Assignment #3 Due date: 21/11/2016

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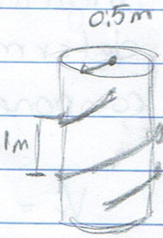
problem 12-183.

$r = 0.5m$, $\theta = 0.5t^3$ rad $z = (2 - 0.2t^2)m$

V and a at $\theta = 2\pi$ rad



lets start by finding all $z, \dot{z}, \ddot{z}, r, \dot{r}, \ddot{r}, \theta, \dot{\theta}, \ddot{\theta}$

$z = 2 - 0.2t^2$, $\dot{z} = -0.4t = V_z$, $\ddot{z} = -0.4 = a_z$

$r = 0.5$ $\dot{r} = 0$ $\ddot{r} = 0$

$\theta = 0.5t^3$ $\dot{\theta} = 1.5t^2$ $\ddot{\theta} = 3t$

$V_r = \dot{r} = 0$ $V_\theta = r\dot{\theta} = 0.5(1.5t^2)$ $V_\theta = 0.75t^2$

$a_r = (\ddot{r} - r\dot{\theta}^2) = (0 - 0.5(1.5t^2)^2) = a_r = -1.125t^4$

$a_\theta = (r\ddot{\theta} + 2\dot{r}\dot{\theta}) = (0.5(3t) + 2(0)(1.5t^2)) = a_\theta = 1.5t$

Since we have to include z in our calculation we define $\dot{z}$ as being V_z and $\ddot{z}$ to be a_z

$V = \sqrt{V_r^2 + V_\theta^2 + V_z^2}$ $a = \sqrt{a_r^2 + a_\theta^2 + a_z^2}$

at instant $\theta = 2\pi$ rad, $t =$

$2\pi = 0.5t^3$ $4\pi = t^3$ $\sqrt[3]{4\pi} = t$

$V = \sqrt{0 + (0.75t^2)^2 + (-0.4t)^2} = \sqrt{0.5625t^4 + 0.16t^2}$

$(-1.125t^4)(-1.125t^4)$ $= \sqrt{0.5625(\sqrt[3]{4\pi})^4 + 0.16(\sqrt[3]{4\pi})^2}$

t^8 $= \sqrt{16.434 + 0.865}$ $V = 4.159$ m/s

magnitude = no directions ?

$a = \sqrt{(-1.125t^4)^2 + (1.5t)^2 + (-0.4)^2}$

$a = \sqrt{1.265625t^8 + 2.25t^2 + 0.16} =$

$a = \sqrt{1.265625(\sqrt[3]{4\pi})^8 + 2.25(\sqrt[3]{4\pi})^2 + 0.16}$

$a = \sqrt{1080.268 + 12.162 + 0.16}$

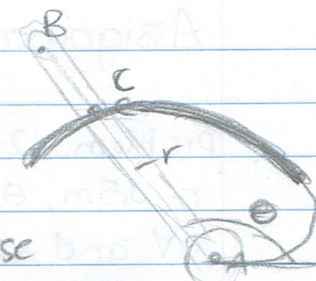
$a = 33.054$ m/s² ← magnitude = no directions ?

problem 12-186.

$r = a\theta$

angular velocity = constant at θ

determine the radial and transverse components of velocity and acceleration of the pin



$$V_r = \dot{r} \quad V_\theta = r\dot{\theta} \quad a_r = (\cancel{\ddot{r}} - r\dot{\theta}^2) \quad a_\theta = (\cancel{r\ddot{\theta}} + 2\dot{r}\dot{\theta})$$

$$R = a\theta \quad \dot{R} = a\dot{\theta} \quad \ddot{R} = a\ddot{\theta} = 0$$

$$\theta = C \quad \dot{\theta} = C \quad \ddot{\theta} = 0$$

Hence, the velocities are

$$V_r = \dot{r} = a\dot{\theta} \qquad V_r = a\dot{\theta}$$

$$V_\theta = r\dot{\theta} = a\theta\dot{\theta} \qquad V_\theta = a\theta\dot{\theta}$$

Also, the accelerations are

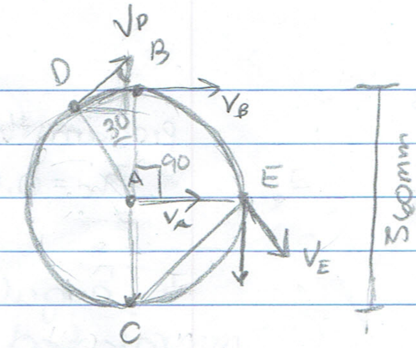
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - a\theta\dot{\theta}^2 \qquad a_r = a\theta\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\theta = 0 + 2(a\dot{\theta})(\dot{\theta}) \qquad a_\theta = 2a\dot{\theta}^2$$

