# ADM2302A Assignment 2: Linear Programming Formulation, Solution, and Sensitivity Analysis

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Date: October 29, 2014

***Personal Ethics Agreement Concerning University Assignments***

By signing this Statement, I am attesting to the fact that I have reviewed not only my own work, but the work of my colleagues, in its entirety.

I attest to the fact that my own work in this project meets all of the rules of quotation and referencing in use at the Telfer School of Management at the University of Ottawa, as well as adheres to the fraud policies as outlined in the Academic Regulations in the University's Undergraduate Studies Calendar. I further attest that I have knowledge of and have respected the "Beware of Plagiarism" brochure (https://www.uottawa.ca/about/sites/www.uottawa.ca.about/files/plagiarism.pdf).

To the best of my knowledge, I also believe that each of my group colleagues has also met the rules of quotation and referencing aforementioned in this Statement.

I understand that if my group assignment is submitted without a signed copy of this Personal Ethics Statement from each group member, it will be interpreted by the Telfer School of Management that the missing student(s) signature is confirmation of non-participation of the aforementioned student(s) in the required work.

Signature — Date: Oct, 29th, 2014

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**1. Develop and solve a LP model to help determine how the initial angel investment of $20,000 should be spent.**

**Objective Function:**

- Maximize Revenue: $\$6.10\ X_{11}+\$5.28\ X_{21}+C_1+\$6.43\ X_{12}+\$5.15\ X_{22} = Z$

**Decision Variables:**

- $X_{11}$ = Number of bottles of Chinon sold in year 1 (2014)
- $X_{12}$ = Number of bottles of Chinon sold in year 2 (2015)
- $X_{21}$ = Number of bottles of Chardonnay sold in year 1 (2014)
- $X_{22}$ = Number of bottles of Chardonnay sold in year 2 (2015)
- $C_1$ = Investment amount remaining after year 1(2014-2015)

**Constraints:**

- $X_{11} \geq 4{,}000$ (minimum bottles of Chinon sold in year 1)
- $X_{12} \geq 4{,}000$ (minimum bottles of Chinon sold in year 2)
- $X_{21} \geq 5{,}000$ (minimum bottles of Chardonnay sold in year 1)
- $X_{22} \geq 5{,}000$ (minimum bottles of Chardonnay sold in year 2)
- $X_{11} = X_{21}$ (equal sales requirement in year 1)
- $X_{12} = X_{22}$ (equal sales requirement in year 2)
- $\$0.90X_{11}+\$0.73\ X_{21} \leq \$20{,}000$ (investment requirement in year 1)
- $\$6.10\ X_{11}+\$5.27\ X_{21} \leq \$0.82\ X_{12}+\$0.85\ X_{22}$ (re-investment requirement in year 2)
- $X_{11}+X_{21}+C_1+ X_{12}+X_{22} \geq 0$ (non-negativity constraint)

**2. Produce a report to your partners outlining TSM Premium Wines' strategy for the first two years.**

Date: October 29th, 2014

To: John Smith, Bank of Nova Scotia Loan Officer

From: TSM Premium Wines

Subject: Business venture loan request

Our organization has recently started operations in the Ottawa Valley by cultivating a vineyard and creating a small bottling distillery. With the help of an angel investment of $20,000, we have been able to manufacture two different types of wine: Chinon and Chardonnay. Each bottle of Chinon wine we produce requires $0.70 worth of Cabernet Franc grapes in 2014 and we are expecting this cost to lower to $0.65 in 2015. Each bottle of Chardonnay wine we produce currently requires $0.60 worth of Chardonnay grapes, although we are expecting this price to rise to $0.75 by 2015. The issue we now face is the lack of funding to be able to expand our operations beyond its' current horizon. It is for that reason that we are approaching you Mr. Smith. We are hoping to receive a loan from the Bank of Nova Scotia to aid our organization in reaching new heights.

