CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section(s)**
Mathematics	205/4	All

Examination	**Date**	**Pages**
Final	April 2008	2

Instructors	**Course Examiner**
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Special Instructions

- ▷ **Only approved calculators are allowed.**
- ▷ **Tables of integrals are not allowed.**

MARKS

[10] **1.** (a) Sketch the graph of the function:

$$f(x) = \begin{cases} 1 + \sqrt{1 - x^2}, & \text{if } -1 \le x < 1 \\ 2 - x, & \text{if } 1 \le x \le 5 \end{cases}$$

Evaluate the definite integral $\int_{-1}^{5} f(x)\,dx$ by interpreting it in terms of area (do not antidifferentiate).

(b) Find the derivative of the function:

$$F(x) = \int_4^{1+3x^2} \frac{1}{2 + e^t}\ dt$$

[15] **2.** Find the indefinite integrals:

(a) $\displaystyle\int \frac{\sec x \tan x}{\sqrt{\sec x}}\ dx$ (b) $\displaystyle\int \frac{\ln x}{x(1 + 4\ln^2 x)}\ dx$ (c) $\displaystyle\int \frac{x + 4}{x^2 + 5x - 6}\ dx$

[15] **3.** Calculate the definite integrals below. Give exact answers.

(a) $\displaystyle\int_1^e x^3 \ln x\ dx$ (b) $\displaystyle\int_0^{\frac{\sqrt{3}}{2}} \frac{4x^2}{(1 - x^2)^{\frac{3}{2}}}\,dx$ (c) $\displaystyle\int_{\ln(\frac{3}{4})}^{\ln(\frac{4}{3})} \frac{e^{2x}}{(1 + e^x)^{\frac{3}{2}}}\ dx$

[15] **4.** (a) Find the total area bounded by the curves $y = 7 - 2x^2$ and $y = x^2 + 4$.

(b) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$.

(c) Find the average value of the function $f(x) = 1 - (\cos \frac{\pi x}{4})^4$ on the interval $[0, 4]$.

[12] **5.** Evaluate the given improper integral or show that it diverges:

(a) $\displaystyle\int_1^\infty \frac{e^x}{x} dx$ (b) $\displaystyle\int_0^2 \frac{x+1}{\sqrt{4-x^2}} dx$

[9] **6.** Find the limit of the sequence or show that it does not exist:

(a) $\left\{\sqrt{\dfrac{2n}{n+1}}\right\}$ (b) $\left\{\dfrac{(-1)^{n+1}(n^2+1)}{(n^2+1000)}\right\}$ (c) $\left\{\left(\dfrac{3}{n}\right)^{\frac{1}{n}}\right\}$

[12] **7.** Test each of the following series to determine if it is convergent or divergent:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$

[12] **8.** (a) Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2}$.

(b) Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}3^n}$.

(c) Find the MacLaurin series for the function $f(x) = x^2 e^{-x}$.
What is the radius of convergence of this series?

[5] **Bonus Question**

Is the solution below correct?

$$\int_{-8}^{1} \frac{1}{x^{\frac{1}{3}}} dx = \int_{-8}^{1} x^{-\frac{1}{3}}\, dx = \left(\frac{3}{2}x^{\frac{2}{3}}\right)\Big|_{-8}^{1} = \frac{3}{2}\left[(1)^{\frac{2}{3}} - (-8)^{\frac{2}{3}}\right] = -\frac{9}{2}$$

If not, what is wrong?