Therefore:

$$V_r = a\dot{\theta} \qquad V_\theta = a\theta\dot{\theta}$$

$$a_r = -a\theta\dot{\theta}^2 \qquad a_\theta = 2a\dot{\theta}^2$$



problem 15.69

$V = 80\,Km/h$

$d = 560mm = 0.56m$

$r = 0.28\,m$

→ $V_A = 80\,Km/h = 22.22\,m/s$

→ $V_c = 0\,m/s.$

$\omega = \frac{V_A}{r} = \frac{22.22\,m/s}{0.28m} = 79.3571\,rad/s \downarrow$

$V_{B/A} = V_{D/A} = V_{E/A} = r\omega$

$= (0.28m)(79.3571\,rad/s) = 22.22\frac{m\,rad}{s}$

$V_B = V_A + V_{B/A} = (22.22m/s \rightarrow) + (22.22m\,rad/s)$

→ $V_B = 44.44\,m\,rad/s$

$V_D = V_A + V_{D/A} =$ ____

$V_{D/A} = 22.22\frac{m\,rad}{s}$ $\angle 30$

$22.22\sin 30 = 11.11$

$22.22\cos 30 = 19.24$

$V_{Dx} = 22.22 + 19.24 = 41.46\,m/s$

$V_{Dy} = 11.11\,m/s$

$\sqrt{41.46^2 + 11.11^2} = 42.92\,m/s$

$\theta = \tan^{-1}\frac{11.11}{42.92m/s}$ $\theta = 14.51°$

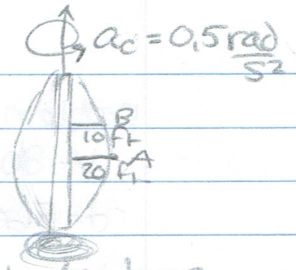
→ $V_D = 42.92\,m/s \angle 14.51°$

$V_E = V_A + V_{E/A} = 22.22m/s \rightarrow + 22.22m/s \downarrow$

$\sqrt{22.22^2 + 22.22^2} = 31.42\,m/s$

$\theta = \tan^{-1}\frac{22.22}{22.22}$ $\theta = 45°$

→ $V_E = 31.42\,m/s \searrow 45°$



problem 16-19.

$\alpha_c = 0.5 \text{ rad/s}^2$

The angular velocity of the blade after blade has rotated $2(2\pi) = 4\pi$ rad

$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

$\omega^2 = 0^2 + 2(0.5)(4\pi - 0)$

$\omega = 3.545 \text{ rad/s}$

$\rightarrow V_A = \omega r_A = 3.545(20) = 70.9 \text{ ft/s}$

$\rightarrow V_B = \omega r_B = 3.545(10) = 35.45 \text{ ft/s}$

Need tangential and normal components of acceleration.

$(a_T)_A = \alpha r_A = 0.5(20) = 10 \text{ ft/s}^2$

$(a_n)_A = \omega^2 r_A = (3.545)^2(20) = 251.34 \text{ ft/s}^2$

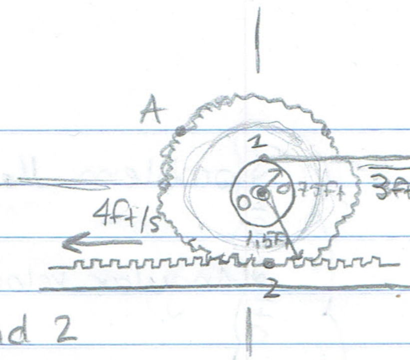
$(a_T)_B = \alpha r_B = 0.5(10) = 5 \text{ ft/s}^2$

$(a_n)_B = \omega^2 r_B = (3.545)^2(10) = 125.67 \text{ ft/s}^2$

magnitude of the acceleration of point A and B

$a_A = \sqrt{(a_T)_A^2 + (a_n)_B^2} = \sqrt{10^2 + 251.34^2} = 251.54 \text{ ft/s}^2$

$a_B = \sqrt{(a_T)_B^2 + (a_n)_B^2} = \sqrt{5^2 + 125.67^2} = 125.77 \text{ ft/s}^2$



problem 16-59

Angular velocity:

$r_{1/2}$ = distance between point 1 and 2

$r_{1/2} = 1.50\,ft + 0.75\,ft = 2.25\,ft$

$V_1 = V_2 + \omega \times r_{1/2}$

$3\bar{i} = -4\bar{i} + \omega\bar{k} \times 2.25ft\,\bar{j}$

$\omega\bar{k} \times 2.25\bar{j} = \begin{vmatrix} 0 & 0 & \omega \\ 0 & 2.25 & 0 \end{vmatrix} = -2.25\omega\bar{i}$

