18 November, 2011
Math 1104, Section B

TEST 4

Consider the matrices: $A = \begin{bmatrix} 1 & 7 & 6 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 & 2 \\ -1 & 2 & -2 \\ 3 & -6 & 7 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 & 6 & 2 & 4 \\ 0 & 1 & 2 & 1 & 1 \\ 2 & 2 & 4 & 0 & 4 \end{bmatrix}$

1. [5 marks] Using matrix A, answer the following:
   (a) The columns of A are: **LI** / **LD**. (*circle the correct response*)
   (b) The rank of A is:
   (c) What is the basis for the *column-space* of A?
   (d) Does $basis\{ColA\}$ also form a basis for R^4? **YES** / **NO** (*circle the correct response*)

2. [5 marks] Using matrix B, answer the following:
   (a) The columns of B are: **LI** / **LD**. (*circle the correct response*)
   (b) The rank of B is:
   (c) What is the basis for the *column-space* of B?
   (d) Does $basis\{ColB\}$ also form a basis for R^3? **YES** / **NO** (*circle the correct response*)

3. [5 marks] Using matrix C, answer the following:
   (a) The columns of C are: **LI** / **LD**. (*circle the correct response*)
   (b) The rank of C is:
   (c) What is the basis for the *column-space* of C?
   (d) Does $basis\{ColC\}$ also form a basis for R^3? **YES** / **NO** (*circle the correct response*)

4. [10 marks] Let $T \colon R^4 \to R^3$ be the linear transformation defined by

$$T\left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}\right] = \begin{pmatrix} 2x_1 - 4x_3 \\ x_2 - x_3 + 3x_4 \\ x_1 + x_2 - 3x_3 + 2x_4 \end{pmatrix}.$$

(a) Find the image of the vector, $\vec{x_D} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ by applying the given transform, T.

(b) Find the standard matrix of T.

(c) Find the image of $\vec{x_D}$ (above) by matrix multiplication of the standard matrix of T and x_D. Compare your answer to (a).

(d) Find the vector from the domain, $\vec{x_D}$, which gives the image $\vec{x_R} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ in the range of T.

5. [5 points] Recall that by definition, a linear transformation must satisfy the following conditions:

(I) $T[X+Y] = T[X] + T[Y]$

(II) $T[\alpha X] = \alpha T[X]$

If $T\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $T\left[\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$, then evaluate $T\left[\begin{pmatrix} 4 \\ -3 \end{pmatrix}\right]$.