1. Light as Waves

Let's first look at some properties of waves. Electromagnetic waves move (propagate) at the speed of light ($c = 3.00 \times 10^8$ m s^{-1}). They have a wavelength denoted by λ, and they have amplitudes. The amplitude is the strength of the electric or magnetic field. Note that the wave can have zero amplitude (every point where it crosses the horizontal axis), but is still propagating.

Example 1. Find the frequency of red light having a wavelength of 700 nm.

$\nu = c/\lambda = (3.00 \times 10^8 \text{ m s}^{-1}) / (700 \times 10^{-9} \text{ m})$

$= 4.29 \times 10^{14} \text{ s}^{-1}$
$(= 4.29 \times 10^{14} \text{ Hz})$

Example 2. Find the frequency of blue light from a mercury lamp having a wavelength of 435.8 nm.

$\nu = c/\lambda = (3.00 \times 10^8 \text{ m s}^{-1}) / (435.8 \times 10^{-9} \text{ m})$

$= 6.88 \times 10^{14} \text{ s}^{-1}$
$(= 6.88 \times 10^{14} \text{ Hz})$

(higher that that of red light - blue light is more energetic than red)

Example 3. Find the frequency (ν) of a γ-ray having a wavelength (λ) of 3.56×10^{-11} m
$\nu = c/\lambda = (3.00 \times 10^8 \text{ m s}^{-1}) / (3.56 \times 10^{-11} \text{ m})$
$= 8.42 \times 10^{18} \text{ s}^{-1}$
(makes sense, since this is a higher frequency than visible light)

Example 4. Find the frequency of a radar wave with $\lambda = 10.3$ cm

$\nu = c/\lambda = (3.00 \times 10^8 \text{ m s}^{-1}) / (1.03 \times 10^{-1} \text{ m})$
$= 2.91 \times 10^9 \text{ s}^{-1}$
(much lower frequency, as expected)

Example 5. Find the wavelength of an FM signal having a frequency of 106.1 MHz (i.e. CHEZ-FM)

$\lambda = c/\nu = (3.00 \times 10^8 \text{ m s}^{-1}) / (106.1 \times 10^6 \text{ s}^{-1})$
$= 2.83$ m

2. Light as waves and as particles

We can also think of light as particles - photons. These will have specific energies, and we can use Planck's equation to relate their wavelike characteristics (frequencies or wavelengths) to particle-like characteristics (energies).

Example 1. Find the energy of photons from:
(a) IR radiation having $\lambda = 1.55 \times 10^{-6}$ m (=1550 nm)
$E = h\nu = hc/\lambda = 6.63 \times 10^{-34}$ J s ((3.00 x 10^{8} m s^{-1}) / 1.55 x 10^{-6} m))
= 1.28 x 10^{-19} J (per photon)

To find the energy per mole of photons, multiply by Avogadro's number:
E = 6.022 x 10^{23} mol^{-1} x 1.28 x 10^{-19} J
= 77,300 J/mol
= 77.3 kJ/(mol photons)

(b) UV radiation having λ = 250 nm

Note that this is a much shorter wavelength than the IR radiation. Thus, we expect the energy to be higher per photon. This is why UV radiation is more damaging to your skin than IR.

As above,

$E = h\nu = hc/\lambda = 6.63 \times 10^{-34}$ J s ((3.00 x 10^{8} m s^{-1}) / 250 x 10^{-9} m))
= 7.96 x 10^{-19} J/photon

To find the energy per mole of photons, multiply by Avogadro's number:
E = 6.022 x 10^{23} mol^{-1} x 7.96 x 10^{-19} J/photon
= 479000 J (mol photons)$^{-1}$
= 479 kJ (mol photons)$^{-1}$

Example 2. Iodine molecules (I_2) can be dissociated into two iodine atoms by light if the energy of the light is sufficient. Experiments show that the wavelength of the light must be less than 499.5 nm.

(a) What is the frequency of 499.5 nm light? What part of the electromagentic spectrum is this light in?

Visible light has wavelengths from 350 to 800 nm, so this is in the visible portion of the spectrum.

(b) Use Planck's equation to calculate the energy of dissociation of iodine. Express your answer in kJ/mol.