After some careful analysis, we have determined that the wholesale selling price of our wines to the Liquor Control Board of Ontario (LCBO). In 2014, we expect to sell one bottle of Chinon wine for $7.00 and a bottle of Chardonnay wine for $6.00. In 2015, we expect the price of Chinon to rise to $7.25, while the price of Chardonnay to remain the same at $6.00 per bottle. Our Chinon wine is projected to sell at least 4,000 bottles per year, while our Chardonnay wine is projected to sell at least 5,000 bottles per year. That being said, we are hoping to be able to sell an equal amount of each type of wine in each of the next two years. Another key factor in our sales is the amount we invest in marketing. With the help of the Canadian Vintners Association, we have determined that for each dollar we spend marketing Chinon in 2014, there will be a consumer demand of five bottles. In 2015, this number changes to six bottles, while Chardonnay receives a consumer demand of eight bottles in 2014 and ten bottles in 2015. Finally, our profits made in 2014 will be fully re-invested in production in the beginning of 2015, allowing output to dramatically increase.

Through the use of smart spending, we can achieve maximal revenue. We have modeled this scenario with the help of a linear program. Our decision variables (factors we can change) are as follows:

$X_{11}$ = Number of bottles of Chinon sold in year 1 (2014)
$X_{12}$ = Number of bottles of Chinon sold in year 2 (2015)
$X_{21}$ = Number of bottles of Chardonnay sold in year 1 (2014)
$X_{22}$ = Number of bottles of Chardonnay sold in year 2 (2015)
$C_1$ = Investment amount remaining after year 1(2014-2015)

As we would like to maximize our revenue over the next two years, we have developed an equation to represent this maximization. It is as follows:

$\$6.10\ X_{11}+\$5.28\ X_{21}+C_1+\$6.43\ X_{12}+\$5.15\ X_{22} = Z$ (Maximized revenue)

This scenario would not be realistic if it did not involve any restrictions and constraints on obtaining maximum revenue. Our restrictions are:

| | | | |
|---|---|---|---|
| $X_{11} \geq 4{,}000$ | $X_{21} \geq 5{,}000$ | $X_{11} = X_{21}$ | $\$0.90X_{11} + \$0.73\ X_{21} \leq \$20{,}000$ |
| $X_{12} \geq 4{,}000$ | $X_{22} \geq 5{,}000$ | $X_{12} = X_{22}$ | $\$6.10\ X_{11} + \$5.27\ X_{21} \leq \$0.82\ X_{12} + \$0.85\ X_{22}$ |

Once everything is placed together, we can create and solve a linear program with the help of Microsoft Excel. This allows for a more clear explanation of all costs and profits involved with producing our wines.

| | Year 1 | | | Year 2 | | | | |
|---|---|---|---|---|---|---|---|---|
| DECISION VARIABLES | Chinon (X11) | Chardonnay (X21) | Amount Remaining (C1) | Chinon (X12) | Chardonnay (X22) | | | |
| Bottles Sold | 12308 | 12308 | 0 | 83963 | 83963 | | | |
| | | | | | | | | |
| OBJECTIVE | | | | | | | | |
| Price ($)/Bottle | $7.00 | $6.00 | $0.00 | $7.25 | $6.00 | | | |
| Marketing costs ($)/Bottle | $0.20 | $0.13 | $0.00 | $0.17 | $0.10 | | | |
| Production Costs ($)/Bottle | $0.70 | $0.60 | $0.00 | $0.65 | $0.75 | | | |
| MAX REVENUE($)/Bottle | $6.10 | $5.27 | $0.00 | $6.43 | $5.15 | $1,112,230.89 | | |
| | | | | | | | | |
| CONSTRAINTS | | | | | | LHS | | RHS |
| Bottles of Chinon sold year 1 (X11) | 1 | 0 | 0 | 0 | 0 | 12308 | >= | 4000 |
| Bottles of Chardonnay sold year 1 (X21) | 0 | 1 | 0 | 0 | 0 | 12308 | >= | 5000 |
| Bottles of Chinon sold year 2 (X12) | 0 | 0 | 0 | 1 | 0 | 83963 | >= | 4000 |
| Bottles of Chardonnay sold year 2 (X22) | 0 | 0 | 0 | 0 | 1 | 83963 | >= | 5000 |
| Equal Sales Requirement year 1 | 12308 | 12308 | 0 | 0 | 0 | 12308 | = | 12308 |
| Equal Sales Requirement year 2 | 0 | 0 | 0 | 83963 | 83963 | 83963 | = | 83963 |
| Investment restriction year 1 | $11,076.92 | $8,923.08 | $0.00 | $0.00 | $0.00 | $20,000.00 | <= | $20,000.00 |
| Re-investment restriction year 2 | $75,076.92 | $64,861.54 | $0.00 | $68,569.85 | $71,368.62 | $139,938.46 | <= | $139,938.46 |

