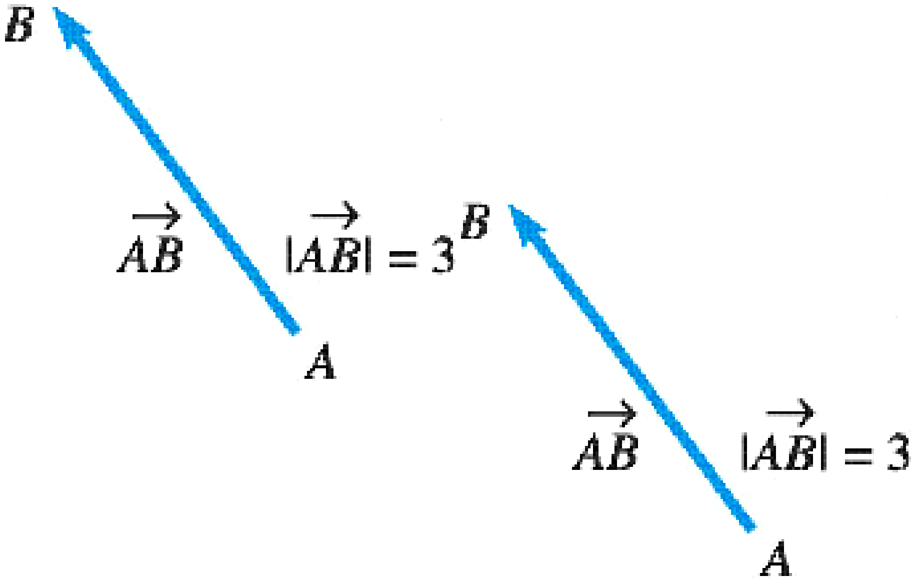


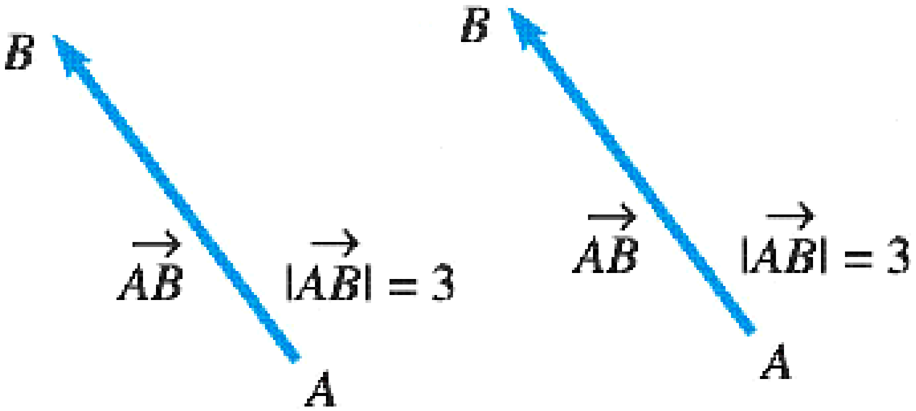
Vectors: Notion and terminology

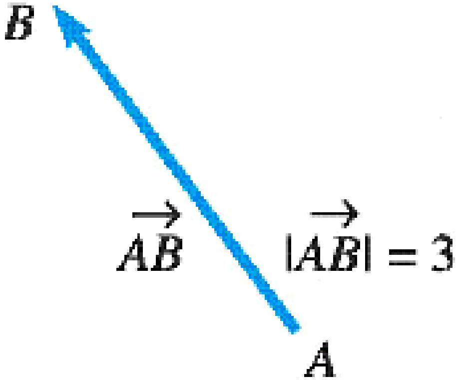
- Parallel vectors: Only absolute direction



