

## Assignment one – Part 2 – Solution

**Total points : 20**

*Please don't forget to complete your signed statement of Academic Integrity within the body of your solution. Submit a PDF of your type-written (i.e., not handwritten) solution via blackboard learn by the due-date. Make sure to show all steps of how you arrived at your answers wherever required (only providing the final answer is not acceptable). Part 1 of the assignment should be done on myStatLab.*

1. **Sample Spaces.** For each of the following, list the sample space and tell whether you think the events are equally likely. **(4 Points)**

- a) Roll two dice; record the sum of numbers appear.
- b) A family has three children; record each child's gender in order of birth.
- c) Toss four coins; record the number of tails.
- d) Toss a coin 10 times; record the longest run of heads.

**a) ( 1 Point)**

$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ . All outcomes are not equally likely. For example, there are four equally likely outcomes that result in a sum of 5 ( $1 + 4$ ,  $4 + 1$ ,  $2 + 3$ , and  $3 + 2$ ), and only one outcome that results in a sum of 2 ( $1 + 1$ ).

**b) (1 Point)**

$S = \{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$ . All outcomes are equally likely .

**c) (1 Point)**

$S = \{0, 1, 2, 3, 4\}$ . All outcomes are not equally likely. For example, there are four equally likely outcomes that produce one tail (HHHT, HHTH, HTHH, and THHH), but only one outcome that produces four tails (TTTT).

**d) (1 Point)**

$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$  All outcomes are not equally likely. A string of three heads is much more likely to occur than a string of 10 heads in a row.

2. **Census.** Canada conducts a census every five years. In 2006, Canada tried something that no other country had ever tried – Census questionnaires that could be completed online. 80% of households received a short census form with only eight questions. The remainder received a long census form with 53 questions. Fifteen percent of households received the short form and completed their census online; 4% of households received the long form and completed their census online. **(4 Points)**

- a) What percent of households filed online ?
- b) What percent of households received the short form **and** filed online ?
- c) What percent of households received the short form **or** filed online ?
- d) What percent of households received the long form **and** did not file online ?

Let S = randomly selected person receives a short census, randomly selected person receives a long census, and O = randomly selected person completes online. We know that  $P(S) = 0.8$ ,  $P(L) = 0.2$ ,  $P(S \text{ and } O) = 0.15$ ,  $P(L \text{ and } O) = 0.04$  (4%).

**a) ( 1 Point)**

$$P(O) = P(S \text{ and } O) + P(L \text{ and } O) = 0.15 + 0.04 = 0.19 (19\%)$$

**b) (1 Point)**

$$P(S \text{ and } O) = 0.15 (15\%)$$

**c) (1 Point)**

$$P(S \text{ or } O) = P(S) + P(O) - P(S \text{ and } O) = 0.8 + 0.19 - 0.15 = 0.84 (84\%)$$

**d) (1 Point)**

$$P(S^c \text{ and } O^c) = P(S \text{ or } O)^c = 1 - P(S \text{ or } O) = 1 - 0.84 = 0.16 (16\%)$$

3. **Blood type.** Canadian Blood Services says that about 46% of the Canadian population have Type O blood, 42% have Type A, 9% have Type B, and the rest have Type AB. Among four potential donors, what is the probability that: **(4 Points)**

- a) All are Type O ?
- b) No one is Type AB ?
- c) They are not all Type A ?
- d) At least one person is Type B?

**a) ( 1 Point)**

$$0.46 \times 0.46 \times 0.46 \times 0.46 = 0.04477456$$

**b) (1 Point)**

$$P(AB) = 1 - 0.46 - 0.42 - 0.09 = 0.03 \text{ and } P(\text{no one is Type AB}) = 0.97 \times 0.97 \times 0.97 \times 0.97 = 0.8853$$

**c) (1 Point)**

$$P(\text{all type A}) = 0.42 \times 0.42 \times 0.42 \times 0.42 = 0.03111696 \text{ and so } P(\text{not all type A}) = 1 - P(\text{all type A}) = 1 - 0.03111696 = 0.96888304.$$