Upon solving the linear program, we can see that with an investment of $20,000 we can achieve an optimal profit of $1,112,230.89 over the course of the next two years. During 2014, $11,076.92 of the funds should be put towards producing Chinon wine and $8,923.08 should be put towards producing Chardonnay wine. The resulting outcome will be 12,308 bottles of each type of wine produced in 2014 and their respective revenue being $75,076.92 and $64,861.54. In 2015, all profits will be reinvested into production, along with anything remaining of the $20,000 angel investment (although there was nothing left). $68,569.85 will be used for the production of Chinon wines and $71,368.62 will be used in the production of Chardonnay wines, creating a total of 83,963 bottles of each type of wine.

**3. After consulting with a wine buyer from the LCBO, you were told that because of forecasted low-priced imports from France and Chile, your price estimate for Chinon in the second year is too high. Assess the price elasticity for Chinon. What would happen if the wholesale price per bottle has to be cut in half? Justify your answer with the results of the sensitivity analysis.**

Currently, the price elasticity for Chinon is extremely elastic. In the first year, we sold 12308 bottles, for $7.00 each. Then, in the second year, we sold 84036 bottles, for $7.25 each. This

makes it evident that since there is a small change in price, and a large change in quantity sold, Chinon has a high elasticity.

Price Elasticity of Demand = % Change in Quantity Demanded / % Change in Price
= [(84037-12308)/12308]/[(7.25-7.00)/7.00]
=163.179

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$3 | Bottles Sold Chinon (X11) | 12307.69231 | 0 | 6.1 | 1E+30 | 90.44824 |
| $C$3 | Bottles Sold Chardonnay (X21) | 12307.69231 | 0 | 5.28 | 1E+30 | 90.44824 |
| $E$3 | Bottles Sold Chinon (X12) | 84036.92308 | 0 | 6.43 | 1E+30 | 11.58 |
| $F$3 | Bottles Sold Chardonnay (X22) | 84036.92308 | 0 | 5.15 | 1E+30 | 11.58 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $G$12 | Bottles of Chinon sold year 1 (X11) LHS | 12307.69231 | 0 | 4000 | 8307.692308 | 1E+30 |
| $G$13 | Bottles of Chardonnay sold year 1 (X21) LHS | 12307.69231 | 0 | 5000 | 7307.692308 | 1E+30 |
| $G$14 | Bottles of Chinon sold year 2 (X12) LHS | 84036.92308 | 0 | 4000 | 80036.92308 | 1E+30 |
| $G$15 | Bottles of Chardonnay sold year 2 (X22) LHS | 84036.92308 | 0 | 5000 | 79036.92308 | 1E+30 |
| $G$16 | Equal Sales Requirement year 1 LHS | 12307.69231 | -1.611609846 | 0 | 13194.44444 | 18620.68966 |
| $G$17 | Equal Sales Requirement year 2 LHS | 84036.92308 | 0.7558 | 0 | 161299.843 | 156935.1433 |
| $G$18 | Investment restriction year 1 LHS | $20,000.00 | 55.66045538 | 0 | 1E+30 | 11875 |
| $G$19 | Re-investment restriction year 2 LHS | $140,061.54 | 6.948 | 0 | 1E+30 | 131728.2051 |

However, if we were to cut the wholesale price of Chinon in half ($3.65) then this would lower the elasticity of the wine. The sensitivity analysis shows that our allowable decrease is equal to our objective coefficient. Therefore, since it is within the allowable range, it will not affect the optimal solution of 84037 for Chinon in year 2, along with the rest of the constraints.