- Equality condition: Both the direction and the magnitude

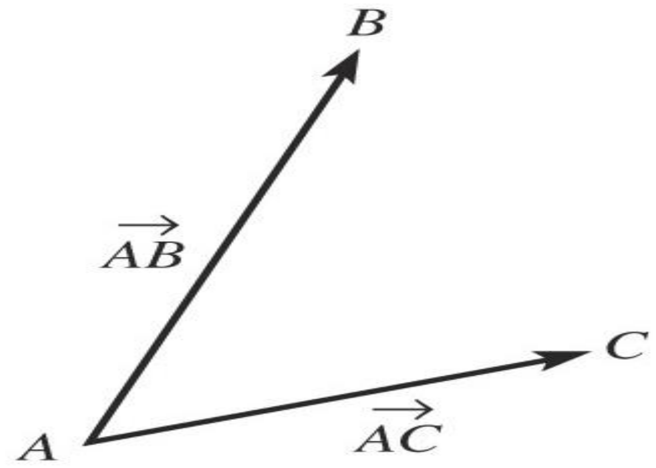






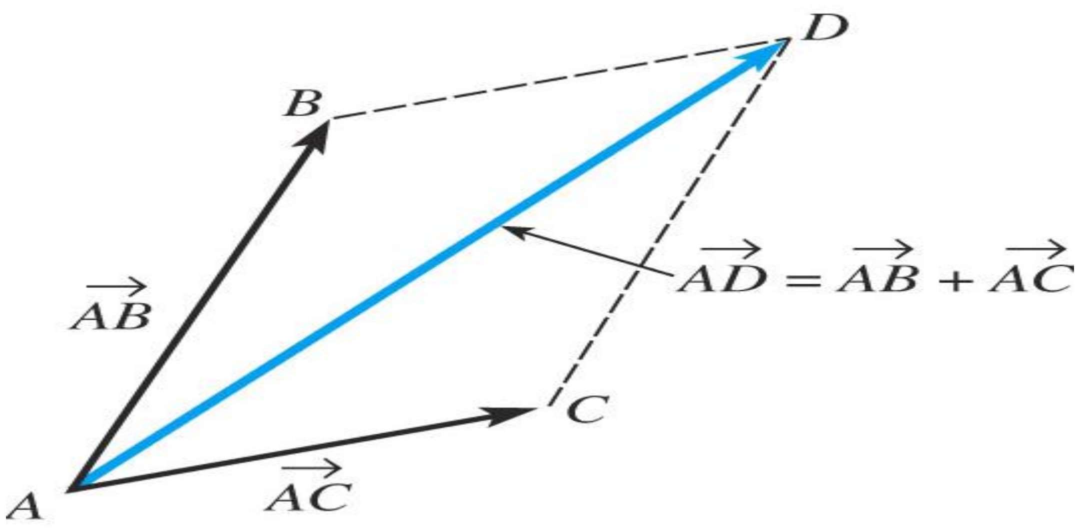
Vector summation

Consider two vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ with common initial point A



The sum of two vectors is the main diagonal of the parallelogram with the vectors as sides

$$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$$



(b)

