Math 208 Sample Midterm (SOLUTIONS)

① (a) Given: $(x_1, p_1) = (7500, 2.28)$, $(x_2, p_2) = (7900, 2.37)$

$$m = \frac{p_2 - p_1}{x_2 - x_1} = \frac{0.09}{400} = 0.000225$$

$\Rightarrow p - p_1 = m(x - x_1)$
$\Rightarrow p - 2.28 = 0.000225(x - 7500)$
$\Rightarrow p - 2.28 = 0.000225x - 1.6875$
$\Rightarrow \underline{p = 0.000225x + 0.5925}$ (price-supply eq'n)

(b) Given: $(x_1, p_1) = (7900, 2.28)$, $(x_2, p_2) = (7800, 2.37)$

$$m = \frac{p_2 - p_1}{x_2 - x_1} = \frac{0.09}{-100} = -0.0009$$

$\Rightarrow p - p_1 = m(x - x_1)$
$\Rightarrow p - 2.28 = -0.0009(x - 7900)$
$\Rightarrow p - 2.28 = -0.0009x + 7.11$
$\Rightarrow \underline{p = -0.0009x + 9.39}$ (price-demand eq'n)

(c) This occurs where (a) and (b) intersect:

$0.000225x + 0.5925 = -0.0009x + 9.39$
$\Rightarrow 0.001125x = 8.7975$
$\Rightarrow x = 7820$ $\Rightarrow p = -0.0009(7820) + 9.39 = \$2.352$

The equilibrium point is $(x, p) = \underline{(7820, 2.35)}$. (x in millions, p in dollars)

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② (a) $64^{x^2} = 256^x \Rightarrow (2^6)^{x^2} = (2^8)^x$ (use a calculator to see this)
$\Rightarrow 2^{6x^2} = 2^{8x}$
$\Rightarrow 6x^2 = 8x$
$\Rightarrow 6x^2 - 8x = 0$
$\Rightarrow 2x(3x - 4) = 0$
$\Rightarrow \underline{x = 0 \text{ OR } x = 4/3}.$

(b) $3^{\log_2 x} = 3^5 \Rightarrow \log_2 x = 5$

$\Rightarrow \underline{x = 2^5 = 32.}$

(c) $e^{-x^2-1} = e^{3x+1} \Rightarrow -x^2-1 = 3x+1$

$\Rightarrow x^2+3x+2=0$

$\Rightarrow (x+1)(x+2)=0$ (or use Quadratic Formula)

$\Rightarrow \underline{x=-1 \text{ OR } x=-2}$

(d) $\log_{10}(x+1) - \log_{10}(10x-3) = 1 \Rightarrow \log_{10}\left(\frac{x+1}{10x-3}\right) = 1$

$\Rightarrow \frac{x+1}{10x-3} = 10^1$

$\Rightarrow x+1 = 10(10x-3)$

$\Rightarrow x+1 = 100x-30$

$\Rightarrow 99x = 31$

$\Rightarrow x = \frac{31}{99}$

CHECK domain! $x+1>0$ ☑ $10x-3>0$ ☑ Thus, $\underline{x = \frac{31}{99}.}$

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③ (a) Given: $a_1 = -5$, $a_{15} = 23$. But $\underline{a_{15} = a_1 + 14d}$!

Hence, $23 = -5+14d \Rightarrow 14d = 28 \Rightarrow d = 2.$

So $a_{73} = a_1 + 72d = -5 + 72(2) = \underline{139.}$

(b) Given: $a_1 = 3$, $a_{10} = 30$. But $\underline{a_{10} = a_1 r^9}$!

Hence, $30 = 3r^9 \Rightarrow r^9 = 10 \Rightarrow r = 10^{1/9}$

So $a_{40} = a_1 r^{39} = 3(10^{1/9})^{39} \approx \underline{64\,633.041.}$

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④ STEP ①: Find the amount that is paid at the end of 90 days to the third party:

$$A = P(1+rt) \qquad (P=5500,\ r=0.08,\ t=\tfrac{90}{360})$$

simple interest

$$= 5500(1+0.08(\tfrac{90}{360}))$$

$$= \$5610$$

STEP ②: Find the rate r required to make \$5560 grow to \$5610 in the last 60 days:

$$A = P(1+rt) \qquad (P=5560,\ A=5610,\ t=\tfrac{60}{360})$$

simple interest

$$\Rightarrow 5610 = 5560(1+r\cdot\tfrac{1}{6})$$

$$\Rightarrow \frac{5610}{5560} = 1+\frac{r}{6}$$

$$\Rightarrow r = \left(\frac{5610}{5560}-1\right)(6) \approx 0.05396.$$

The annual rate is 5.396%.

⑤ Use $FV = PMT\left[\frac{(1+i)^n-1}{i}\right]$, where $PMT = \$500$ and $i = \frac{0.08}{4} = 0.02$.

(and n = # of periods)

after 1st year: $FV = 500\left[\frac{(1.02)^4-1}{0.02}\right] = \$2060.80$

after 2nd year: $FV = 500\left[\frac{(1.02)^8-1}{0.02}\right] = \$4291.48$

after 3rd year: $FV = 500\left[\frac{(1.02)^{12}-1}{0.02}\right] = \$6706.04$

[FV in 1 year] − [year 1 deposits]

$\Rightarrow$ [interest in year 1] = \$2060.80 − \$2000

= \$60.80

[FV in year 2] − [FV in year 1] − [year 2 deposits]

$\Rightarrow$ [interest in year 2] = \$4291.48 − \$2060.80 − \$2000

= \$230.68

[FV in year 3] − [FV in year 2] − [year 3 deposits]

$\Rightarrow$ [interest in year 3] = \$6706.04 − \$4291.48 − \$2000

= \$414.56

⑥ (a) Use $PMT = PV\left[\frac{i}{1-(1+i)^{-n}}\right]$, where $\begin{cases} PV = \$240000 \\ i = \frac{0.0575}{12} = 0.0047916\ldots \\ n = (12)(20) = 240 \text{ periods} \end{cases}$

$$= 240000\left[\frac{0.0047916\ldots}{1-(1.0047916\ldots)^{-240}}\right]$$

$$= \underline{\$1685.00} \leftarrow \text{(monthly payment)}$$

$\Rightarrow$ [total interest] = [all payments] − [initial loan]

$$= (240)(\$1685.00) - (\$240,000)$$

$$= \underline{\$164,400} \leftarrow \text{(wow)}$$

(b) After 8 years, there are 12 years left on the mortgage. We want the PV of a \$1685 per month, 12-year annuity.

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right] \quad n = (12)(12) = 144 \text{ periods}$$

$$= 1685\left[\frac{1-(1.0047916\ldots)^{-144}}{0.0047916\ldots}\right]$$

$$= \underline{\$174,980.97} \leftarrow \text{(unpaid balance after 8 years)}$$