**4. Your partners are worried about the rather optimistic minimal sales level target for Chardonnay. How would changes in these targets affect the strategic plan? Justify your answer with the results of the sensitivity analysis.**

Upon analyzing the results of the sensitivity analysis, it is evident that changes in these minimal sales level targets for Chardonnay would affect the strategic plan. This is primarily due to the fact that the results could essentially all decrease, however, the values will no longer remain optimal since the allowance decrease can go to infinity. (As highlighted in the table below)

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$3 | Bottles Sold Chinon (X11) | 12307.69231 | 0 | 6.1 | 1E+30 | 90.44824 |
| $C$3 | Bottles Sold Chardonnay (X21) | 12307.69231 | 0 | 5.28 | 1E+30 | 90.44824 |
| $E$3 | Bottles Sold Chinon (X12) | 84036.92308 | 0 | 6.43 | 1E+30 | 11.58 |
| $F$3 | Bottles Sold Chardonnay (X22) | 84036.92308 | 0 | 5.15 | 1E+30 | 11.58 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $G$12 | Bottles of Chinon sold year 1 (X11) LHS | 12307.69231 | 0 | 4000 | 8307.692308 | 1E+30 |
| $G$13 | Bottles of Chardonnay sold year 1 (X21) LHS | 12307.69231 | 0 | 5000 | 7307.692308 | 1E+30 |
| $G$14 | Bottles of Chinon sold year 2 (X12) LHS | 84036.92308 | 0 | 4000 | 80036.92308 | 1E+30 |
| $G$15 | Bottles of Chardonnay sold year 2 (X22) LHS | 84036.92308 | 0 | 5000 | 79036.92308 | 1E+30 |
| $G$16 | Equal Sales Requirement year 1 LHS | 12307.69231 | -1.611609846 | 0 | 13194.44444 | 18620.68966 |
| $G$17 | Equal Sales Requirement year 2 LHS | 84036.92308 | 0.7558 | 0 | 161299.843 | 156935.1433 |
| $G$18 | Investment restriction year 1 LHS | $20,000.00 | 55.66045538 | 0 | 1E+30 | 11875 |
| $G$19 | Re-investment restriction year 2 LHS | $140,061.54 | 6.948 | 0 | 1E+30 | 131728.2051 |

**5. The loan officer from the Bank of Nova Scotia is not convinced that your strategy of selling an equal number of bottles of the two wine varieties makes sense. Which one is better and why? In response you need to re-work the model**

Selling a greater quantity of Chinon then Chardonnay is a better strategy then selling an equal number of the two wines, as this alternative will ultimately maximize revenue. When an equal number of Chardonnay and Chinon are sold, revenue is at $1,112,230.89, but as show in figure 1.1, when a greater quantity of Chinon are sold (166,300 bottles), and only 5000 bottles of Chardonnay are sold, revenue is maximized at $1,235,119.53. Furthermore, in figure 1.2, the opposite is tested: 22,621 bottles of Chardonnay are sold, whereas only 4000 bottles of Chinon are sold. In this case, when a greater quantity of Chardonnay is sold then Chinon, revenue is only at $1,143,218.39. Thus, after analyzing all of the alternatives, choosing to sell a greater quantity of Chinon then Chardonnay is the best option, as this will ultimately maximize revenue.