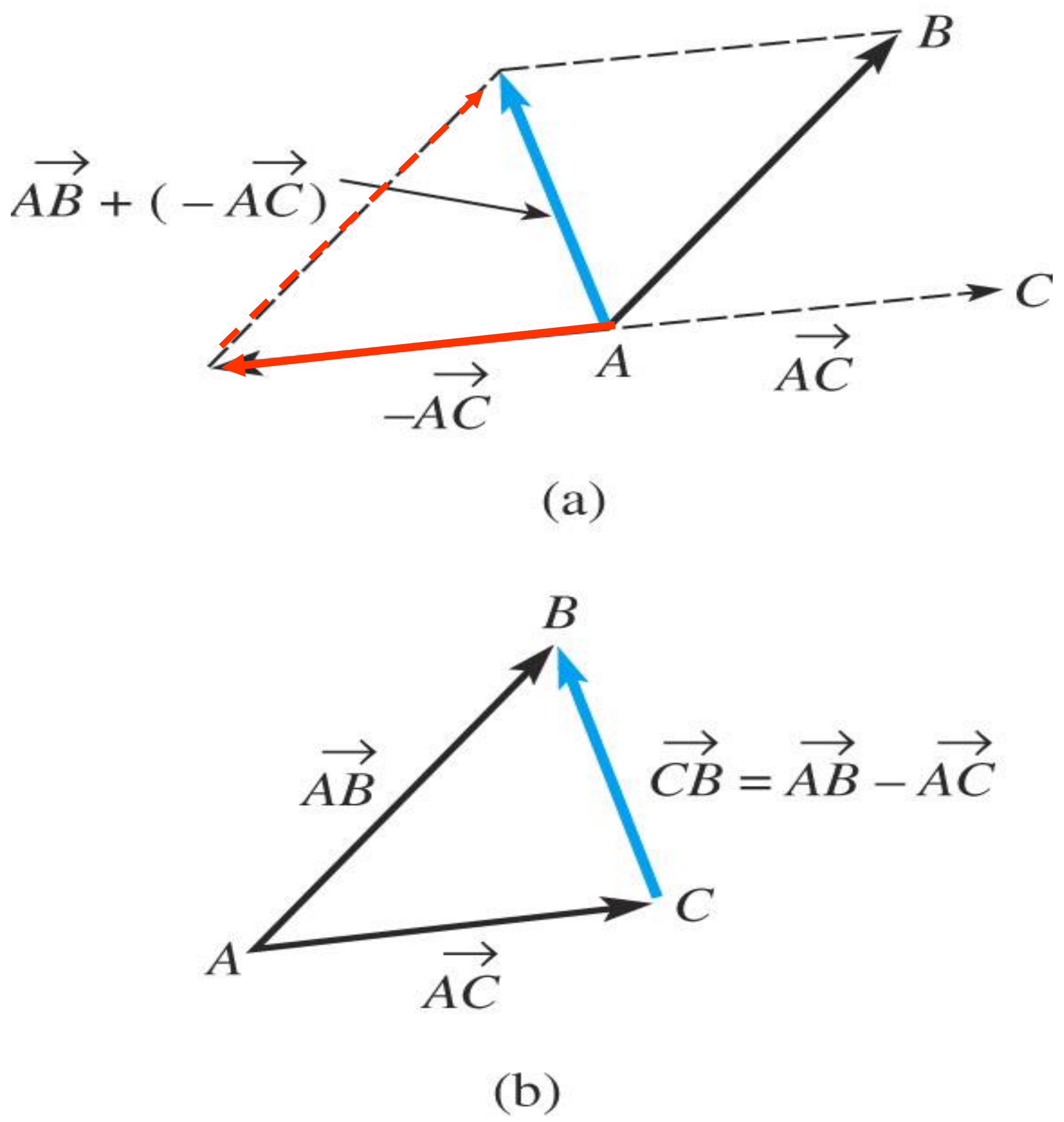
Vector subtraction

The difference of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is defined by

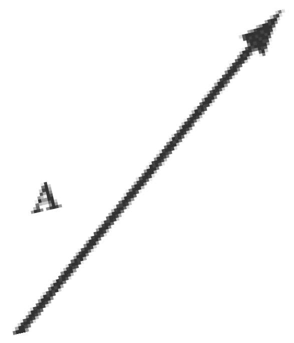
$$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{AB} + (-\overrightarrow{AC})$$

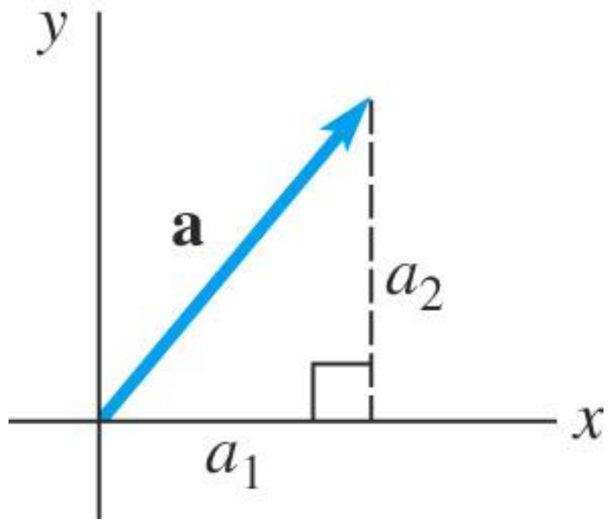
$\overrightarrow{AB} - \overrightarrow{AC}$ is the main diagonal of the parallelogram with sides $\overrightarrow{AB}$ and $-\overrightarrow{AC}$

