

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	Date	Duration
Midterm	23 October 2016	1 h 30 min
Special	Only approved calculators are allowed	
Instructions:	Show all your work for full marks	

[Marks]

- [11] **1. (a)** Write the sigma notation formula for the right Riemann sum R_n of the function $f(x) = 4 - x^2$ on the interval $[0, 2]$ using n subintervals of equal length, and calculate the definite integral $\int_0^2 f(x) dx$ as the limit of R_n at $n \rightarrow \infty$.

$$(\text{Reminder: } \sum_{k=1}^n k = n(n+1)/2, \quad \sum_{k=1}^n k^2 = n(n+1)(2n+1)/6)$$

- (b)** Use the Fundamental Theorem of Calculus to calculate the derivative of $F(x) = \int_{e^{-x}}^x \ln(t^2 + 1) dt$

- [15] **2.** Calculate the following indefinite integrals

$$\text{(a)} \quad \int \frac{3x^3}{\sqrt{16-x^2}} dx \quad \text{(b)} \quad \int \frac{x^2+3}{x^2-2x+5} dx \quad \text{(c)} \quad \int \frac{2^x}{2^{2x}-4} dx$$

- [6] **3.** Find the antiderivative $F(x)$ of $f(x) = \frac{\sec^2(x)}{\sec^2(x)+3}$ such that $F(0) = 0$.

- [12] **4.** Evaluate the following definite integrals (give the exact values, **do not approximate**):

$$\text{(a)} \quad \int_0^4 \frac{x^2}{\sqrt{2x+1}} dx \quad \text{(b)} \quad \int_0^1 x^2 \cos(\pi x) dx$$

- [6] **5.** Find the area enclosed by the graphs of the functions $f(x) = x^2 - 3x - 5$ and $g(x) = 3 - x$.

- [3] **Bonus question.** Given that

$$\int_0^\pi [f(x) + f''(x)] \sin x dx = 2$$

and $f(\pi) = 1$, find $f(0)$