

This test paper has two parts and total of 30 marks.

Part I has 4 multiple choice questions. Part II has 3 long answer questions.

It cannot be taken from the examination room.

Only nonprogrammable calculators are allowed. Duration: 50 Minutes.

NAME :

STUDENT NO :

**PART I: Multiple Choice Questions. Circle the correct answer. No partial marks.**

[2] 1. Consider the following augmented matrix of a system of linear equations:

$$\left[ \begin{array}{cccc|c} 1 & 2 & 3 & 0 & 4 \\ 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]. \text{ The system has}$$

- (a) a unique solution
- (b) infinitely many solutions with one free variable
- (c) infinitely many solutions with two free variables
- (d) infinitely many solutions with three free variables
- (e) no solutions

[2] 2. The augmented matrix of a linear system that corresponds to the matrix equation

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = b \text{ has been transformed by row operations into the form below.}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{array} \right]. \text{ What is the value of } y \text{ in the solution of the system?}$$

- (a)  $y = 0$
- (b)  $y = 4$
- (c)  $y = 2$
- (d)  $y = 1$
- (e)  $y = 3$

[2] 3. Let  $A = \begin{bmatrix} 1 & 1 & 5 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \end{bmatrix}$ .

If reduced row echolon form of  $A$  is  $\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix}$ , what is the value of  $b_{13}$ ?

- (a) 0      (b) 1      (c) 4      (d) 5      (e) 6

[2] 4. Let  $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 2 & 6 \end{bmatrix}$  and  $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$  in  $R^3$ .

Give a description of the set of all  $b$  for which the equation  $Ax = b$  is consistent.

- (a) for all vectors  $b$  in  $R^3$ .  
(b) for all  $b_1, b_2, b_3$  satisfying  $b_3 - 3b_1 = 0$ .  
(c) for all  $b_1, b_2, b_3$  satisfying  $b_3 + 3b_2 = 0$ .  
(d) for all  $b_1, b_2, b_3$  satisfying  $b_3 - 2b_2 = 0$ .  
(e) for all  $b_1, b_2, b_3$  satisfying  $b_3 + 2b_1 = 0$ .

**PART II: Long answer questions. Show all your work.**

[12] 1. Consider the following system of linear equations.

$$\begin{aligned}x_1 - x_2 + 2x_4 &= 1 \\x_1 - x_2 - x_3 &= -1 \\x_1 + 2x_2 + 2x_3 + 3x_4 &= 2\end{aligned}$$

(a) Write the augmented matrix of the above system. Find the general solution of the system. Write the solution in parametric vector form.

(b) What is the general solution of the corresponding homogeneous system of linear equations?

[5] 2. Let  $u = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ ,  $v = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$  and  $b = \begin{bmatrix} 2 \\ -2 \\ k \end{bmatrix}$ .

Determine the value(s) of  $k$  such that the vector  $b$  is in the span of the vectors  $u$  and  $v$ ?

[5] 3. Let  $u = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ ,  $v = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 2 \end{bmatrix}$  and  $w = \begin{bmatrix} 1 \\ -1 \\ -2 \\ -4 \end{bmatrix}$ .

Determine whether the vectors  $u$ ,  $v$  and  $w$  are linearly independent. State your reason clearly.