$3\bar{i} = -4\bar{i} - 2.25\omega\bar{i}$

$7 = -2.25\omega$

$\omega = -3.111$ rad/s

$V_0 = V_2 + \omega \times r_{0/2}$

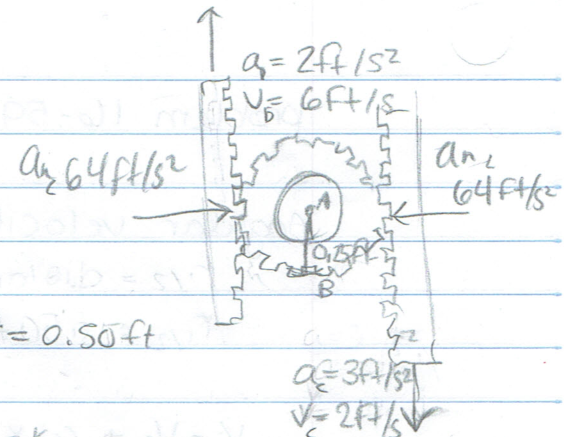
$V_0 = -4\bar{i} + -3.111\bar{k} \times 1.5\bar{j}$

$3.111\bar{k} \times 1.5\bar{j} = \begin{vmatrix} 0 & 0 & -3.111 \\ 0 & 1.5 & 0 \end{vmatrix} = 4.666\bar{i}$

$V_0 = -4\bar{i} + 4.666\bar{i} = 0.666\bar{i}$

$V_0 = 0.6666$ ft/s

Ans $\bar{a}_A = -0.5j \ ft/s^2$, $\bar{a}_B = (-2.5i + 63.5j) \ ft/s^2$



problem 16-127.

need it to cancel $\omega^2(R)$ out (the i component) (even if the video it says otherwize)

Angular velocity : $\omega = \frac{V_D}{r_{D/ic}} = \frac{V_c}{r_{c/ic}}$

$\frac{6}{r_{D/ic}} = \frac{2}{r_{c/ic}}$ where $r_{D/ic} + r_{c/ic} = 0.50ft$

$r_{D/ic} = 0.5 - r_{c/ic}$

$r_{c/ic} = \frac{2r_{D/ic}}{6}$ $\quad r_{c/ic} = \frac{2(0.5 - r_{c/ic})}{6}$

$r_{c/ic} = \frac{1 - 2r_{c/ic}}{6}$ $\quad r_{c/ic} = \frac{1}{6} - \frac{1}{3}r_{c/ic}$ $\quad r_{c/ic} + \frac{1}{3}r_{c/ic} = \frac{1}{6}$

$1.33 r_{c/ic} = \frac{1}{6}$ $\quad \boxed{r_{c/ic} = 0.125ft}$ $\boxed{r_{D/ic} = 0.375ft}$

thus $\omega = \frac{6}{0.375} = \frac{2}{0.125} = 16 rad/s$

$a_{cn} = 16^2(0.25) = 64i$ $\qquad a_B = a_c + \alpha R + \omega^2 R$

$a_A = a_c + \alpha R_{AC} - \omega^2 R_{AC}$

$a_D = a_c + \alpha R_{DC} - \omega^2 R_{DC}$

$64i + 2j = -64i - 3j + (-\alpha k) \times -0.5i - 16^2 \cdot 1(-0.5)i$

$64i + 2j = -64i - 3j - \alpha k \times -0.5i + 128i$

$-\alpha k \times -0.5i \quad \begin{vmatrix} i & j & k \\ 0 & 0 & -\alpha \\ -0.5 & 0 & 0 \end{vmatrix} = i(0-0) - j(0-0.5\alpha) + (0-0)k$

$64i + 2j = -64i + 3i + 0.5\alpha + 128i$ = i = cancel out

$2 = -3 + 0.5\alpha = 5 = 0.5\alpha \quad \alpha = 10 rad/s^2$

$a_A = a_c + \alpha \times R_{A/c} - \omega^2 R_{A/c}$

$a_A = -64i - 3j + (-10k) \times (-0.25i) - 16^2(-0.25i)$

$\begin{vmatrix} i & j & k \\ 0 & 0 & -10 \\ -0.25 & 0 & 0 \end{vmatrix} = -j(0-2.5) = 2.5j$

$a_A = -64i - 3j + 2.5j + 64i$ $\quad$ i = cancel out

$a_A = 0.5j \ ft/s^2$ $\quad a_A = 0.5 \ ft/s^2 \downarrow$

problem 16-127 (suite)

$r_{B/C} = -0.25i - 0.25j$

$a_B = a_C + \alpha + r_{B/C} - \omega^2 r_{B/C}$

$= -64i - 3j + \underbrace{(-10k) \times (-0.25i - 0.25j)} - 16^2(-0.25i - 0.25j)$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & -10 \\ -0.25 & -0.25 & 0 \end{vmatrix} = \hat{i}(0 - 2.5) - \hat{j}(0 - 2.5) + k(0)$$

$= -2.5\hat{i} + 2.5\hat{j}$

$= \cancel{-64i} - 3j - 2.5i + 2.5j \cancel{+ 64i} + 64j$

$= -3j - 2.5i + 2.5j + 64j$

$= -2.5\hat{i} + 63.5j \; ft/s^2$

$a_B = \sqrt{(-2.5^2) + (63.5)^2} = 63.5 \; ft/s^2$

$\theta = \tan^{-1} \frac{63.5}{2.5} = 87.7°$

$a_B = 63.5 \; ft/s^2 \; 87.7°$