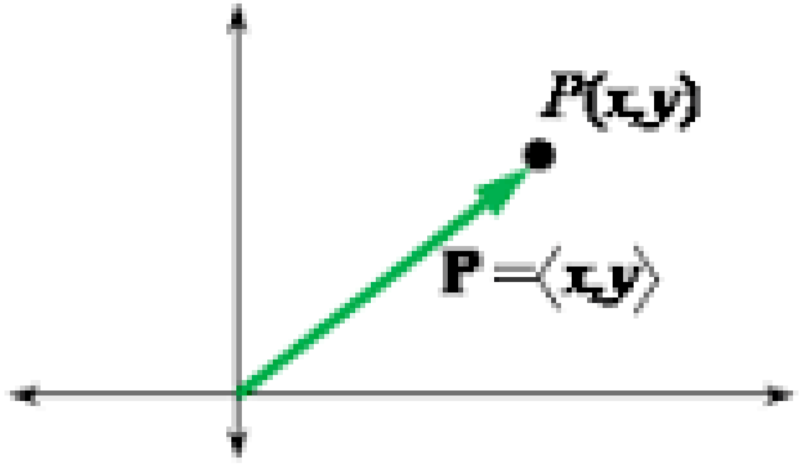
Or $\overrightarrow{CB} = \overrightarrow{AB} - \overrightarrow{AC}$ is a vector from the end of the second vector toward the end of the first vector



Conditioning vectors into a coordinate system







$$\vec{a} = a_1\hat{i} + a_2\hat{j} \quad = < a_1, a_2 > \quad = [a_1, a_2]$$

$$\text{Magnitude}: \ \|\vec{a}\| = \sqrt{a_1^2 + a_2^2}$$

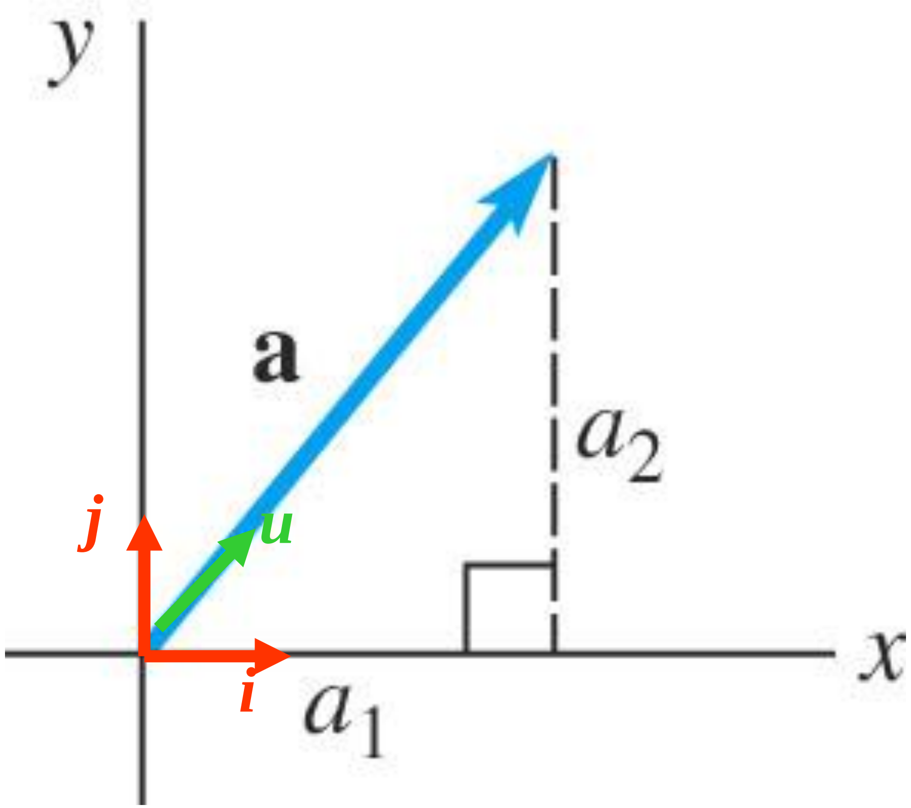
Independent components of a vector in a coordinate system

- **Magnitude**, length, or norm of a vector **a**: ||**a**||
- If $\mathbf{a} = < a_1, a_2 >$ then: $\| \mathbf{a} \| = \sqrt{a_1^2 + a_2^2}$
- **unit vector:**

