Lecture one $\quad$ sept 10, 2012

Matrix: (Plural of matrix is matrices)
A table of numbers that have m rows n columns

The dimension of a matrix equal $m \times n$

ex: 1) $A = \begin{bmatrix} -3 & 7 & 4 \\ 4 & 1 & 0 \end{bmatrix}$ 2 rows, 3 columns

Diamension $A = 3 \times 2$

2) $A = \begin{bmatrix} -1 & 3 & 4 \\ 5 & 2 & -1 \\ 4 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ $\underset{\text{rows}}{4} \times \underset{\text{columns}}{3}$ matrix

<u>(i,j) - Entry of a matrix</u>

Let A an $m \times n$ matrix. Let i, j 2 integers such that
$1 \le i \le m$
$1 \le j \le n$

The entry of A is the number in A in the i^{th} row and j^{th} column of A.

Ex 1) $\begin{bmatrix} -1 & 5 & 4 & 3 \\ 3 & -2 & 1 & 2 \\ 4 & 4 & -5 & 3 \end{bmatrix}$
(3,2) - Entry of A = 4.
(2,4) - Entry of A = 2
(5,3) - Entry of A = no such thing

3 elementary row operations on matrices

- We define 3 operations that can be performed on the rows of a matrix so that we can obtain another matrix

* Master these operations

Interchanging 2 rows of a matrix A

Interchange the i^{th} row and j^{th} row of A
(written $R_i \Longleftrightarrow R_j$)

$$\begin{bmatrix} 7 & 2 & 3 \\ 5 & 1 & 4 \\ 0 & 3 & 2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 0 & 3 & 2 \\ 5 & 1 & 4 \\ 7 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & 7 & 2 \\ 5 & 2 & 1 & 4 \\ 1 & 0 & -1 & 2 \\ 3 & 2 & 1 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} -1 & 2 & 7 & 2 \\ 1 & 0 & -1 & 2 \\ 5 & 2 & 1 & 4 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

Multiplication of a multiple of a row by a number K

(written $R_i = KR_i$) all Entries of a particular row by a number $K \neq 0$

$$\begin{bmatrix} 5 & 2 & -4 \\ 1 & -2 & 3 \\ -2 & 4 & 2 \end{bmatrix} \; R_2 = 2R_2 \; \begin{bmatrix} 5 & 2 & -4 \\ 2 & -4 & 6 \\ -2 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 2 & 8 \\ -2 & 4 & 1 & 5 \\ 4 & 8 & 1 & -2 \end{bmatrix} \xrightarrow{R_3 = \frac{1}{2}R_3} \begin{bmatrix} 0 & 3 & 2 & 8 \\ -2 & 4 & 1 & 5 \\ 2 & 4 & \frac{1}{2} & -1 \end{bmatrix}$$

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3) Addition of a multiple of a row to another given row

Replace the i^{th} row by the sum of the ith row and a number $K \neq 0$ times the j^{th} row

(written as $R_i = R_i + KR_j$)

Ex1: $A = \begin{bmatrix} 1 & -2 & 5 \\ 2 & -3 & 1 \\ 4 & 0 & 2 \end{bmatrix} \xrightarrow{R_2 = R_2 - 2R_1} \begin{bmatrix} 1 & -2 & 5 \\ 0 & -7 & -9 \\ 4 & 0 & 2 \end{bmatrix}$

R_2: 2 −3 1
$-2R_1$ −2 4 −10
$R_2 - 2R_1 =$ 0 1 −9

2) $\begin{bmatrix} 3 & -1 & 4 & 0 \\ 2 & -2 & 4 & 1 \\ 4 & 3 & -1 & 6 \end{bmatrix} \xrightarrow{R_3 = R_3 - R_1} \begin{bmatrix} 3 & -1 & 4 & 0 \\ 2 & -2 & 4 & 1 \\ 1 & 4 & -5 & 0 \end{bmatrix}$

- we can combine all these operations together

Ex. $\begin{bmatrix} 3 & -1 & 2 \\ -1 & 0 & 4 \\ 2 & 1 & 2 \end{bmatrix} \xrightarrow{R_1 = R_1 + 2R_2} \begin{bmatrix} 1 & 1 & 10 \\ -1 & 0 & 4 \\ 2 & 1 & 2 \end{bmatrix} \xrightarrow{R_3 = \frac{1}{2}R_2} \begin{bmatrix} 1 & -1 & 10 \\ -\frac{1}{2} & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}$

2) $\begin{bmatrix} 4 & 1 & -3 & 2 \\ -2 & 3 & 4 & 6 \\ 3 & -1 & 2 & -1 \end{bmatrix} \xrightarrow[R_3 = R_3 - R_1]{R_2 = R_2 - R_1} \begin{bmatrix} 4 & 1 & -3 & 2 \\ -6 & 2 & 7 & -2 \\ -1 & -2 & 5 & -3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} -1 & -2 & 5 & -3 \\ -6 & 2 & 7 & -2 \\ 4 & 1 & -3 & 2 \end{bmatrix}$

$* \xrightarrow{R_2 = -\frac{1}{3}R_2} \begin{bmatrix} -1 & -2 & 5 & -3 \\ 2 & -\frac{2}{3} & -\frac{7}{3} & \frac{2}{3} \\ 4 & 1 & -3 & 2 \end{bmatrix}$

Definitions:

Row of zero's : A row of matrix whoses entries are only zero

non-zero row of a matrix: A row of a matrix that is, at least one entry is not equal to zero

Leading entry of a row: The first Entry starting from the Left of a non-Zero row

Note: A row of zero doesn't have a leading entry

Ex: $A = \begin{bmatrix} -1 & 4 & 0 & 1 & 2 & 1 \\ 0 & 0 & 3 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \end{bmatrix}$

row of Zeros: 3

non Zero row: 1, 2, 4

Leading Entry: row 1: -1

row 2: 3

row 4: 2

Echelon form of matrix (or row echelon form)

An $m \times n$ matrix is said to be an echelon form matrix if it satisfies the following:

1) Every row of zero's is below every non-zero rows

2) Suppose the rows $1, 2, \ldots, k$ (where $k \leq m$) are the non-zero rows of A

Let c_i = the column in which the leading Entry of the i^{th} row is

Then we must have $c_1 < c_2 < \ldots c_k$.

* Look at Def. in text Book for echelon form matrix