$E = h\nu = hc/\lambda = 6.63 \times 10^{-34}$ J s ((3.00 x 10^{8} m s^{-1}) / 499.5 x 10^{-9} m))
= 3.98 x 10^{-19} J/photon

To find the energy per mole of photons, multiply by Avogadro's number:
E = 6.022 x 10^{23} mol^{-1} x 3.98 x 10^{-19} J/photon
= 239700 J (mol photons)$^{-1}$
= 239.7 kJ (mol photons)$^{-1}$
= 239.7 kJ (mol I_2)$^{-1}$

3. If an electron can have wavelike properties, so can any moving object:

1. Calculate the wavelengths of:

(a) an electron moving at 1/20th the speed of light:
λ = h/mv = (6.63 x 10^{-34} J s) / ((9.11 x 10^{-31} kg) x (3.00 x 10^{8} m s^{-1} / 20)
= 4.85 x 10^{-11} m (The units work out to be m, since a Joule (J) is a kg m^2 s^{-2}.)

(b) a neutron moving at the same speed:
λ = h/mv = (6.63 x 10^{-34} J s) / ((1.67 x 10^{-27} kg) x (3.00 x 10^{8} m s^{-1} / 20)
= 2.65 x 10^{-14} m

(more massive particle = more energy = LOWER wavelength)

2. A major league baseball weighs approximately 142 g. Find the wavelength of a 95 mph fastball.

here, v = 95 mile/h x (1.62 km/mile) x (1000 m/km) x (1 h / 3600 s) = 42.8 m s^{-1}
thus λ = h/mv = (6.63 x 10^{-34} J s) / ((0.142 kg) x (42.8 m s^{-1})
= 1.09 x 10^{-34} m (much more massive particle; even though the velocity is small, the wavelength is very small, i.e. very high energy)

3. A photon strikes a metal surface and causes an electron to be ejected at 2.2 x 10^{3} km s^{-1}. If the electron behaves as a wave, what part of the electromagnetic spectrum is it in?

λ = h/mv = (6.63 x 10^{-34} J s) / ((9.11 x 10^{-31} kg) x (2.2 x 10^{5} m s^{-1})
= 3.31 x 10^{-9} m = 3 nm

ν = c/λ = 3.00 x 10^{8} m s^{-1} / 3.31 x 10^{-9} m = 9.1 x 10^{16} s^{-1} (which is in the far UV region, close to x-ray frequencies)

4. Atomic Spectra

Energy levels in a simple hydrogen atom can be calculated using the Balmer-Rydberg equation:

1/λ = R[1/m^2 - 1/n^2] where R is the Rydberg constant, 0.01097 nm^{-1}, n is the upper energy level and m is the lower one.

Note that three series of lines have been identified, differing only in the value of m. If m=2, we have the Balmer series, in the visible range. If m=1, we have the Lyman series, in the UV range. And if m=3, we have the Paschen series in the IR.

Note also that if n=infinity, this is the largest energy transition in the series, and $1/\lambda = R[1/m^2]$.

For example, lines in the Brackett series have m=4. Find the wavelength of the "first line" in the series (which means the lowest energy transition). Also find the photon energy in kJ/mol.

$1/\lambda = R[1/m^2 - 1/n^2]$

The first line has n=5.

Thus, $1/\lambda = 0.01097\ \text{nm}^{-1}\ [1/4^2 - 1/5^2] = 0.000247\ \text{nm}^{-1}$. Thus $\lambda = 1/0.000247\ \text{nm}^{-1} = 4051$ nm (which is in the IR portion of the spectrum).
$E = h\nu = hc/\lambda = 6.63 \times 10^{-34}$ J s $(3.00 \times 10^{8}\ \text{m s}^{-1}) / 4051 \times 10^{-9}$ m $= 4.90 \times 10^{-20}$ J/photon
$\times\ 6.02 \times 10^{23}\ \text{mol}^{-1}$ = 29,600 J/mol photons = 29.6 kJ/mol

For example, find the ionization energy of a hydrogen atom whose electron is is initally in the ground state (which means the lowest energy level).

To solve this, note that to ionize the atom, we must remove the electron completely. The energy required to do this is the energy the electron will gain moving UP from m = 1 to n = infinity, which is the same as the energy the electron would lose if it fell from n = infinity to m = 1:

$1/\lambda = 0.01097\ \text{nm}^{-1}\ [1/1^2 - 1/\infty^2] = 0.01097\ \text{nm}^{-1}$. Thus $\lambda = 91.1$ nm (UV)
$E = h\nu = hc/\lambda = 6.63 \times 10^{-34}$ J s $(3.00 \times 10^{8}\ \text{m s}^{-1}) / 91.16.63 \times 10^{-9}$ m $= 2.18 \times 10^{-18}$ J/photon
$\times\ 6.02 \times 10^{23}\ \text{mol}^{-1}$ = 1314356 J/mol photons = 1,314 kJ/mol.