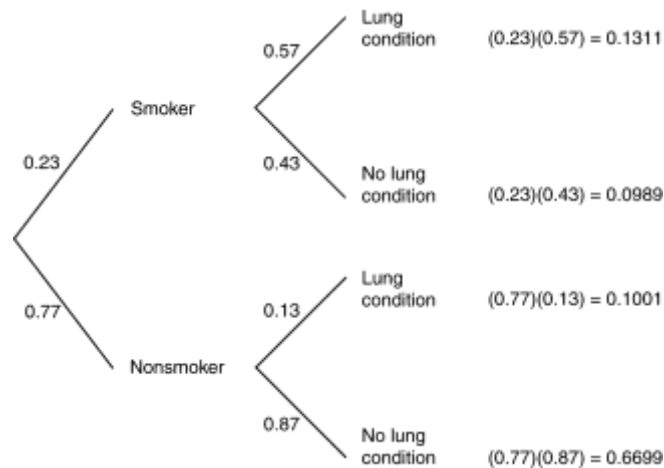
**d) (1 Point)**

$$P(\text{none has type B}) = (1 - 0.09) \times (1 - 0.09) \times (1 - 0.09) \times (1 - 0.09) = 0.68574961 \text{ and so } P(\text{at least one Type B}) = 1 - P(\text{none has type B}) = 1 - 0.68574961 = 0.31425039.$$

4. **Lungs and smoke.** Suppose that 23% of adults smoke cigarettes. It is known that 57% of smokers and 13% of non-smokers develop a certain lung condition by age 60. Based on the statistics, answer the following questions. **(4 Points)**

- Draw a “tree diagram” and organize the information into the tree.
- Explain how these statistics indicate that lung condition and smoking are not independent.
- What is the probability that a randomly selected 60-year-old has his lung condition?
- What is the probability that someone with the lung condition was a smoker ?

**a) ( 1 Point)**



**b) (1 Point)**

The lung condition and smoking are not independent, since rates of the lung condition are different for smokers and nonsmokers. 57% of smokers have the lung condition by age 60, while only 13% of nonsmokers have the condition by age 60.

**c) (1 Point)**

$$P(\text{lung condition}) = P(\text{smoker and lung condition}) + P(\text{nonsmoker and lung condition}) = (0.23)(0.57) + (0.77)(0.13) \approx 0.231$$

The probability that a randomly selected 60-year-old has the lung condition is about 0.231.

**d) (1 Point)**

$$P(\text{smoker lung cond.}) = P(\text{smoker and lung cond.}) / P(\text{lung cond.}) = (0.23)(0.57) / ((0.23)(0.57) + (0.77)(0.13)) \approx 0.567$$

The probability that someone who has the lung condition by age 60 is a smoker is approximately 56.7%.

5. **Men's health.** The table shows the approximate probabilities that an adult Canadian male has high blood pressure and/or high cholesterol. Based on this data answer the following questions. (4 Points)

		Blood Pressure	
		High	OK
Cholesterol	High	0.11	0.21
	OK	0.16	0.52

- What is the probability that a man has both conditions ?
- What is the probability that he has high blood pressure ?
- What is the probability that a man with high blood pressure has high cholesterol?
- What is the probability that a man has high blood pressure if it is known that he has high cholesterol ?
- Are high blood pressure and high cholesterol independent ? Explain.

**a) (0.5 Point)**

$$P(\text{both conditions}) = 0.11$$

**b) (0.5 Point)**

$$P(\text{high blood pressure}) = 0.11 + 0.16 = 0.27$$

**c) (1 Point)**

$$P(\text{high chol.} | \text{high BP}) = P(\text{high chol. and high BP}) / P(\text{high BP}) = 0.11 / 0.27 = 0.407$$

Consider only the High Blood Pressure column. Within this column, the probability of having high cholesterol is 0.11 out of a total of 0.27.

**d) (1 Point)**

$$P(\text{high BP} | \text{high chol.}) = P(\text{high BP and high chol.}) / P(\text{high chol.}) = 0.11 / 0.32 = 0.344$$

Consider only the High Cholesterol row. Within this row, the probability of having high blood pressure is 0.11, out of a total of 0.32.

**e) (1 Point)**

High blood pressure and high cholesterol are not independent events. 28.8% of men with OK blood pressure have high cholesterol, while 40.7% of men with high blood pressure have high cholesterol (or compare 40.7% with the unconditional 32% of men who have high cholesterol). If having high blood pressure and high cholesterol were independent, these percentages would be the same.