CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/2		All
Examination	**Date**	**Time**	**Pages**
Midterm	November 2015	1 Hour 30 minutes	2

Instructors	Course Examiner
E. Duma, F. Romanelli, H. Greenspan, J. Ortmann, L. Aggoun, L. Dube M. Padamadan, P. Gauthier, U. Tiwari, V. Kalvin	D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\frac{(1+i)^n - 1}{i}, \quad PV = PMT\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[5+5] **1.** A manufacturer has been selling 1200 television sets a week at \$480 each. A market survey indicates that for each \$30 rebate offered to a buyer, the number of sets sold will increase by 300 per week.

(A) Find the demand function $p(x)$, where x is the number of the television sets sold per week and $p(x)$ is the price of one set.

(B) How large rebate should the company offer to a buyer, in order to maximize its revenue?

[$2\frac{1}{2} \times 4$] **2.** Solve for x in the following equations:

(A) $7^{x^2+x-9} = 343^{-3x+5}$

(B) $\log_2(x-3) + \log_2(2x-4) = 2$

(C) $e^{x^2-2x+5} = \left(\frac{1}{\sqrt[4]{e}}\right)^{-8x^2+12x+28}$

(D) $\log_4(x^2+x+4) = 2$

PLEASE TURN OVER

[(3+4)+3] **3.**

(A) If the 8th and 19th terms of an arithmetic sequence are 9 and -24 respectively, find the 50th term and the sum of the first 61 terms of the sequence.

(B) Find the sum of the entire infinite geometric sequence $2, -\frac{1}{2}, \frac{1}{8}, \ldots\ldots\ldots\ldots$, if it exists.

[5+5] **4.** A radio commercial for a loan company states: "You only pay \$0.29 a day for each \$500 borrowed." If you borrow \$1,500 for 120 days, what amount will you repay, and what annual interest rate is the company actually charging?

[5+5] **5.** A bond issue is approved for building a marina in a city. The city is required to make regular payments every 3 months into a sinking fund paying 5.4% compounded quarterly. At the end of 10 years, the bond obligation will be retired with a cost of \$5,000,000.

(A) What should each payment be?

(B) How much interest is earned during the 10th year?

[5+5] **6.** Consider a \$21,281.27 loan for 7 years at 8% interest compounded quarterly and a payment of \$1000 per quarter-year.

(A) Compute the unpaid balance after 5 years.

(B) How much interest is paid during the fifth year?

Math 208 / ~~Midterm~~ Nov. 2015

1, a)

Price (P)	# of Television (x)	
$480	1200	
$450	1500	→ $30 rebate, 300 more T.V. Sold

$P(x) = mx + b$

$$m = \frac{480-450}{1200-1500} = \frac{30}{-300} = -0.1$$

pt (1200, 480)

$480 = -0.1(1200) + b$

$= -120 + b$

$600 = b$

$P(x) = -0.1x + 600$

b) Revenue $= R(x) = x\,P(x)$

$= x(-0.1x + 600)$

$= -0.1x^2 + 600x$

$= -0.1(x^2 - 6000x)$

$= -0.1(x^2 - 6000x + 9\,000\,000 - 9\,000\,000)$

$= -0.1(x - 3000)^2 + 900\,000$

$= -0.1(x-3000)^2 + 900\,000$

with 3000 TV gives max Revenue $ 900 000

To find the price when $x = 3000$

$P(3000) = -0.1(3000) + 600$

price/TV $= \$300$ to max the $R(x)$.

Then the rebate $= 480 - 300 = \$180$

2) a) $7^{x^2+x-9} = 343^{-3x+5}$ ($343 = 7^3$)

$= 7^{3(-3x+5)}$

$x^2+x-9 = -9x+15$

$x^2+10x-24 = 0$

$(x+12)(x-2) = 0$

$x = -12, 2$

b) $\log_2(x-3) + \log_2(2x-4) = 2$

$\log_2(x-3)(2x-4) = 2\log_2 2 = \log_2(2^2)$

$(x-3)(2x-4) = 4$

$2x^2-4x-6x+12 = 4$

$2x^2-10x+8 = 0$

$2(x^2-5x+4) = 0$

$(x-1)(x-4) = 0$

$x = 1, 4$

$\boxed{x=4}$ → check.

c) $e^{x^2-2x+5} = \left(\frac{1}{\sqrt[4]{e}}\right)^{-8x^2+12x+28}$

$= e^{-1/4(-8x^2+12x+28)}$

$x^2-2x+5 = 2x^2-3x-7$

$0 = x^2-x-12$

$0 = (x-4)(x+3)$

$x = -3, 4$

2 d) $\log_4(x^2+x+4) = 2$.

