Mat2377 Midterm

D. McDonald

October 22, 2008

Instructions: This is an open book, one hour and twenty minute test. Write your answers in the spaces provided. Calculators are permitted. There are 42 points in all. There are points for explanation.

Name: Prof

Student number :

Exam on 40

1) We want to know the probability you receive two aces as hole cards while playing Texas Holdum. More prosaically, what is the probability of getting two aces if you draw two cards from a deck of 52.

a - 3 point) Write down an equiprobable sample space describing this experiment.

$S = \{ \{x_1, x_2\} : x_1 \neq x_2 \text{ chosen from 52 different cards} \}$

$\# S = \binom{52}{2} = \frac{52 \times 51}{2}$

b - 2 point) List the outcomes in the event of interest,

$A = \{ \{A\heartsuit, A\spadesuit\}, \{A\heartsuit, A\clubsuit\}, \{A\heartsuit, A\diamondsuit\}, \{A\spadesuit, A\clubsuit\}, \{A\spadesuit, A\diamondsuit\}, \{A\clubsuit, A\diamondsuit\} \}$

$\# A = \binom{4}{2} = \frac{4 \times 3}{2} = 6$

c - 1 point) Calculate the probability getting two aces as hole cards.

$P(A) = \frac{\# A}{\# S} = \frac{6}{52 \times 51 / 2} = \frac{3}{13 \times 51} = \frac{1}{13 \cdot 17}$

2) Let A be the event that an item is scratched and let B be the event that an item is discoloured. Suppose $P(A) = .4$, $P(B) = .3$ and $P(A \cap B) = .1$. Calculate

a - 1 point) $P(A') = 1 - P(A) = 1 - .4 = .6$

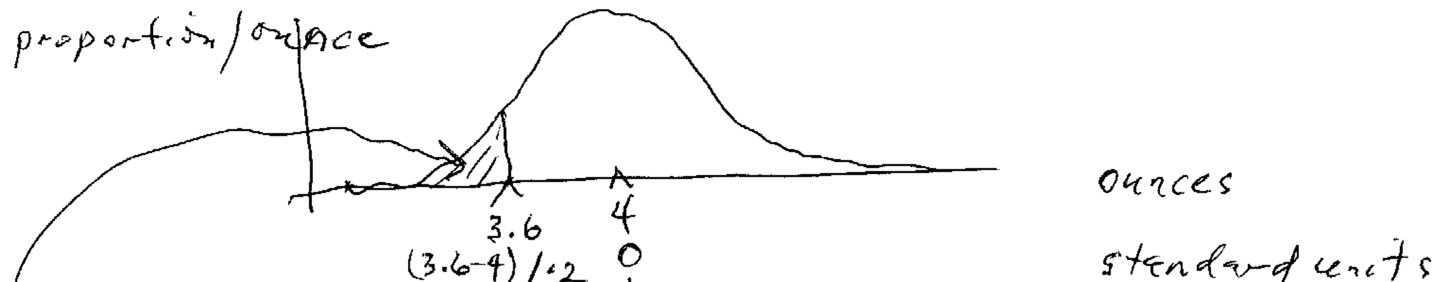
b - 2 points) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .4 + .3 - .1 = .6$

c - 2 points) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.1}{.3} = \frac{1}{3}$

d - 1 point) $P(A \cup B|B) = \frac{P((A \cup B) \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$

3) A machine makes quarter pound hamburger patties by volume but due to variations in the density of meat the true weight is normally distributed with a mean of 4 ounces and a standard deviation of .2 ounces.

a - 2 points) Sketch the density of the population of pattie weights. Give the units of the x and y axes.



b - 2 points) Use the sketch in part a) to find the probability a pattie chosen at random weighs less than 3.6 ounces?

-2 0 standard units

From Tables the proportion of the population < 3.6 is $\boxed{0.02275}$ Or else $P(X < 3.6) = P\left(\frac{X-\mu}{\sigma} < \frac{3.6-4}{.2}\right) = P(Z < -2)$

c - 3 points) It is possible to increase the mean weight of a pattie by adjusting the dial on the machine. The standard deviation doesn't change. What mean weight setting would assure that 95% of patties weight a least 4 ounces?

Let x be the weight set on the dial.
Hence the weight X of a pattie is $N(x, (.2)^2)$

$$.95 = P(X \geq 4) = P\left(\frac{X-x}{\sigma} \geq \frac{4-x}{.2}\right) = P\left(Z \geq \frac{4-x}{.2}\right)$$

-1.64 from normal Table gives area to left $= .05$

$$\therefore \frac{4-x}{.2} = -1.64$$

$$x = 4 + (.2)(1.64)$$
$$= 4.328$$

Let A be the event a flyer comes from A
" B " " " " " " " B

4) Suppose 1% of the printed flyers from printer A contain flaws while 2% of the printed flyers from printer B contain flaws. We buy 75 percent of our flyers from A and 25 percent from B.

a - 2 points) What is the probability a flyer we sent out will have a flaw?

