Math 208 Sample Midterm (SOLUTIONS)

① (a) y-int: Set $x=0 \Rightarrow y_{int}=5$

x-int: Set $f(x)=0 \Rightarrow 0.5x^2-2x+5=0$

$$\Rightarrow x=\frac{-(-2)\pm\sqrt{(-2)^2-4(0.5)(5)}}{2(0.5)} \quad \text{(quadratic formula)}$$

$$\Rightarrow x=2\pm\boxed{\sqrt{-6}} \leftarrow \text{does not exist!}$$

$\Rightarrow$ There are no x-intercepts.

(b)
$$\begin{aligned} f(x) &= 0.5x^2-2x+5 \\ &= 0.5(x^2-4x)+5 \qquad \left(\tfrac{-4}{2}\right)^2=4 \\ &= 0.5(x^2-4x+\boxed{4}-\boxed{4})+5 \\ &= 0.5[(x-2)^2-4]+5 \\ &= 0.5(x-2)^2-2+5 \\ &= 0.5(x-2)^2+3 \end{aligned}$$

(c) vertex $=(h,k)=(2,3)$.

Since $a=0.5>0$, the parabola opens up:

The min value is 3. There is no max value.

② (a) $5^{7x-x^2}=125^{-6} \Rightarrow 5^{7x-x^2}=(5^3)^{-6}$

$$\begin{aligned} &\Rightarrow 5^{7x-x^2}=5^{-18} \\ &\Rightarrow 7x-x^2=-18 \\ &\Rightarrow 0=x^2-7x-18 \\ &\Rightarrow 0=(x-9)(x+2) \\ &\Rightarrow x=9 \text{ OR } x=-2. \end{aligned}$$

(b) $\ln x + \ln(x+1) = \ln 6$

$\Rightarrow \ln[x(x+1)] = \ln 6$

$\Rightarrow \ln(x^2+x) = \ln(6)$

$\Rightarrow x^2+x = 6$

$\Rightarrow x^2+x-6 = 0$

$\Rightarrow (x+3)(x-2) = 0$

$\Rightarrow x=-3$ OR $x=2$. Since we need $x>0$, $\underline{x=2}$ is the only solution.

(c) $e^{x^2-5x} = 1 \Rightarrow e^{x^2-5x} = e^0$

$\Rightarrow x^2-5x = 0$

$\Rightarrow x(x-5) = 0$

$\Rightarrow \underline{x=0 \text{ OR } x=5.}$

(d) $\log_4(x^2-9) = 2 \Rightarrow x^2-9 = 4^2$

$\Rightarrow x^2-9 = 16$

$\Rightarrow x^2 = 25$

$\Rightarrow \underline{x = \pm 5}$ (both answers work)

③ (a) Given: $a_{11} = 20$ and $a_{19} = -28$. But $\underline{a_n = a_1 + (n-1)d}$!

Hence, $a_1 + 10d = 20$ ① and $a_1 + 18d = -28$ ②

②−① gives: $8d = -48 \Rightarrow \underline{d=-6.}$

Plug this in ① to get: $a_1 - 60 = 20 \Rightarrow \underline{a_1 = 80.}$

Finally, $S_n = \frac{n}{2}[2a_1 + (n-1)d]$ (where $n=76$, $a_1=80$, $d=-6$)

$= 38[160 + (75)(-6)]$

$= \underline{-11{,}020}$

(b) Given: $a_1 = 100$, $r = \frac{1}{2}$, $n = 10$, so...

$$a_n = a_1 r^{n-1} \Rightarrow a_{10} = 100\left(\tfrac{1}{2}\right)^9 = \underline{0.1953125}$$

Since the sequence is infinite, its sum is:

$$S_\infty = \frac{a_1}{1-r} = \frac{100}{1-\frac{1}{2}} = \underline{200}.$$

④ (a) Given: $i = \frac{r}{m} = \underset{\uparrow}{0.01}$ (basically, $\underline{r = 0.52}$ is the annual interest rate,
crazy high! and it's being compounded $\underline{m = 52}$ times a year)

$$APY = \left(1+\tfrac{r}{m}\right)^m - 1 = (1.01)^{52} - 1 \approx 0.67769 \approx \underline{67.769\%}$$

(b) Given: $i = \frac{r}{m} = 0.01$, $t = \frac{1}{2}$ (year), $P = 500$

$$A = P\left(1+\tfrac{r}{m}\right)^{mt} = 500(1.01)^{26} \approx \underline{\$647.63}.$$

(c) $A = P\left(1+\frac{r}{m}\right)^n$ where $\underline{n = mt}$ = total # of compounding periods

$\Rightarrow 2P = P(1.01)^n$

$\Rightarrow 2 = (1.01)^n$

$\Rightarrow \ln 2 = \ln\left[(1.01)^n\right]$

$\Rightarrow \ln 2 = n \cdot \ln(1.01)$

$\Rightarrow n = \frac{\ln 2}{\ln 1.01} \approx \underbrace{69.66\ldots}_{\text{round up!}}$ It will take $\underline{70 \text{ weeks}}$.

⑤ Use $\underline{FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]}$, where $PMT = \$100$ and $i = \frac{0.06}{12} = 0.005$.
(and n = # of periods)

- after year 1: $FV = 100\left[\frac{(1.005)^{12}-1}{0.005}\right] = \$1233.56$

- after year 2: $FV = 100\left[\frac{(1.005)^{24}-1}{0.005}\right] = \$2543.20$

- after year 3: $FV = 100\left[\frac{(1.005)^{36}-1}{0.005}\right] = \$3933.61$

$\Rightarrow [\text{interest in year } 1] = \overbrace{\$1233.56 - \$1200}^{[\text{FV in 1 year}]-[\text{year 1 deposits}]}$
$= \underline{\$33.56}$

$\Rightarrow [\text{interest in year } 2] = \overbrace{\$2543.20 - \$1233.56 - \$1200}^{[\text{FV in year 2}]-[\text{FV in year 1}]-[\text{year 2 deposits}]}$
$= \underline{\$109.64}$

$\Rightarrow [\text{interest in year } 3] = \overbrace{\$3933.61 - \$2543.20 - \$1200}^{[\text{FV in year 3}]-[\text{FV in year 2}]-[\text{year 3 deposits}]}$
$= \underline{\$190.41}$

⑥ (a) Given: $P = 6{,}000,\ i = \frac{\overset{\textcircled{r}}{\textcircled{0.035}}}{\underset{\textcircled{m}}{\textcircled{12}}} = 0.0029166\ldots,\ t = 2$

$\Rightarrow A = P\left(1+\frac{r}{m}\right)^{mt} = 6000(1.0029166\ldots)^{24} = \underline{\$6434.39} \leftarrow$ PV of the amortization!

Then $\underline{PMT = PV\left[\frac{i}{1-(1+i)^{-n}}\right]}$ where $n = (12)(4) = 48$ periods

$= 6434.39\left[\frac{0.0029166\ldots}{1-(1.0029166\ldots)^{-48}}\right]$

$= \underline{\$143.85} \leftarrow$ (monthly payment)

(b) $[\text{total interest}] = [\text{all payments}] - [\text{initial loan}]$
$= (48)(\$143.85) - (\$6{,}000)$
$= \underline{\$904.80}$