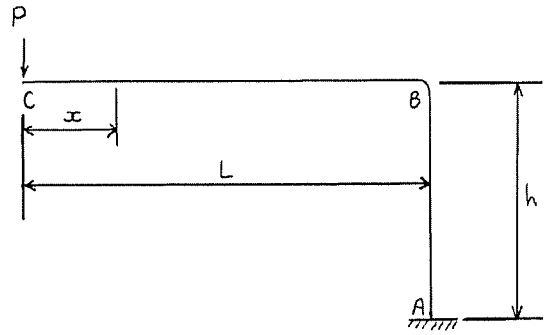
24 marks

MCG 3131 - Assignment 1

Problem 1

Consider the 90° bent cantilever beam shown in the figure below. The beam has constant properties E, I, and A. Assume elastic deflections and negligible deflection due to transverse shear. Determine the rotation (careful, not deflection) at point C due to load P using Castigliano's theorem.



Solution

1 We need to add an imaginary bending moment M to get the rotation at point C.

1 Bending in portion AB, where $M_{AB} = PL + M$.

1 Bending in portion BC, where $M_{BC} = Px + M$.

1 Compression in portion AB, of magnitude P.

1 $\theta = \int_0^h \frac{M_{AB}(\partial M_{AB}/\partial M)}{EI} dy + \int_0^L \frac{M_{BC}(\partial M_{BC}/\partial M)}{EI} dx + \int_0^h \frac{P(\partial P/\partial M)}{EA} dy,$

1 $\theta = \int_0^h \frac{(PL+M)(1)}{EI} dy + \int_0^L \frac{(Px+M)(1)}{EI} dx + \int_0^h \frac{P(0)}{EA} dy.$

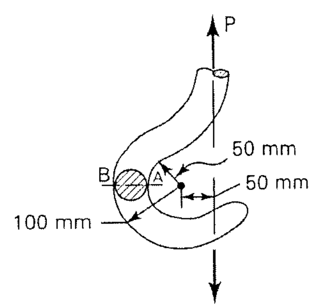
1 Setting $M = 0$,

1 $\theta = \frac{PLh}{EI} + \frac{PL^2}{2EI}.$

8 marks

Problem 2

For the hook of circular cross section shown in the figure below, (a) determine the maximum load P that may be supported without exceeding a stress of 161 MPa at point A; (b) determine the stress at point B in section $A-B$ for the load obtained in (a).



Solution

Realize that section $A-B$ does not only experience bending stresses (the hook wants to open up under the load), but also normal stresses (all the load is transferred through the section). At A, both stresses will be tensile (plus sign). At B, the bending stress will be compressive (minus sign), and the normal stress will be tensile. When you do this thinking, it becomes easier to pick the right equations to use. Note that the diameter of section $A-B$ is 50 mm (outer hook radius of 100 mm - inner hook radius of 50 mm).

(a) Location A :

0.25 $r_i = 50$ mm

0.25 $r_c = 75$ mm

0.25 $R = 25$ mm

1 $\sigma_A = \frac{P}{A} + \frac{Mc_i}{Aer_i}$ with

0.25 $A = \pi R^2 = 1963.5$ mm^2

1 $M = P(r_c + 50) = 125P$ (calculated at centroid)

1 $e = r_c - r_n = r_i + R - \frac{R^2}{2(r_c - \sqrt{r_c^2 - R^2})} = 2.1$ mm, from Page 10, Lecture 1, class notes

0.25 $r_n = r_c - e = 72.9$ mm

0.25 $c_i = R - e = 22.9$ mm

One wants $\sigma_A = \frac{P}{A} + \frac{Mc_i}{Aer_i} \leq 161$ Nmm^{-2} or MPa

1 $\frac{P}{1963.5} + \frac{125P \times 22.9}{1963.5 \times 2.1 \times 50} \leq 161 \Leftrightarrow P = 11185.5$ N, or 11.2 kN

(b) Location B :

1 $\sigma_B = \frac{P}{A} - \frac{Mc_o}{Aer_o}$

0.25 $r_o = 100$ mm

0.25 $c_o = R + e = 27.1$ mm

1 $\sigma_B = \frac{11185.5}{1963.5} - \frac{125 \times 11185.5 \times 27.1}{1963.5 \times 2.1 \times 100} = -86.2$ Nmm^{-2} or MPa

8 marks

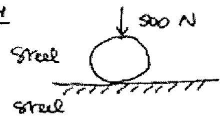
Problem 3

A 50-mm diameter ball is pressed against a flat surface by a force of 500 N. The material is steel for both ($E = 200$ GPa, $\nu = 0.3$). Determine (a) the radius of the contact area; (b) the maximum contact pressure. Same questions when the ball is pressed against an identical ball.

Solution

Problem 4

(1) a)



$E = 200$ GPa
$\nu = 0.3$
$d = 50$ mm

Sphere - plane contact is a sphere - sphere contact, with one sphere diameter being infinitely large, ie $d_2 = \infty$

$$b = \sqrt[3]{\frac{3F}{8} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

with $\nu_1 = \nu_2 = \nu$, $E_1 = E_2 = E$ $d_1 = 50$ mm, $d_2 = \infty$

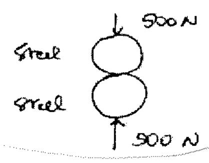
$$b = \sqrt[3]{\frac{6F}{8} \frac{(1-\nu^2)/E}{1/d}}$$

$$b = \sqrt[3]{\frac{6(500)}{8} \frac{(1-0.3^2)/200\times10^3}{(1/50)}}$$

$b = 0.44$ mm

b) $p_{max} = \sigma_{z\,max} = \dfrac{3F}{2\pi b^2} = \dfrac{3(500)}{2\pi(0.44)^2} = 1234$ MPa ie 1.2 GPa

(2) a)



$d_1 = d_2 = d = 50$ mm

$$b = \sqrt[3]{\frac{6(500)}{8} \frac{(1-0.3^2)/200\times10^3}{(1/50 + 1/50)}}$$

$b = 0.35$ mm

b) $p_{max} = \dfrac{3F}{2\pi b^2} = \dfrac{3(500)}{2\pi(0.35)^2} = 1.95$ GPa

8 marks