$x^2+x+4 = 4^2 = 16$.

$x^2+x+4-16 = 0$

$x^2+x-12 = 0$

$(x+4)(x-3) = 0$

$\boxed{x = 3, -4}$ check.

$\boxed{a_n = a_1 + (n-1)d}$

3) $a_8 = 9$; $a_{19} = -24$, $a_{50} =$ ——

a)

$\left\{\begin{matrix} a_8 = a_1 + (8-1)d \\ a_{19} = a_1 + (19-1)d \end{matrix}\right\}$ $\begin{matrix} 9 = a_1 + 7d \\ -24 = a_1 + 18d \end{matrix}$ $\left\{\begin{matrix} a_1 = 9 - 7d \\ a_1 = -24 - 18d \end{matrix}\right.$

$9 - 7d = -24 - 18d$

$-7d + 18d = -24 - 9$

$11d = -33$

$\boxed{d = -3}$

$a_1 = 9 - 7d = 9 - 7(-3) = 30$

$a_{50} = 30 + 49(-3) = \boxed{-117}$

$a_{61} = 30 + 60(-3) = \boxed{-150}$

$S_{61} = \frac{61}{2}(a_1 + a_n) = (30 - 150)\frac{61}{2}$

$= \boxed{-3660}$

b) G.P. $r = \frac{-1/2}{2} = -\frac{1}{4}$; $\frac{1/8}{-1/2} = -\frac{1}{4}$. $\boxed{r = -\frac{1}{4}}$

$\boxed{-1 < r < 1}$

$S_\infty = \frac{a_1}{1-r} = \frac{2}{1-(-1/4)}$

$= \frac{2}{1+\frac{1}{4}} = \frac{2}{1.25}$

$= 1.6$.

4) $0.29 for $500 is the daily interest

$\therefore$ $15,00 for 120 days $= (29)(3)(120) = \$104.40$

500 × 3

A $= 1500 + 104.40$
$= 1604.40$

$I = Prt$; $r = \frac{I}{Pt}$; $t = \frac{120}{360} = \frac{1}{3}$

$= \frac{104.40}{1500(\frac{1}{3})}$

$= 20.8\%$

5) $FV = Pmt\left[\frac{(1+i)^n - 1}{i}\right]$

$t = 10$ yrs.

$i = \frac{.054}{4} = .0135$

$n = 10 \times 4 = 40$

$FV = \$5\,000\,000$

A) $5\,000\,000 = Pmt\left[\frac{(1+.0135)^{40} - 1}{.0135}\right] = \frac{(1.709818931 - 1)}{.0135} Pmt$

$= (52.157918)\ Pmt$

$Pmt = \frac{5\,000,000}{52.157918}$

$= 95094.67$

B) FV in 9 yrs $= Pmt\left[\frac{(1+.0135)^{36} - 1}{.0135}\right]$

$= 95094.67\ (.6205226)$ 4370992.44

$\div .0125$

5) B

FV in 10 yrs − FV in 9 yrs } is the growth in the 10th yr.

5001 000 − 4370 992.44

= 629, 007.56

payment in the 10th yr

= $ 95094.64 × 4 = $ 380378.56

Interest in 10th yr =

= 629 007.56 − 380,378.56

= 248,628.55

$$5B = (FV_{10} - FV_9) - pmT(4)$$

↓ payment for 10th year

#6. 8% · quarterly : $i = .02$

$$PV = pmT \left[\frac{1-(1+i)^{-n}}{i}\right]$$

A, unpaid balance after 5 yrs = PV for 2 yrs. out of 7 yrs.

$$= 1000 \left[\frac{1-(1.02)^{-8}}{.02}\right]$$

$$= 7325.48$$

B, To find interest during 5th yr:

7 − 4 = 3

$$PV \text{ after } 4 \text{ yrs} = 1000 \left[\frac{1-(1.02)^{-12}}{.02}\right]$$

$$= 10575.34$$

unpaid after 4 yr $= 10575.34 - 7325.48$

$= 3245.86$

payment in 5th yr $= 1000 \times 4 = 4000$. (capital + interest)

interest $= 4000 - 3245.86$.

$= 754.14$