**Figure 1.1:**

| | Year 1 | | | Year 2 | | | | |
|---|---|---|---|---|---|---|---|---|
| DECISION VARIABLES | Chinon (X11) | Chardonnay (X21) | Amount Remaining (C1) | Chinon (X12) | Chardonnay (X22) | | | |
| Bottles Sold | 12308 | 12308 | 0 | 166300 | 5000 | | | |
| | | | | | | | | |
| OJBECTIVE | | | | | | | | |
| Price ($)/Bottle | $7.00 | $6.00 | $0.00 | $7.25 | $6.00 | | | |
| Marketing costs ($)/Bottle | $0.20 | $0.13 | $0.00 | $0.17 | $0.10 | | | |
| Production Costs ($)/Bottle | $0.70 | $0.60 | $0.00 | $0.65 | $0.75 | | | |
| MAX REVENUE($)/Bottle | $6.10 | $5.28 | $0.00 | $6.43 | $5.15 | $1,235,119.53 | | |
| | | | | | | | | |
| CONSTRAINTS | | | | | | LHS | | RHS |
| Bottles of Chinon sold year 1 (X11) | 1 | 0 | 0 | 0 | 0 | 12308 | >= | 4000 |
| Bottles of Chardonnay sold year 1 (X21) | 0 | 1 | 0 | 0 | 0 | 12308 | >= | 5000 |
| Bottles of Chinon sold year 2 (X12) | 0 | 0 | 0 | 1 | 0 | 166300 | >= | 4000 |
| Bottles of Chardonnay sold year 2 (X22) | 0 | 0 | 0 | 0 | 1 | 5000 | >= | 5000 |
| Equal Sales Requirement year 1 | 12308 | 12308 | 0 | 0 | 0 | 12308 | >= | 12308 |
| Equal Sales Requirement year 2 | 0 | 0 | 0 | 166300 | 5000 | 166300 | >= | 5000 |
| Investment restriction year 1 | $11,076.92 | $8,923.08 | $0.00 | $0.00 | $0.00 | $20,000.00 | <= | $20,000.00 |
| Re-investment restriction year 2 | $75,076.92 | $64,984.62 | $0.00 | $135,811.54 | $4,250.00 | $140,061.54 | <= | $140,061.54 |

**Figure 1.2:**

| | Year 1 | | | Year 2 | | | | |
|---|---|---|---|---|---|---|---|---|
| DECISION VARIABLES | Chinon (X11) | Chardonnay (X21) | Amount Remaining (C1) | Chinon (X12) | Chardonnay (X22) | | | |
| Bottles Sold | 4000 | 22621 | 0 | 86302 | 86302 | | | |
| | | | | | | | | |
| OJBECTIVE | | | | | | | | |
| Price ($)/Bottle | $7.00 | $6.00 | $0.00 | $7.25 | $6.00 | | | |
| Marketing costs ($)/Bottle | $0.20 | $0.13 | $0.00 | $0.17 | $0.10 | | | |
| Production Costs ($)/Bottle | $0.70 | $0.60 | $0.00 | $0.65 | $0.75 | | | |
| MAX REVENUE($)/Bottle | $6.10 | $5.28 | $0.00 | $6.43 | $5.15 | $1,143,218.39 | | |
| | | | | | | | | |
| CONSTRAINTS | | | | | | LHS | | RHS |
| Bottles of Chinon sold year 1 (X11) | 1 | 0 | 0 | 0 | 0 | 4000 | >= | 4000 |
| Bottles of Chardonnay sold year 1 (X21) | 0 | 1 | 0 | 0 | 0 | 22621 | >= | 5000 |
| Bottles of Chinon sold year 2 (X12) | 0 | 0 | 0 | 1 | 0 | 86302 | >= | 4000 |
| Bottles of Chardonnay sold year 2 (X22) | 0 | 0 | 0 | 0 | 1 | 86302 | >= | 5000 |
| Equal Sales Requirement year 1 | 4000 | 22621 | 0 | 0 | 0 | 4000 | <= | 22621 |
| Equal Sales Requirement year 2 | 0 | 0 | 0 | 86302 | 86302 | 86302 | <= | 86302 |
| Investment restriction year 1 | $3,600.00 | $16,400.00 | $0.00 | $0.00 | $0.00 | $20,000.00 | <= | $20,000.00 |
| Re-investment restriction year 2 | $24,400.00 | $119,437.24 | $0.00 | $70,480.25 | $73,356.99 | $143,837.24 | <= | $143,837.24 |