Let F be the event a flyer has a flaw

$$P(F) = P(F \cap A) + P(F \cap B) = P(F|A)P(A) + P(F|B)P(B) = (.01)(.75) + (.02)(.25) = \boxed{.0125}$$

b - 2 points) If a customer receives a flyer from us that has a flaw, what is the probability the flyer was produced by printer A?

$$P(A|F) = \frac{P(A \cap F)}{P(F)} = \frac{P(F|A)P(A)}{P(F)} = \frac{(.01)(.75)}{.0125} = \frac{.0075}{.0125} = \frac{3}{5} = \boxed{.6}$$

5) Samples of 20 parts from a metal punching process are selected every hour at random from the production line. Typically 10% of the parts require rework. Let X denote the number of parts in the sample of 20 that require rework.

a - 3 points) What is the distribution of X and give the mean and variance if indeed we are in a typical situation. State your assumptions.

X is the number of successes (needs rework) in $n=20$ trials

$\boxed{X \sim \text{Binomial}(20, p=.1)}$ $\boxed{EX = 2}$ $\text{Var}\,X = 20(.1)(.9) = \boxed{1.8}$

b - 2 points) What is the probability X is more than 5 in a typical situation.

$$P(X>5) = 1 - P(X \le 5) = 1 - .9887 = \boxed{.0113}$$

Binomial Table pg 711

c - 2 points) Let T be the number of samples until we finally observe more than 5 parts that need reworking in a sample. In a typical situation what is the distribution of T and what is the mean of T.

T is the number of trials to obtain a success (sample >5 defectives)

Each trial has prob success .0113

$\therefore$ T is geometric $p = .0113$

$$ET = \frac{1}{p} = \boxed{\frac{1}{.0113}} \approx 88.5$$

5) The time to wire a specialized circuit board cannot be predicted because several steps may have to be repeated when a component breaks during soldering. The probability mass function of the wiring time is approximately equal to $f(x)$:

x hours	1	2	3	4
$f(x)$	.4	.3	.2	.1

Let X represent the time to wire a board

a - 2 points) Calculate the expected amount of time to produce this circuit board.

$$EX = \sum x f(x) = 1(.4) + 2(.3) + 3(.2) + 4(.1) = \boxed{2}$$

b - 3 points) Calculate the standard deviation of amount of time to produce this circuit board.

$$\text{Var } X = \sum (x - EX)^2 f(x) = (1-2)^2(.4) + (2-2)^2(.3) + (3-2)^2(.2) + (4-2)^2(.1)$$
$$= .4 + .2 + .4 = \boxed{1.0}$$

$\sigma_X = 1$

We have an order for 100 circuit boards and want to know how long it will take to produce this many. Assume the times to wire different boards are independent.

c - 2 points) Calculate the mean total time to build 100 boards and calculate the standard deviation of this time.

Let X_1 represent the time for board 1
Let X_{100} represent " " " " 100

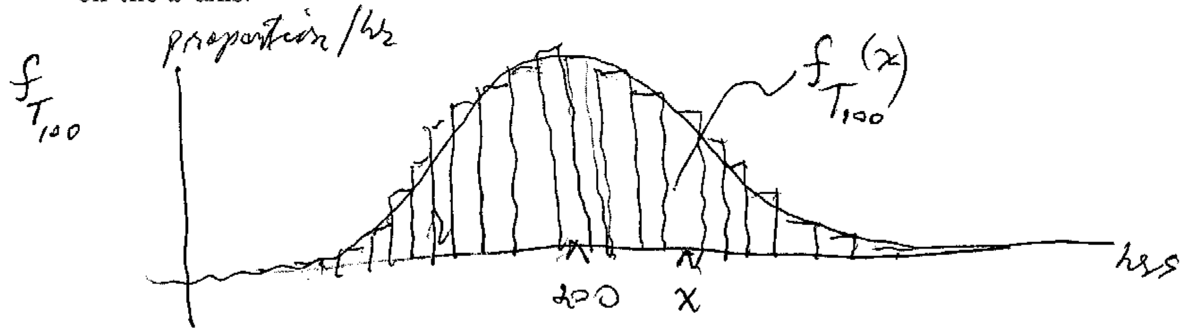
assumed iid with p.m.f f

Let $T_{100} = X_1 + \cdots + X_{100}$

$$ET_{100} = 100\, EX = \boxed{200 \text{ hrs}}$$

$$\text{Var } T_{100} = 100 \text{ Var } X = 100 \qquad \sigma_{T_{100}} = \sqrt{100} = \boxed{10 \text{ hrs}}$$

d - 3 points) Sketch the probability mass function of the total time to build 100 boards. Indicate the scale on x-axis and the y axis and identify one point on the x-axis.



e - 2 points) Estimate the probability the time to build 100 boards will exceed 120 hours.

By the CLT the density $f_{T_{100}}$ is approximately $N \sim$ Normal with same mean & variance as T_{100}

ie $N \sim \text{Normal}(200, 100)$

$$P(T_{100} > 120) \approx P(N > 120.5)$$

$$= P\left(\frac{N-200}{10} > \frac{120.5-200}{10}\right)$$

$$= P(Z > -7.95)$$

$\therefore$ Almost sure $\overset{\approx 1}{\text{it}}$ will take 120 hours or more