  A vector that has magnitude 1

  A unit vector in the direction of **a** is:

$$\hat{\mathbf{u}} = \left(\vec{\mathbf{a}} \big/ \| \mathbf{a} \| \right) \text{ with } \|u\| = 1$$



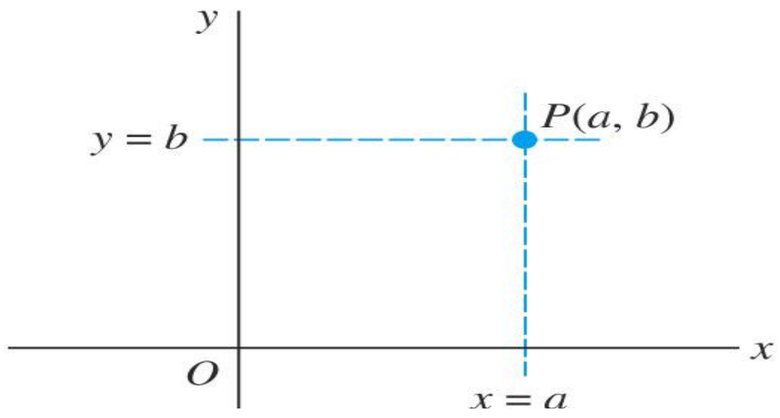
Vectors in a 3 dimensional space

- Rectangular or Cartesian Coordinate

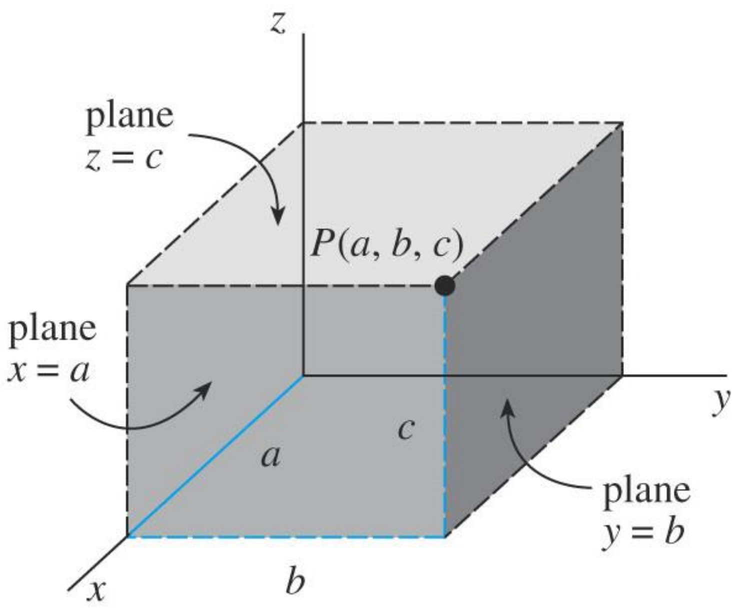
  2D-Space: Two orthogonal axes

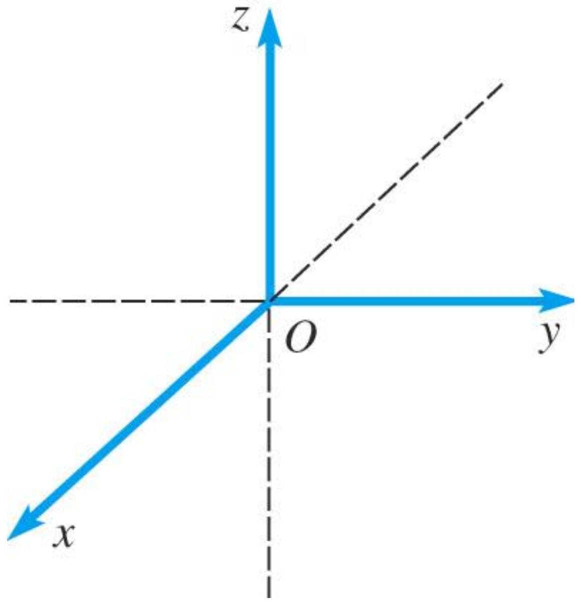
  3D-Space: Three mutually orthogonal axes

