NUMERICAL METHODS IN ENGINEERING
ENGR 391

QUIZ I

OCTOBER 22nd, 2015

CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE

Student's Name: _____

I.D.: _____

Notes:

Material allowed: single-sided hand-written letter size crib sheet
Duration: 4:15 pm to 5:30 pm
Total marks: 100
An ENCS-approved hand calculator is allowed

Instructions:

Read each question carefully
Answer all questions
Write your solutions on the question sheet
Write down all your intermediary steps

QUESTION 1: (Total of 30 points)

Given the following function:

$$f(x) = x^3 - 7x^2 + 8x - 0.35$$

a) Use a calculator to obtain the exact solution with a value of at least six digits for $x = 1.37$
b) Evaluate the same function using three-digit arithmetic with chopping.
c) Employ a third order Taylor series expansion to approximate $f(1.37)$ using a base point $x = 1$ and keeping 5 significant digits.
d) Compute the true percent relative errors for the results of parts (b) and (c)

a) $f(x) = (1.37)^3 - 7(1.37)^2 + 8(1.37) - 0.35$

$= 0.043053$ — 2pts

b) 3 digit chopping

$(1.37)^3 \longrightarrow 2.571353 \longrightarrow 2.57$

$-7(1.37)^2 \longrightarrow -7(1.87) \longrightarrow -13.0$

$8(1.37) \longrightarrow 10.96 \longrightarrow 10.9$ → 2pts on individual operations

$\Rightarrow f(x) = 2.57 - 13 + 10.9 - 0.35 = 0.12$ → 2pts numerical value

c) Third order Taylor Series

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \frac{f'''(x_i)}{3!}h^3$$ → 4pts 3rd order equation

where $x_{i+1} = 1.37$

$x_i = 1$

$h = x_{i+1} - x_i = 0.37$. 2pts stepsize

$f(x_i) = f(1) = (1)^3 - 7(1) + 8(1) - 0.35 = 1.65$

$f'(x) = 3x^2 - 14x + 8$ 2pts

$f'(x_i) = f'(1) = 3 - 14 + 8 = -3$

$f''(x) = 6x - 14$ 2pts

$f''(x_i) = f''(1) = -8$

$f'''(x) = 6$ 2pts

$f'''(x_i) = 6$

$$\Rightarrow f(1.37) \simeq 1.65 - 3(0.37) + \frac{(-8)}{2}(0.37)^2 + \frac{6}{6}(0.37)^3$$

$f(1.37) \simeq 1.65 - 1.11 + (-0.5476) + 0.0506 = 0.043$ 4pts

d) $\varepsilon_T = \left|\frac{0.12 - 0.043053}{0.043053}\right| \times 100 = 178.8\%$. 2pts numerical value

$\varepsilon_T = \left|\frac{0.043 - 0.043053}{0.043053}\right| \times 100 = 0.123\%$ 2pts

4pts on ε_T formula

QUESTION 2: (Total of 30 points)

Consider the following 2 × 2 linear system of the form $[A]\{x\} = \{b\}$:

$$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 3 \end{Bmatrix}$$

(a) Decompose the matrix A into LU = A using the LU method with Gaussian elimination (without pivoting).
(b) Solve the system for the solution vector ***x***.

Note: Keep 5 significant digits for the results.

a) $A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \begin{matrix} (1) \\ (2) \end{matrix}$

Set $m_{21} = \frac{4}{2} = 2$

$$\begin{matrix} (1) \\ (2) - m_{21}(1) \end{matrix} \begin{bmatrix} 2 & 3 \\ 0 & -7 \end{bmatrix} \begin{matrix} (1) \\ (2') \end{matrix}$$

$L = \begin{bmatrix} 1 & 0 \\ m_{21} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ 5pts $\quad U = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a'_{22} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & -7 \end{bmatrix}$ 5pts

$[L][U] = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -7 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} = [A]$

b) Set vector $\{z\} = \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix}$ such that

5pts $[L]\{z\} = \{b\}$ or $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 3 \end{Bmatrix}$

$\Rightarrow z_1 = 2$

$2(2) + z_2 = 3 \rightarrow z_2 = -1$ } 5pts

$[U]\{x\} = \{z\}$ or

$$\begin{bmatrix} 2 & 3 \\ 0 & -7 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$$ 5pts

$-7x_2 = -1 \rightarrow x_2 = 1/7$.

$2(x_1) + 3(1/7) = 2$.

$x_1 = \frac{1}{2}\left(2 - 3/7\right) = 0.7857$

$$\Rightarrow \{x\} = \begin{Bmatrix} 0.78571 \\ 0.14285 \end{Bmatrix}$$ 5pts

Double check.

$$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \begin{Bmatrix} 0.78571 \\ 0.14285 \end{Bmatrix} = \begin{Bmatrix} 1.99997 \\ 2.99999 \end{Bmatrix}.$$

QUESTION 3: (Total of 40 points)

The following equation $f(x) = 2x^3 - 11.7x^2 + 17.7x - 5$ has a root between $x_L = 2.5$ and $x_U = 4.0$

1) Determine the root using the bisection method; carry out three iterations. Compute the approximate relative error at ~~each~~ iterations (2) & (3)
2) Determine the root using the fixed-point iteration method. Prior to performing the computations, make certain that you develop a solution that converges to the root. Then, carry out three iterations and use initial guess $x_0 = 3$. Compute the approximate relative error at each iteration.
3) Discuss the advantages and limitations of both methods.

Note: Keep 5 significant digits for the results.

1) Bisection $f(x_L) = f(2.5) = -2.625$ $f(x_U) = f(4) = 6.6$

$i=1$ $\frac{x_L + x_U}{2} = \frac{2.5+4}{2} = 3.25 = x_{m_1}$ → 4pts.

$f(x_{m_1}) = -2.\cancel{4}$ → $x_L = x_{m_1}$

$i=2$ $\frac{x_L + x_U}{2} = \frac{3.25+4}{2} = 3.625 = x_{m_2}$ → 4pts

$f(x_{m_2}) = 0.686718$ → $x_U = x_{m_2}$ $\varepsilon_{a_2} = \left|\frac{x_{m_2} - x_{m_1}}{x_{m_2}}\right| \times 100 = 10.34\%$ (1)

$i=3$ $\frac{x_L + x_U}{2} = \frac{3.25+3.625}{2} = 3.4375 = x_{m_3}$ → 4 pts

$f(x_{m_3}) = -1.1704$ $\varepsilon_{a_3} = 5.45\%$ (1)

ε_a formula = 4pts

2) fixed point iteration

$$x = g(x) = \frac{5 - 2x^3 + 11.7x^2}{17.7}$$ → 4pts

$$g'(x) = \frac{1}{17.7}(-6x^2 + 23.4x)$$

convergence criterion:

$|g'(x_0)| = 0.915 < 1$ → 4pts

$i = 1$

$x_1 = g(x_0) = g(3) = 3.180791$ 2pts $\quad \varepsilon_{a_1} = 5.68\%$ (1)

$i = 2$

$x_2 = g(x_1) = g(3.180791) = 3.333959$ 2pts $\quad \varepsilon_{a_2} = 4.59\%$ (1)

$i = 3$

$x_3 = g(x_2) = g(3.333959) = 3.442543$ 2pts $\quad \varepsilon_{a_3} = 3.15\%$ (1)

3) Bisection is bracketing technique

advantages: guaranteed to convergence

drawback: slow.

fixed point is Open method.

advantages: fast

drawback: not guaranteed to converge

fixed point has restrictive convergence criterion

↳ 5pts discussion