  The three axes follow the Right Hand Rule



right hand

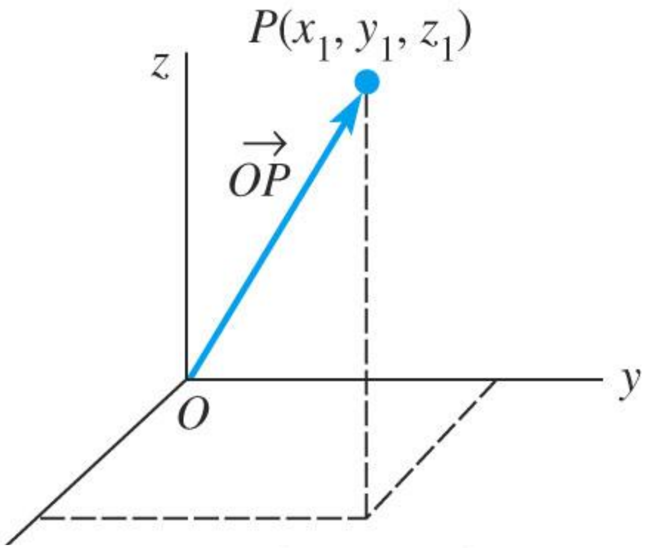




Locating a point in a 3D space

- Position vector

$$\vec{r}_1 = \vec{OP} = < x_1, y_1, z_1 >$$



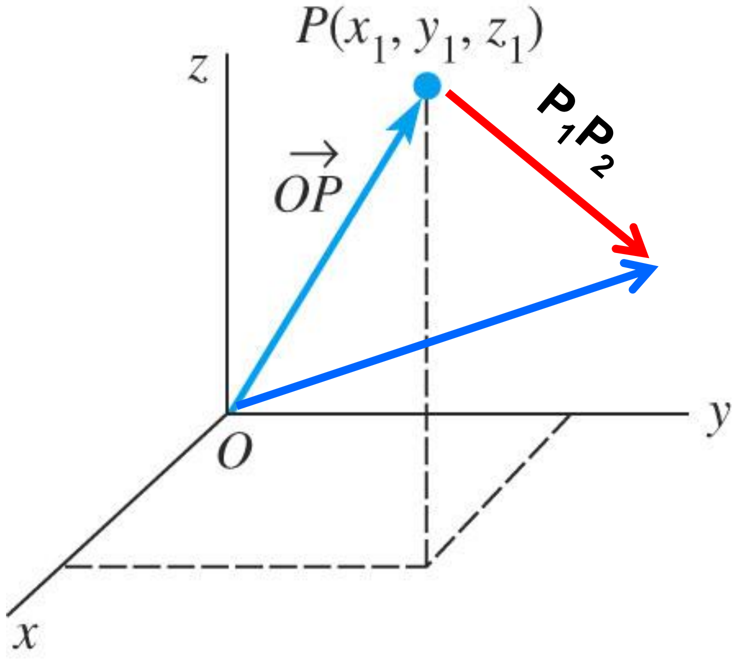
- Distance between points

$$P_1(x_1, y_1, z_1) \;\; \& \;\; P_2(x_2, y_2, z_2$$

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\vec{P_1P_2} = \vec{OP_2} - \vec{OP_1}$$

$$= < x_2 - x_1, y_2 - y_1, z_2 - z_1 >$$



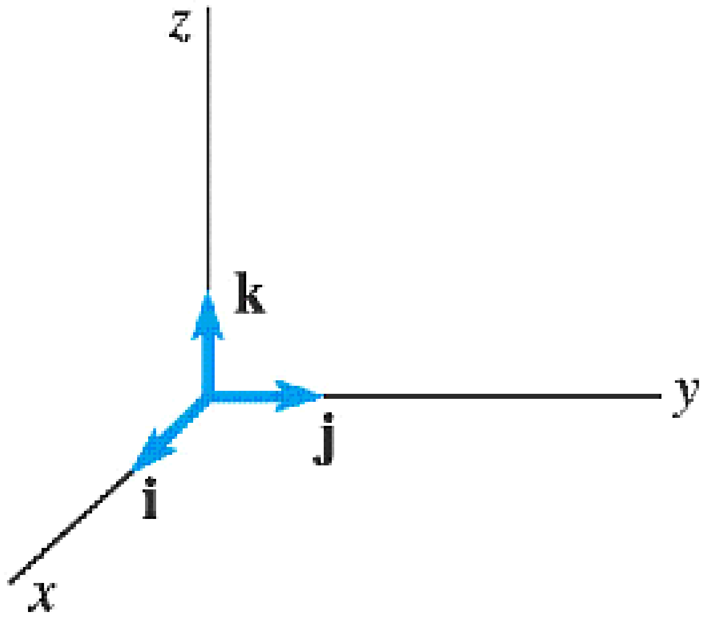
Unit Vectors in 3D space

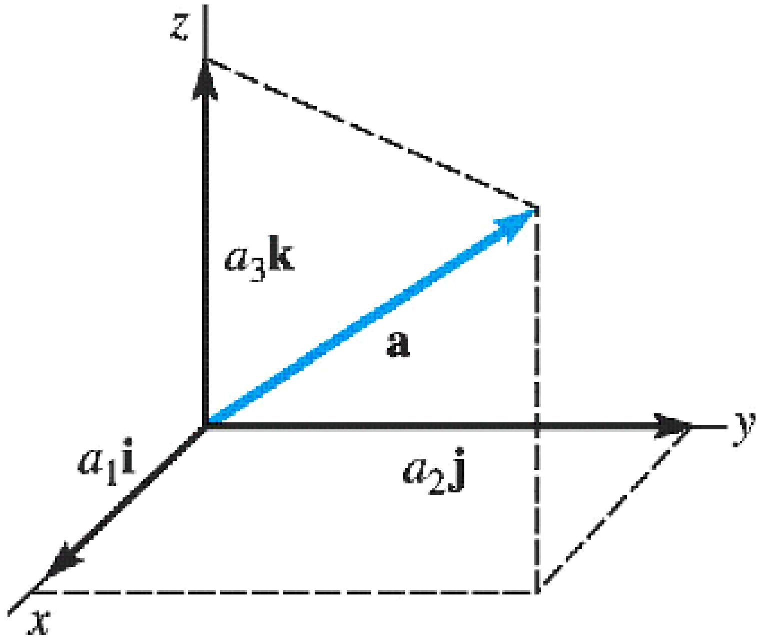
i = <1,0,0>,

j = <0,1,0>,

k = <0,0,1>

$$\mathbf{a} = < a_1, a_2, a_3 > = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$





Component Definitions in a 3D-Space

Let $\mathbf{a} = < a_1, a_2, a_3 >$ and $\mathbf{b} = < b_1, b_2, b_3 >$ be vectors in R^3

(i) **Addition**: $\mathbf{a} + \mathbf{b} = < a_1 + b_1, a_2 + b_2, a_3 + b_3 >$

(ii) **Scalar Multiplication:** $k\mathbf{a} = < ka_1, ka_2, ka_3 >$

(iii) **Equality:** $\mathbf{a} = \mathbf{b}$ if an only if $a_1 = b_1, a_2 = b_2, a_3 = b_3$

(iv) **Negative of a vector:** $-\mathbf{b} = < -b_1, -b_2, -b_3 >$

(v) **Subtraction:** $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b}) = < a_1 - b_1, a_2 - b_2, a_3 - b_3 >$

(vi) **Zero vector:** **0** = <0,0,0>

(vii) **Magnitude:** $\mathbf{a} = < a_1, a_2, a_3 > = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$

$$\| \mathbf{a} \| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Extension into a *n*-dimensional space

Vectors are not restricted to two dimensional or three dimensional space. In fact, vectors can exist in a general n-dimensional space. At this juncture, the general notation for a n-dimensional vector is,

$$\vec{v} = \{a_1, a_2, a_3, \ldots, a_n\}$$

and each of the a_i's are called **components** of the vector.

We will be mainly working in 2D and 3D spaces because of the physical restrictions of the engineering problems

Resources

- Check sections 7.1 and 7.2 in the textbook and go through assignments and other exercises
- This presentation is available through Moodle.
- An additional lecture note is also placed on Moodle.
- As an example of online resources check Oregon State's Lecture Notes: http://math.oregonstate.edu/home/programs/undergrad/CalculusQuestStudyGuides/vcalc/vector/vector.html